AN INTRODUCTORY TEXT-BOOK OF LOGIC

WITH NUMEROUS EXAMPLES AND EXERCISES

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PREFACE TO THE FIRST EDITION.

In the present volume the author's aim has been to provide a text-book at once elementary and philosophical. More specifically, he has endeavoured, in the first place, to give an *accurate* exposition of the essentials of "the Traditional Logic"; in the second place, to connect the traditional doctrine with its Aristotelian fountainhead,—not only because of the value and clearness of Aristotle's own treatment (as compared with later accretions), but in order to make various doctrines and phrases intelligible, which in the ordinary text-book are simply "shot from a pistol" as it were; in the third place, to show the open door leading from the traditional doctrine into the more modern and more strictly philosophical treatment of the subject. The book is intended to stop short of giving what is supplied in Professor Bosanquet's *Essentials of Logic* (not to mention larger works), but to lead on naturally to that and to a serious study of "Modern Logic."

A text-book constructed on this plan seems to correspond closely to the treatment of the subject required by the course of instruction for the ordinary Degree in many of our Universities and Colleges.

The author's plan has certain difficulties which he has at least endeavoured to avoid. The chief of these is the danger of leaving an unbridged gap between the traditional or formal and the philosophical parts of the book. To some extent the author found that this difficulty was diminished by keeping as close as possible to the Aristotelian exposition, which is in itself thoroughly philosophical. By treating the formal part of the subject in this way, the gap seemed almost to disappear. For the rest, the most practically convenient course seemed to be to indicate, in the earlier chapters, by footnotes or otherwise, those points at which more fundamental questions arise; and in a concluding chapter, to bring these references together and develop them. The author hopes that he will at least be found to have avoided a mistake too common in books of this kind: of making the treatment of the traditional Logic perfunctory or even inaccurate; of expounding it *de haut en bas*, so to speak, leaving on the student's mind the impression that it is not worth his attention—a mistake equally serious from the educational and the philosophical point of view. It is hoped also that some freshness will be found in the choice of examples and illustrations, as well as in other respects. If Logic seems trivial to the student, the fault is not necessarily in Logic; it may be because the student's range of know-

ledge is trivial, so that he is prevented from understanding the application of logical principles to material of real importance.

In the preparation of this book the author received many valuable criticisms and suggestions from two friends, to whom his cordial thanks are due—Professor A. Seth Pringle-Pattison, of Edinburgh University, and Professor D. G. Ritchie, of St Andrews University, both of whom read the manuscript and most of the proofs. In the chapter on Immediate Inference, the author owes some paragraphs to a privately printed treatment of this part of the subject, prepared by Professor Pringle-Pattison. The two chapters dealing with Induction have also benefited by suggestions made by Miss Margaret Drummond, M.A., of Edinburgh. Of his obligations to previous writers on Logic, there are some which require special mention. He has made constant reference to the works of Bosanquet, Jevons, Mill, Creighton, Minto, Stock, and Welton (*Inductive Logic*). Most of the questions contained in the *Exercises* have been set in Examinations for Degrees and other purposes, in the Universities of Oxford, Cambridge, Edinburgh, Glasgow, St Andrews, or London. For some of these, the author is indebted to a little book entitled *Questions in Logic*, by Holman and Irvine. An additional word seems called for as regards Jevons's *Elementary Lessons in Logic*. The freshness and force with which this book is written have kept it high in the favour of teachers and students, notwithstanding its frequent looseness and faults both of too much and too little, and its occasional

logical mistakes. Some of its doctrines are freely criticised in the following pages; but the present writer fully concurs in the general acknowledgment of its real suggestiveness and value.

S. H. MELLONE.

PREFACE TO THE SECOND EDITION.

THE demand for a second edition of this book has given the author the opportunity of making a number of alterations in it, which have been suggested by its actual use in teaching. Most of the changes have been made in order to render the book more useful to those who are taking up the study of the subject for the first time. Passages which may be omitted on a first reading are now marked with an asterisk; the order of the exposition in some places has been changed (ch. iv. §§ 4-6, ch. vii. §§ 7, 8, ch. viii. §§ 1, 5-8, first edition); a number of passages have been simplified or expanded: in this edition, ch. ii. § 8 (Connotation of Singular names), ch. ii. § 11 and ch. iv. § 2 (Law of Identity), ch. v. §§ 4, 5 (two views of Definition), ch. vii. § 6 (Mill's view of Syllogistic Inference), ch. viii. § 1 (Definition of Induction), § 6 (Mill's theory of Cause), ch. ix. § 11 (the relation of Hypothesis to the Inductive Methods); a section on "Uniformities

of Coexistence" (ch. viii. § 8) has been added; the questions for Exercise have been carefully revised and their number largely increased. They will be found to fall into three groups: those marked "elementary," which can be answered directly from the portions of the text not indicated by an asterisk; those marked "more advanced," for which the text will at least suggest full answers; and those marked with an asterisk, on most of which the text will only throw light.[1] Some of them may profitably be taken as subjects of discussion by the teacher with his class.

The original aims of the book have in no respect been departed from; a few additional paragraphs have been added in order further to contribute to them, and references for advanced reading are given.

In some parts of the book—not, however, where questions of the first importance arise—there has been a change of doctrine. The treatment given to Exclusive and Exceptive Propositions (ch. iii. § 3) is fundamentally altered. In discussing the bearing of the "abstract identity" theory on the interpretation of Judgment (ch. iv. § 2, ch. xi. § 2), all reference to Jevons's so-called "equational view" of Inference is omitted. In so far as Jevons rests his theory on abstract identity of Subject and Predicate, it is a fundamental error to connect it with the notion of a mathematical equation;

[1] A number of questions on the Connotation of Singular Terms have been inserted (p. 48) to illustrate the different ways in which this problem, which covers one of the most fundamental difficulties in the Theory of Knowledge, may be approached.

for an equation, as actually used in any form of symbolic reasoning, is never an identity of the form $a = a$. The symbols on either side of the sign of equality stand for different operations, and the equation asserts that these different operations lead to the same result. In ch. ix. §§ 5, 7, what was previously called the Double Method of Difference is more correctly described by a modification of one of Mill's expressions—as the Joint Method of Difference and Agreement; its chief characteristic is the construction of independent "negative instances" from which the suspected cause is, not subtracted, but *ab initio* excluded. In ch. ix. §§ 10, 13, the author has modified some of the phrases used regarding "Explanation" in Science, in order more clearly to bring out the limits of the process so named.

The author wishes to acknowledge the assistance given by friends who have supplied valuable criticisms and suggestions; and more especially by Professor A. S. Pringle-Pattison, Edinburgh University; the Rev. Professor A. Caldecott, King's College, University of London; Professor S. Alexander, University of Manchester; Mr J. Solomon, Royal Holloway College; and Mr H. Tero, of Edinburgh. The author is also indebted to Mrs S. H. Mellone, M.A., and Miss M. Drummond, M.A., for valuable assistance in reading the proofs.

Two suggestions which the author received are deserving of mention here, although he was unable to carry them out lest the bulk of the volume should be unduly increased: the addition of a chapter on

logical aspects of the relation between Thought and Language, and a chapter on the Classification of the Sciences. For each of these topics references are given to guide the student. The Classification of the Sciences is a subject to which the Teacher might very profitably direct the attention of his class; for it illustrates, in a most instructive manner, on the one hand the different results to which the adoption of different bases of Classification will lead, and on the other hand the significance of varying conceptions of the nature and aims of Science.

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NOTE TO THIRD EDITION.

IN the third edition a few merely verbal changes have been made.

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CONTENTS.

CHAPTER I.

THE GENERAL AIM OF LOGIC.

CHAPTER II.

THE NAME, THE TERM, THE CONCEPT, AND THE LAWS OF THOUGHT.

CHAPTER III.

THE PROPOSITION, THE OPPOSITION OF PROPOSITIONS, AND THE FORMS OF IMMEDIATE INFERENCE.

Part i.—*The Logical Proposition.*

Part ii.—*Opposition of Propositions.*

Part iii.—*Immediate Inference.*

CHAPTER IV.

THE IMPORT OF PROPOSITIONS AND JUDGMENTS.

CHAPTER V.

THE PREDICABLES, DEFINITION, AND CLASSIFICATION.

Part i.—*The Predicables.*

Part ii.—*Definition.*

Part iii.—*Classification.*

Part iv.—*The Categories or Predicaments.*

CHAPTER VI.

MEDIATE INFERENCE AND THE ARISTOTELIAN SYLLOGISM.

CHAPTER VII.

CONDITIONAL ARGUMENTS AND THE VALIDITY OF THE SYLLOGISM.

CHAPTER VIII.

THE GENERAL NATURE OF INDUCTION.

CHAPTER IX.

THE THEORY OF INDUCTION OR SCIENTIFIC METHOD.

CHAPTER X.

FALLACIES.

* CHAPTER XI.

THE PROBLEMS WHICH WE HAVE RAISED.

NOTE.

Passages marked with an asterisk are to be omitted on a first reading. When this mark stands at the head of a section it refers to the whole of that section; otherwise it refers only to the particular paragraph (or question) so marked.

INTRODUCTORY TEXT-BOOK OF LOGIC

CHAPTER I.

THE GENERAL AIM OF LOGIC.

§ 1. WHEN we begin the exposition of any science, it is usual to frame a definition of it. But the beginning is not the point at which we can give a completely satisfactory explanation of the ground to be covered or the nature of the questions to be asked. For the words in which such a definition would be expressed would not be fully intelligible until the student became acquainted with the study which it defines. Hence we shall not for the present attempt any formal definition of Logic, beyond observing that to study Logic is to **think about thought**, in order to distinguish between correct or valid and incorrect or invalid thoughts. Thus, we have to think about that which, in science and common life, we do not think about but *use*,—*i.e.*, thought itself.

§ 2. When we say that Logic aims at *distinguishing correct or valid thoughts*, we do not mean "discovering truths or facts"; for this would make Logic a name

for all the various sciences collectively, which is absurd. By *correct*, or *valid* thoughts we mean those which are correct or valid with reference to a type or pattern, regarded as a rule or regulative principle to be followed. Hence, far from giving us means by which to discover new particular facts, the function of Logic is entirely general. It shows that **the thinking process is essentially the same, whatever be the particulars thought about.** The process of calculation may be explained in Arithmetic without regard to what the numbers represent; and, similarly, thinking may be reduced to general types which are the same in all particular applications. It is the aim of Logic to discover these types, and to show how to regulate thought by them; hence it deals with reasoning as a process common to all the sciences, without regard to their subject-matter. Only in this sense is Logic "the Science of sciences"; and in this sense also, Logic deals with "the form and not the matter of thought."[1]

§ 3. The manner in which the subject has been presented in the more elementary works hitherto, depends partly on its history: and the student will find that a brief consideration of some of the chief stages in that history will clear up his general idea of the logical point of view.

The Greeks invented the very idea of Science, in that sense of the word in which science is an Ideal,—the pursuit of knowledge for the sake of knowing: and to the Greeks also we owe the origin and development of Logic. Aristotle considered that logical inquiries began with the disputations of Zeno the Eleatic (towards the end of the fifth century B.C.), who found a number of

[1] The philosophical aspect of this definition will be considered in our concluding chapter, § 1.

difficulties in the beliefs of common sense, and in the then prevalent philosophical conceptions, as to the reality of time, space, and motion; the discussions, to which these arguments gave rise, began to awaken a conscious interest in *methods of reasoning*, an essential part of Logic. This interest was carried much further by the work of the Sophists and of Socrates. The **Sophists** met a growing demand for means of enlarging and improving human nature, by giving instruction in the arts and accomplishments useful to a citizen in practical life. They gave special attention to what may be called the Art of Persuasion, in a wide sense. This involved the beginnings of Grammar, Rhetoric, and Logic, as distinct studies. Thus Logic first appears as the *art of arguing*. The Sophists were more interested in persuasion than in true instruction, in victories through verbal discussion than in scientific investigation. Some of them, such as Protagoras, were thorough Sceptics, denying the possibility of knowledge. Socrates went with them in their interest in humanity; but he was moved by an invincible faith that knowledge of the truth is possible for us all. His method of arriving at truth was so simple that its deep significance is somewhat hidden. He observed that in ordinary thought people are much more sure of the particular objects to which a name belongs than they are of the qualities in the objects, on account of which the name is given; thus, when we speak of such a thing as "an oak-tree" or "a rose"—or "a beautiful object," "a good action"—it is more easy to bring forward actual instances of these things than to explain what we *mean* (what idea we have in our minds) when we use the name. But to arrive at consistency with ourselves and agreement with others, we must not only be able to

point to the things; we must know the meaning, the thought, which the name expresses. Socrates considered that this could be done by comparing the things, to ascertain the common qualities on account of which they received a common name. His chief contribution to Logic, therefore, was to make people see the importance of Definition, as a means of knowing *things*. **Plato** made further contributions to the analysis of the methods of discussion and scientific procedure; but in Aristotle's works these questions gain distinctness and receive a separate treatment.

Aristotle is the real founder of Logic as a science, for he worked it out systematically in all its parts. His doctrines are contained in six small but masterly treatises, which afterwards, on account of their affinity, were collectively referred to as the *Organon*.[1] The treatises of which the *Organon* consists are the following:—

1. *The Categories.* A philosophical introduction to Logic (see below, ch. V. § 11).
2. *De Interpretatione* (On Expression in Words). An account of terms and propositions.
3. *Prior Analytics.* An account of formal reasoning (see below, ch. VI.)
4. *Posterior Analytics.* An account of the processes by which demonstrative Science may be worked out (as in Mathematics).
5. *Topics.* An account of reasoning in matters where complete demonstration is unattainable.
6. *Sophistical Refutations* (*Sophistici Elenchi*). An account of fallacious arguments.

He founded a logical tradition which has lasted to our own day, although the tradition has departed from

[1] Usually quoted by the English or Latin titles here given.

the spirit of the Aristotelian doctrine, and has made additions to its form. Very few of the additions are improvements.

Aristotle has no *one* name for all the investigations of the *Organon*.[1] This title is in one sense admissible, for it indicates the position of Logic in treating thought as the *instrument* (ὄργανον) which all the sciences use. But in another sense the title is a very unfortunate one, suggesting —to Francis Bacon, for example—the absurd notion that Logic aims at supplying tools for making discoveries, instead of analysing the methods of reasoning. Aristotle seems to have worked at Rhetoric first of all; through his study of the means of *expressing* political, judicial, and disputatious argumentation, he was led to examine the *principles* of controversial discussion; then he passed on to examine inference as such.

In the Middle Age Aristotle's logical writings were known only through imperfect translations. On these some of the most powerful and subtle intellects of the time set themselves to work, and built up a Logic which, though it was accurate and systematic, was also abstract and artificially formal in the extreme. This result was natural; for the spirit of the age left no room for original investigation; its motto was—"Bring your beliefs into harmony with traditional authority." With the Renaissance, a new spirit arose, whose motto was "Bring all beliefs into harmony with the facts of Nature"; and when observation of Nature and her laws became a prevailing pursuit, the deficiency, for this purpose, of the formal Logic of mediæval writers was perceived, and the need was felt of some principles to regulate the observation and explanation of Nature. In this work—inquiry

[1] Aristotle's own name for "logical inquiries," so far as he has any, is "Analytics" (τὰ ἀναλυτικά).

into principles of scientific method—Roger Bacon (1214-1294) was a brilliant forerunner of writers much later in date. Francis Bacon, the Chancellor, carried on the work, and wrote his *Novum Organum* in rivalry with what he thought was the Aristotelian system of Logic. It was natural that as this seemed to be a new beginning in Logic, a new name should be found for it; and during the nineteenth century, "Inductive Logic," as it is called, has received much attention. The most important works in which it has been developed are those of Herschel, Whewell, and John Stuart Mill.

Hence the usual treatment of Logic lays out the subject in two branches. The first of these is founded on the Logic which the mediæval writers developed out of such acquaintance with Aristotle as they possessed. This is usually called "Deductive Logic" or "Formal Logic." The second division is the "Inductive Logic" of which we have spoken, which is often called "Material Logic." So far as the distinction implies a difference in principle between the two kinds of knowledge, it has no foundation in the facts of thought; otherwise, there are advantages in not departing from it.[1]

§ 4. Logic has to consider Language; but only so far as **differences of expression in language** are the embodiment of **differences of type in the process of thought.** The word *logos* (λόγος) had a double meaning in Greek: (*a*) the *thought*, (*b*) the word (or rather, phrase or sentence) which is the expression of the thought,—*ratio* and *oratio*. Aristotle distinguished these, calling the former the "inward" and the latter the "outward" *logos*. This ambiguity has given rise to a dispute as to whether Logic has to do with thought or with language. Whately has been referred to as

[1] The recent philosophical development of Logic will be referred to in our concluding chapter, § 1.

holding the latter view. It is true that when defining Logic he says that it is "entirely conversant about language"; but elsewhere he speaks of the processes of reasoning—*i.e.*, processes of *thought*—as the subject-matter of Logic. No other view can be seriously taken; but the stress which is laid on the *verbal expression* of these processes varies in different works.

We cannot entirely separate the two aspects of the *logos;* for, while thought is prior to language, **thought could make no progress without embodying itself in language.** As soon as we have an idea there is an irresistible impulse to give it bodily shape in a word.

The thought is purely inward and in a sense abstract; the word has an external existence as a sound or a written symbol, and is therefore a thing of *sense;* but the thought would dissolve again were it not stereotyped in a word. Hamilton (*Logic*, i. p. 138) has illustrated this reciprocal dependence as follows. An army may overrun a country, but the country is only conquered by the establishment of fortresses; words are the "fortresses" of thought. And in tunnelling through a sandbank it is impossible to proceed until the present position is made secure by an arch of masonry; words are such "arches" for the mind.

Questions connected with the foregoing, and deserving of the student's attention, are, the extent to which language may be a hindrance, as well as a help, to thought; and the reason why *spoken* language has become universal rather than gesture language. And we may remark, in passing, that Grammar, dealing with the thought-structure of language, lays stress on the other side of the λόγος, the outward expression. Hence Grammar has been called a "concrete Logic." When it is said (as it usually is) that Grammar deals with *language* as such, it is not and cannot be denied that it deals with the thought expressed in lan-

guage, but only in so far as this is needed to make clear the distinctions which the particular language recognises (*e.g.*, between nouns and adjectives), to the end that one may use the language with conventional propriety.

§ 5. We shall find a convenient centre from which to start if we ask—"What is the *simplest* type of thought which may be either true or false?" Evidently this cannot be less than a single **assertion or statement of fact,** affirmative or negative. Let us call a thought of this kind a **Judgment**; and the expression of it in language a **Proposition.** It would be well if the term "proposition" could be kept for "the Judgment expressed so as to bring out its logical character"—*i.e.*, expressed in a grammatically complete sentence, with subject and predicate; but common usage is too strong, and we must take the term as meaning "the sentence which contains (or, as containing) a Judgment," whether it is properly formulated (see below) or not.

Not every judgment is naturally expressed in the form of a complete proposition: a single word, *e.g.*, "Fire!" may suffice to express a judgment. The judgments of children are often of this kind.

Again, "every sentence (λόγος) is *significant*, but only such as can be true or false are *assertive*" (Ar. *De Interpretatione*, iv.) In other words, not every sentence is a *proposition*; thus, "go away!" is not a statement of fact,—the notion of truth or falsity does not belong to it. Even the enunciative sentence contains emotional elements over and above the mere judgment; *e.g.*, "there's the door" may express much more than a judgment concerning the place, &c., of the door. Just as "Fire!" contains a judgment, but a great deal besides.

The Judgment may be called the **Unit of Thought**;

for all our deliberate thinking consists in making statements or assertions, and if we are to have truth or falsity we must have at least a judgment.

§ 6. In any judgment we may distinguish two relatively simpler thoughts, which for the present we will vaguely call *ideas*. An idea by itself cannot be either true or false; it must enter into a judgment first. "An example of this is, that 'unicorn' *means* something, but is not true or false until affirmation or denial of its existence is added" (Ar. *De Int.* i.) This does not mean that judgments are built up by putting together ideas that were separate. Whether we can even entertain a significant idea as such without judging, or at least framing possible judgments on the basis of that idea, is very doubtful. In Logic we may assume that **ideas exist only as elements in a judgment.**

We have a corresponding relation in the proposition. **A proposition affirms or denies something of something else**: *e.g.*, "Some useful metals are becoming rarer." The Subject is that about which the assertion is made (*i.e.* "Some useful metals"); the Predicate, that which is asserted (*i.e.* "are becoming rarer"). It is a standing convention in elementary Logic to express the statement which is made, by the verb *is* or *is not* (*are* or *are not*); and the predicate of a proposition is always understood to be expressed in a form admitting of the use of this verb, which is called the *Copula* (*i.e.*, in our example, "Some useful metals *are* things which are becoming rarer"). The subject and predicate are the *terms* (*termini*, limits) of the proposition; and we shall understand by a "term," any word, phrase, or sentence which is standing as the subject or predicate of a proposition. A Term which is not in its place in a proposition we shall call a "name."

Just as every sentence is not a proposition, so every word is not a term. A term will be either a noun, an adjective, or a participle, or some word, phrase, or sentence equivalent to one of these. Words which cannot by themselves be used as terms are distinguished as "syncategorematic," while terms are called "categorematic." A "syncategorematic" word may become a term in a proposition which makes some statement about its use as a "part of speech": *e.g.*, "*When* is an adverb, and sometimes a conjunction also."

The student must remember that there is no separate existence in thought (no third idea coming between the subject and predicate) corresponding to the separate existence of the copula in the typical proposition, *S is P*.

§ 7. Judgments may be combined into reasonings or inferences. What is an inference? To **infer** is to arrive at a truth not directly through experience, but **as a consequence of some truth or truths already known**; as when I see a circle of stones, and *infer* that they were arranged by human hands; or when I believe that nothing proceeding from a pure moral intention can be utterly condemned, and that some deviations from the common rules of morality have proceeded from this source, and accordingly *infer* that those deviations are not to be altogether condemned. J. S. Mill defines inference thus: "We start from known truths to arrive at others really distinct from them." The truths from which we start are the **premises**, that which we reach is the **conclusion**. Both Mill and Whately point out that the chief work of practical life is concerned with "drawing inferences" in this sense.

Hence we have three *main* divisions of Logic—

I. The doctrine of Terms, leading on to that of "ideas," the element in the Judgment to which the Term corresponds.

II. The doctrine of the Judgment.

III. The doctrine of Inferential Thought.

§ 8. We have seen that Ideas are not prior to Judgments; for a Judgment is not built up by putting separate Ideas together. Ideas are distinguishable though not separate elements in a Judgment. Hence when we treat Ideas as the subject of a first division of Logic, as if they could exist independently in the mind, we are treating them *in abstraction* from their natural environment. It is, however, a necessary abstraction. It is the same with Terms. In the origin of language the sentence is prior to the word, and the "parts of speech" were originally sentences; but we may give separate logical treatment to Terms apart from Propositions, if we remember that in living speech the Term only exists as a part of a Proposition expressed or understood.

* This statement of the relation between the three "divisions" of Logic differs from what Jevons and some other writers say. Jevons speaks thus: "*Simple apprehension* is the act of mind by which we merely become aware of something, or have a notion, idea, or impression of it brought into the mind. The adjective *simple* means, apart from other things; and *apprehension*, the taking hold by the mind. Thus the name or term 'iron' makes the mind think of a strong and very useful metal, but does not tell us anything about it, or compare it with anything else. . . . *Judgment* is a different action of mind, and consists in comparing together two notions or ideas of objects derived from simple apprehension, so as to ascertain whether they agree or differ." And similarly, he continues, when we have already made judgments, a third activity of mind may come in and combine them into processes of argument or reasonings. According to Jevons' account, the three "activities of mind," apprehension, judgment, reasoning, are three different *kinds* of operation, which simply come after one

another. The later forms use the finished products of the earlier; but knowledge is made to resemble a process of adding part to part from the outside. This view of the logical processes of the mind, and of the growth of knowledge, is fundamentally a mistake; the further the student pursues the study of modern logic the more clearly he will see that it is so. The point of view adopted in modern logic is, that in the formation of ideas, in judgment, in reasoning, we have not three separate processes but a development or expansion of one and the same process; and the full significance of this statement will be seen at a later stage. We may add that the statements made earlier in this chapter imply that there is no such thing as "*simple* apprehension" as Jevons defines it. We "apprehend" or mentally take hold of an idea, only by making judgments about a *thing;* we form the idea of "iron" through the *judgments* that it is "hard," "heavy," "malleable," &c., and the idea of "iron" is at once a product of such judgments and the subject of further judgments.

CHAPTER II.

THE NAME, THE TERM, THE CONCEPT, AND THE LAWS OF THOUGHT.

§ 1. THE student will notice that the word "name" is often used in Logic instead of "term." What is the relation between them? Hobbes, in his *Computation or Logic*, Part I. ch. ii. § 4, defines a name as "a word [we must add, "or a group of words"] taken at pleasure to serve for a mark, which may raise in our minds a thought like to some thought which we had before, and which, being disposed in speech and pronounced to others, may be to them a sign of what thought the speaker had before in his mind" (Croom Robertson, *Hobbes*, p. 83). The first part of the definition brings out the fact that language is necessary even for our own private thoughts, in identifying our ideas (cf. ch. I. § 4; ch. II. § 6); the second part brings out the purpose of the name as a sign to others, a means of communication. But the phrase "taken at pleasure" is objectionable, as implying a conscious arbitrary choice; whereas in the formation and use of names, *laws* can be discerned. Aristotle observes more truly that a name (*ὄνομα*) is a sound which has signification "according to convention,"—*i.e.*, by agreement (*De Interpretatione*, ch. ii.) A **term** is a name considered as part of a proposition, as Subject or Predicate; and a **name** is

any word or combination of words which can serve as a term, but is considered without special reference to its use in a proposition as a term.

Aristotle had already remarked that the Term (ὅρος, *terminus*) is—not something out of which a proposition is built up, but—"that into which a proposition is analysed, as its subject or predicate" (*Prior Analytics*, I. 1).[1] All that Logic has to do with terms is to distinguish their various kinds, so far as these throw light *on the process of thinking*. Now if we take the Aristotelian conception of the Term as always either subject or predicate of a proposition, a great deal of what English logicians say about "terms"—and some of them, especially Jevons, use the word in a loose sense as equivalent to "names" or "words" or "phrases"—falls outside Logic. It belongs to Grammar or Rhetoric, or to special sciences. Hence when dwelling on the distinctions usually given, we shall speak of "names" as above defined, and not of "terms"; for only one of these distinctions is of primary *logical* importance,—that between "singular" and "general," which is the only one that applies strictly to logical *terms*, as parts of a proposition.

§ 2. Our first division is into abstract and concrete names.

Mill explains **a concrete name** as the name of an object or "thing" viewed as possessing attributes; an **abstract name**, as the name of an attribute (a quality, property, or action) viewed apart from the object to which it belongs. The ground of this distinction in the use of names lies in the fact that we may think of things as having attributes — *i.e.*, qualities predicated of them,

[1] In *De Interpretatione*, ch. i., Aristotle seems to give more countenance to the view that the judgment is a "combination or separation" (σύνθεσις or διαίρεσις) of *concepts*, as though it were built up out of them; but this is for the special purpose of urging that only the judgment, as distinguished from the concept, can have truth or falsehood.

when the names by which we signify the things are concrete; or we may think of the qualities apart from their attribution to things, when the names by which we signify them are abstract. The distinction concerns the *use* of names; for some names may be used now as abstract, now as concrete. Hence before we can determine to which of the two classes any term belongs, we must consider a proposition or statement in which it is contained. Thus, all *adjectives* are concrete; for an adjective can be a logical term *only* when standing as a predicate of a proposition,—if it is not predicated of a noun it must be prefixed to a noun. This will make the noun a concrete term, and the adjective will share this character with it; "the light of certain stars is *coloured*."

Abstract names are generally marked by a suffix: "whiteness," "manhood," "hospitality." A phrase or sentence may be an abstract term: "*that this rumour is false* is evident on the face of it." The names of attributes are sometimes used to signify instances of their occurrence, and then they must be considered as concrete names: "*unpunctuality* is irritating." In this connection, Mill refers to the apparent use of abstract names in the plural; but the name of an attribute can be described as common and put in the plural number, only in so far as it can be regarded as varying, as being itself the subject of attributes; and then it becomes a concrete name. A purely abstract name—*e.g.*, *colour* when it means simply *colouredness*—cannot be used in the plural. When we speak of "colours" we use the term as a concrete which has different attributes or varieties. Hence the distinction of abstract and concrete has no fixity as applied to names; a name may pass from one class to the other.

Some names which are used in two senses may be abstract in one, concrete in the other—*e.g.*, "introduction" (the opening of a discourse,—the act of introducing). This is an example of an "equivocal" or ambiguous term. We cannot make a separate class of names out of these, as Jevons and others do, calling them "equivocal" or ambiguous names; for each of them is really two or more names. Thus, "vice" (meaning an immoral action) is a different name from "vice" (the mechanical instrument).

§ 3. Concrete names are ordinarily divided into singular, common (or general), and collective; and although such a classification really implies two principles of division,—since collective names may be either singular or common,—there is some practical convenience in following it.

(*a*) A **singular name** can denote only a single object, as long as its meaning does not change. All proper names belong to this class. If the singular name is not a proper name, it is always indicated by a demonstrative, or by an equivalent expression giving the object a definite position in time or place. This does not apply to *abstract* names as such (§ 2). These also are singular. They denote qualities as qualities and not as quasi-things.

The following are singular names which are not proper names: "the writer of the letters of Junius," "the year in which Queen Victoria died," "the present Government," "the earth," "the largest planet of the solar system"; and all names introduced by singular demonstrative adjectives, "this," "that," &c. A proper name may be described as a "particularised demonstrative." It is a mark used for the sake of distinguishing one particular object, and not (at first) for what it *means*. It may have almost no meaning when first applied (see below, § 8).

There is great vagueness in the explanation of singular names in logical text-books, through neglect to notice that the characteristic of such names (with the exception above noted) is to specify the object by limiting it or "individualising" it in space and time.

Singular terms can only be used as *subjects* in a proposition. The student may find *apparent* exceptions to this rule.

(*b*) A **common name** is applicable without change of meaning to a number of objects, because they resemble one another in some characteristic features or aspects, called in Logic **attributes.** When a name is thus applicable to every one of a class in turn, it is said to be **distributively** used. The name is applied to the individuals, because they have in common certain attributes. These attributes are what the name *means;* together they form what is called the **connotation** of the name, or the **intension, content,** or **comprehension** of the idea: and the objects to which it is applied constitute the **denotation** of the name, or the **extension** of the idea. Thus the denotation of the name "man" consists of the whole group or class of beings which this name *denotes*—that is, which it points out and distinguishes from other groups; and the name is applicable to each member of the group. The connotation of the same name consists of the attributes by which all these beings are distinguished,—the attributes constituting "humanity." Or, to give a mathematical example, the connotation of the name "circle" may be accepted in the form in which Euclid states it; while its denotation consists of all the cases of motion, form, &c., which are "circular."

It has been objected that in names such as "unicorn," "dragon," we have connotation, but the attributes which are signified do not exist, and therefore we have no

denotation. But by denotation we do not mean only existence in the real world; existence in any kind of world which is being spoken of as the subject of discourse is sufficient—*e.g.*, the ideal world, or the world of heraldry or folklore. Hence every common name has both connotation and denotation, and is in short the name of a *class*. It is none the less a class name even if there is only one instance to which it is applied; for if it signifies certain characteristic attributes of the thing which it denotes, it is *potentially* common; "sun" is an instance of this. On the other hand, the class denoted need not be numerically definite or limited; it is known by the attributes, and any instance of these, whether a known or an unknown instance, constitutes a member of the class. At a later stage of our present discussion, we shall consider the connotation and denotation of singular names (§ 8).

Names of materials, the so-called "homogeneous" names, are in a doubtful position. Names such as "water," "wood," "iron," are singular as used of the mass as a whole, but common as applicable in the same sense to different portions of the mass. Aristotle had already noticed this (*Topics*, I. ch. vii.): "The case of water from the same well differs from the usual case of objects being members of the same class only in that the degree of resemblance between the objects is higher in the former."

(*c*) A **collective name** is the name of a group of similar things regarded as a whole, the name not being applicable to the things taken one by one. Collective names may be singular, as, "the British Army in South Africa," "the present House of Commons"; or common, as "a committee," "a library." Where a name may be

used in both ways, the collective and distributive meanings must be carefully distinguished. Thus the name "committee" is used distributively as being applicable to each one of the many different groups formed in the manner, and with the object, which the name signifies. But as applied to any particular one of these groups, its use is not distributive but collective; it cannot be given to each or to any member composing the group, but only to all the members together.

This distinction is of great importance; and the neglect of it may lead to serious fallacies or mistakes in reasoning. The word "all," for instance, may be used either collectively or distributively; "all men" may mean "any man," or "all men together"—*i.e.*, the human race as a whole. And what is true of "all" collectively may not be true of "all" distributively, or *vice versâ*. It is not easy to give simple examples where the distinction covers a really deep difference of meaning, for such cases usually occur in the discussion of difficult questions in ethics or philosophy. Consider Kant's *dictum*, "ought implies can." We may interpret this in the sense that "man is capable of realising every ideal which he is capable of presenting to himself." Understood distributively, this means that each man is capable of being and doing everything which he sees that he ought to be and do. Understood collectively, it means that though you or I may not always be able to do everything which we see that we ought to do, yet the human race can, in the course of time, realise every genuine ideal which any man is capable of conceiving.

Some logicians—*e.g.*, Hamilton, followed by Dr Fowler—treat collective names as always singular; "the committee," "the library," "the regiment" are treated as the true collective terms, while "committee," "library," "regiment" are ordinary common terms.

§ 4. Another division of names is into **positive** and **negative**.

Positive names imply the presence, negative names the absence, of a given attribute. Sometimes two different words are used to express the two implications; sometimes the negative name is formed from the positive by a prefix.

Positive names.	*Negative names.*
Light.	Darkness.
Gratitude.	Ingratitude.
Agreeable.	Disagreeable.
Manly.	Unmanly.

The negative name, as Mill points out, does not imply mere negation, but the presence of some other quality; in each of the above instances the negative name implies the presence of an actual quality which is the opposite of the one excluded. Hence, as Jevons says, it is often "a matter of accident whether a positive or negative name is used to express any particular notion."

This leads us to a distinction which is of the highest importance, and which must be clearly grasped before the student proceeds further. A pair of names which, as in the instances already given, represent **opposites,** but which do not exhaust between them the whole universe, are called **contraries.** Any name may have a "contrary" in this sense. In the case of two contrary names, a thing need not be of necessity either the one or the other. But every name must have a **contradictory,** which is expressed by prefixing "not" to the original name. The "contradictory" denotes everything which is not denoted by the original name—*e.g.*, "not-white" denotes whatever in the heavens, the earth, or in the mind of man is not "white."

Hence, of two contradictory names. either one or

the other must be applicable to anything that exists or that we choose to think of. Hence, also, there is no one definite idea or thought corresponding to the contradictory of a term. Aristotle observed that "'not-man' is not a name, and there is no name for it, for it is not an idea." He calls it an "indefinite name" (*ὄνομα ἀόριστον, nomen indefinitum*), to distinguish it from ordinary names.

Objection has been made to any use of contradictory names in Logic: "If 'not-man' means all that it ought logically to mean,—triangle, melancholy, sulphuric acid, as well as brute and angel,—it is an utterly impossible feat to hold together this chaotic mass of the most different things in any *one* idea, such as could be applied as predicate to a subject" (Lotze, *Logic*, § 40). This may be granted, to the extent of admitting that the contradictory is not any one idea. To obviate the apparent absurdity of bringing together such different things under a single term, some logicians have introduced the term "universe of discourse," the whole sphere or class of things which we have in view in actually making the judgments whose terms are under consideration; and they define contradictory terms as those which exhaust between them the universe *of discourse*, not the whole universe of thought and existence. Thus, "white" and "not white" are contradictories in the world of colour; and only those things which may have colour must be either the one or the other. Sometimes we have a pair of names which themselves denote a particular sphere; "British" and "Alien" are limited to the sphere of human beings, and within that sphere would be considered as contradictories, if the view to which we have referred is to be accepted. But it is preferable to keep to the older view and take the "contradictory" in the widest possible sense, as this brings out more forcibly the nature of pure logical contradiction. We may interpret the pure contradictory in such a way that it involves no logical absurdity. We need not, for instance, use the name "not-man" as meaning all things together which are not man, but as being applicable to *any thing*

which is not man: it is exactly therefore what Aristotle called it,—an *indefinite name*. If we try to express its denotation, we must think, not of "a chaotic mass of the most different things" together, but of "either this, or this, or this, or . . . " and so on indefinitely, through everything which is not denoted by the original term. Those who take the narrower view of *contradictory* names, explain *contrary* terms as representing opposites without exhausting between them the particular sphere of reference or "universe of discourse"; thus, "white" and "black" are contraries in the world of colour.

According to our view, contraries do not exhaust between them the universe of thought and existence; and the *opposition* which they express is of various kinds. The type to which Aristotle restricts the name of "contrary opposition" is the relation of "things which stand furthest apart among those of the same genus" (*Categories*, ch. vi., and elsewhere); as "white" and "black," "virtuous" and "vicious." A more general case is *incompatibility*, *i.e.*, the opposition of qualities which cannot be possessed by the same thing in the same way, as "round" and "square," "one" and "many," "red" and "green"; while "red" and "round," "large" and "square," &c., are perfectly compatible. The opposition of *positive and negative* names approaches more nearly to that of contradictory names. In those, the formation of the words indicates that the opposition is one of the presence and absence of a certain quality. Names which indicate *contrasted classes*, "British" and "foreign," "male" and "female," &c., are analogous to positive and negative names; and these are a frequent type of contrary opposition. But the different kinds of opposition which pairs of contrary names express, depend on the things denoted by the names, and our understanding of the opposition depends

on our knowledge of the things. Logic can give no general account of all the types of contrariety. Hence contrary opposition is *real* or *material*, while contradictory opposition is *formal.*

§ 5. Names may also be divided into relative and absolute.

A **relative name** has been defined as denoting an object which cannot be thought of without reference to another object, or can only be thought of as part of a larger whole. But in this sense, there are no **non-relative** or **absolute** names. Everything is related to other things, even on a superficial view; and if we imagine ourselves to be knowing or investigating its connections as completely as possible, "root and all, and all in all," its relations to other things would be found to have increased in extent and complexity, the further our knowledge had penetrated.

Hence every conception which we form is relative to something else; whenever we think of a thing we are distinguishing it from other things. We think of a table, and the table is at once opposed at least to vacuity, if not to other articles of furniture. In this sense, every name is relative. It is possible, however, to distinguish "relative names" in a narrower sense, as Mill has done. "A name is relative, when over and above the object which it denotes, it implies the existence of another object deriving its denomination from the same fact which is the ground of the first name": *e.g.*, "father, child," both terms implying the facts of parentage; "king, subject," both implying one of the modes of government. Such pairs of names are called **correlatives.**

§ 6. Let us now characterise more precisely the kind of *idea* which we use in judgment.

Why do we *express* our thoughts at all? Because thought forms a common ground in which different minds can meet, and which affords them a means of mutual understanding. Every judgment gives *information;* it points outwards by means of language to other minds, to whom, actually or in imagination, it is always addressed. Hence when we express a judgment in the form of a proposition, S is P, there are two conditions which the terms must fulfil:—

(*a*) Each term ought to have the same meaning for the mind using it, at one time, as it has at every other time; otherwise it would not be the genuine *identification of a thought;*

(*b*) Each term ought to have a meaning for other minds beside the one which judges, otherwise no information is conveyed; and it ought to have identically the same meaning for all these various minds, for otherwise the information conveyed is confused or misunderstood.

Thus we see that the *meaning of a term* in judging, is not and cannot be the private possession of any one mind. There is, so to speak, an aspect of *generality* in the meaning of every term. We may even say that *every term is a general term.*[1] But so far, we have grasped only one aspect, so to speak, of the meaning. It is not only identical in meaning for each individual mind and identical in meaning for different minds; it is also the thought of the same object, whoever may think it; in other words, it always *means the same thing.* Thus

[1] The student should carefully notice that *not every general term is a common term;* for a common term is the name of a *class.* Common terms are one division of general terms. Some writers have caused confusion by identifying the two.

when I speak of "the earth," "the British Constitution," "English writers on Logic," "a library," &c., &c., in each case I refer to *something real* which I am thinking about, but which continues to be what it is and mean what it means whether I am thinking about it or not; and I intend the *same* reference to be understood whenever I use the words. For this reason the logical term has also been described as an "identical reference."

In the case of common terms, the identical reference is to the common qualities of the objects to which the name is applicable. Common terms signify an idea which is formed usually by *comparison;* and the general idea of the points in which the things resemble one another is fixed by the common term. Consider, for example, the two well-known heavenly bodies called Jupiter and Sirius. "Bringing them into comparison, I observe that they agree in being small, bright, shining bodies which rise and set and move round the heavens with apparently equal speed. By minute examination, however, I notice that Sirius gives a twinkling or intermittent light, whereas Jupiter shines steadily. More prolonged observation shows that Jupiter and Sirius do not really move with equal and regular speed, but that the former changes its position upon the heavens from night to night in no very simple manner. If the comparison be extended to others of the heavenly bodies, I shall find that there are a multitude of stars which agree with Sirius in giving a twinkling light and in remaining fixed in relative position to each other, whereas several other bodies may be seen which resemble Jupiter in giving a steady light, and also in changing their position from night to night among the fixed stars. I have now formed

in my mind the general idea of *fixed stars* by bringing together mentally a number of objects which agree; while from several other objects I have formed the general idea of *planets*." This example, from Jevons, illustrates in a simple case the formation (by comparison) of the idea of a class. A class is "the indefinite number of individual objects or cases characterised by the possession by each of a certain set of definite marks" (see below, ch. V. §§ 7 to 10).

We may illustrate the process also by reference to a few of the general qualities of bodies. Among the qualities which our sense of sight reveals to us, there is a group connected by an obvious resemblance to which we give the name of "colour." It is not easy to explain precisely what is common to all the different colours, unless we are acquainted with the psychology and physiology of "visual sensation," and the physical theory of light; nevertheless we are convinced that they have something in common, and we refer to this by the general idea named "colour." Similar observations apply to the general idea of "brilliancy." Again, the universal property of Gravitation, which is common to all the different degrees of heaviness, is named "weight"; and similarly with "density." Now to take a more complex case. Metals, such as gold, silver, copper, lead, &c., resemble one another in certain definite ways; each of them has colour of one kind or another, each has some degree—more in one case, less in another—of brilliancy, weight, and density: hence the universal, "metal," includes the general ideas, "*some kind* of colour, *some degree* of brilliancy, of weight, and of density." If we pursue the subject scientifically, we have of course to include the ideas of other qualities in the universal—*e.g.*, that metal is an "element," is a "good conductor of heat and electricity." Once more, we observe that some animals walk, others fly, and so on; that some breathe through lungs, others through gills, others through the skin; that some produce young alive, others lay eggs, others multiply by division. Hence we form the universals, "locomotion," "respiration," "repro-

duction," which are included in the general idea "animal," —"some kind of reproduction, of respiration, and of locomotion." When the "general idea" of a class is defined with precision, it is called a *class-concept* or, briefly, a *concept*.

We will now compare the relation between changes of connotation and changes of denotation in common terms which are related—*i.e.*, which denote related kinds of things. The connotation of the term "ship" is definite enough for an illustration. Increase the connotation to "steam-ship"; what change have we made in the denotation? Obviously there are fewer "steam-ships" than "ships." Increase the connotation to "screw steam-ship"; the denotation is further decreased. We may arrange such related terms in a series of increasing connotation and decreasing denotation, or *vice-versa: Ship*, *Steam-ship*, *Screw steam-ship*, *Iron screw steam-ship*, *British iron screw steam-ship*. Here the connotations form an increasing series, the denotations or applications a diminishing series. Hence the following rule is given: **As connotation increases, denotation decreases; as denotation increases, connotation decreases.** The rule applies only to terms which can be arranged in a **classificatory series**, as is the case with the series of terms given above. This implies that *the connotations of the terms are fixed, and accepted as practically adequate* (see § 7, *ad finem*); and that the terms are arranged in a series, in ascending or in descending order of divisions and subdivisions. The rule is sometimes wrongly stated, and is so exposed to objections which are really irrelevant. Jevons states it as though it applies *to the same term*. If so, the rule might fail in two ways. We might, through increase of knowledge, expand the connotation of a term without decreasing

its denotation; and we might find new individuals to which the term is applicable without decreasing the connotation—*e.g.*, increase of population does not change the meaning of *man*. But the rule was never meant to apply to what happens to a single term through increasing knowledge or increasing number of individuals.

The best illustrations of the law are found in the sciences of classification. Thus, the adequate definitions of *Dicotyledon*, *Thalamifloræ*, *Ranunculaceæ*, *Ranunculus*, *Ranunculus ficaria* form an increasing series; the applicability of these terms is a diminishing series. The older logicians were fond of the following illustration, which has therefore acquired a certain historic importance:—

Connotation least, denotation highest.

Being
(*i.e.*, anything existing,—"being" in the most general sense)
|
material being
(*i.e.*, *matter* in the widest sense)
|
organic material beings
(*i.e.*, the whole world of life,—animal and vegetable)
|
Sentient organic material beings
(*i.e.*, animal life)
|
Rational sentient organic material beings
(*i.e.*, humanity)
|
This Man.

Connotation highest, denotation least.

In this case each term is predicable of the following

one. By reversing the order we make each term predicable of the preceding one:—

This man
|
men
|
animals
|
living beings
|
material being
|
being.

In each series the rule is strictly observed.

The rule is sometimes expressed in mathematical language which is not appropriate: "connotation and denotation *vary in inverse ratio*." But *qualities* cannot be separately numbered like the individuals in a class, and compared as regards their quantity with the latter. And there need not be any kind of proportion between an increase of connotation and the resulting decrease of denotation: thus, from "civilised man" to "native of Switzerland" is not a great increase of connotation, and the decrease of denotation is enormous; while from "civilised man" to "European" the increase of connotation does not carry with it nearly so large a decrease of denotation. *A single change in the connotation will result sometimes in a great and sometimes in a small change of denotation.* Hence the mathematical terminology should be avoided.

Finally, the rule will not apply unless the "increase of connotation" means the addition of a really new predicate. This is not the case if we change "man" to "mortal man," or "metal" to "elementary metal." These changes make no difference in the denotation; for the property of mortality belongs to all men, and

that of being an element to all metals. As we shall see (ch. V. § 2), this is equivalent to saying that the new quality must not be a Definition or a *Proprium;* it must be a *Differentia.*

§ 7. The next necessary question is as to the *limits of connotation.*

The traditional view is that the connotation consists of a perfectly definite group of attributes which are neither more nor less than sufficient to mark off a class from all other classes. These attributes are expressed in the *definition* of the term. On this view of connotation some important logical distinctions depend, as that between "verbal" and "real" predication (ch. III. § 2). But what the student has to notice is the implication that *to each term there belongs a fixed and definite meaning.* This is a logical ideal rather than a psychological fact; and for this reason many of the rules of the Aristotelian Logic seem artificial,—they are not intended to have reference to the shifting connotations of many of our ordinary terms. Logically, it is our business to make the meanings of our terms definite, and to keep them so, changing them only when a real advance in knowledge requires it. Thus, in Plato's time the connotation of the term "sun" was—"the brightest of the heavenly bodies which move round the earth." This clear and definite idea had to be changed to what we now mean by "the sun" in consequence of advancing knowledge. *The connotation of a term should be made clear and distinct, and then remain fixed as long as possible, being revised only when revision is inevitable.*

How little attention is paid to this logical requirement in the ordinary affairs of life was shown by Locke in a vigorous passage in his *Essay concerning Human Understanding* (Bk. III. ch. xi.): "He that should well con-

sider the errors and obscurity, the mistakes and confusion that are spread in the world by an ill use of words, will find some reason to doubt whether language, as it has been employed, has contributed more to the improvement or hindrance of knowledge amongst mankind. How many are there that, when they would think on *things*, fix their thoughts only on *words*, especially when they would apply their minds to moral matters; and who then can wonder if the results of such contemplations and reasonings,—whilst the ideas they annex to them are very confused and very unsteady, or perhaps none at all,—who can wonder, I say, that such thoughts and reasonings should end in nothing but obscurity and mistake, without any clear judgment or knowledge? This inconvenience in an ill use of words men suffer in their own private meditations; but how much more manifest are the discords which follow from it in conversation, discourse, and arguments with others. For language being the great conduit whereby men convey their discoveries, reasonings, and knowledge from one to another; he that makes an ill use of it, though he does not corrupt the fountains of knowledge which are in things themselves; yet he does, as much as in him lies, break or stop the pipes whereby it is distributed to the public use and advantage of mankind."

The only remedy for this condition of things is to realise clearly what are the ideas for which words stand, and to take care that for each term there shall always be the same definite idea.

Some logicians have proposed to give a wider meaning to "connotation," and to understand by it, *all the known* qualities of the thing, or (if the term denotes a class) all the known qualities common to the members of the class. But with the growth of experience and knowledge.

we usually find that many of these known qualities are unessential, and some are insignificant from every point of view, and we simply leave them out of account in forming our idea; hence they do not form part of the connotation. It is sufficient if the connotation includes the "important" or "essential" attributes. The connotation of "man" does not include an idea of the peculiar shape of the ears, of the capacity for laughter, and other "known qualities common to the class."

There is a third possible meaning of connotation,—that it is all the qualities of the thing (or class), whether known to man or not. The word is not employed in this sense, for it would introduce fundamental confusion into Logic. If we assume that Tennyson's well-known lines on the "flower in the crannied wall" express a philosophical truth,—that the complete and perfect knowledge of the flower would involve the knowledge of "what God and Man is,"—then (using "connotation" in the sense that we now speak of) God, Man, and the whole universe would be part of the connotation of the flower. But "complete and perfect knowledge" is an ideal so far beyond our present attainment, that we have no right to say what it would or would not imply.

Our result is therefore as follows. The question for Logic is never what a name means for you or me, but always what it ought to mean. And what it ought to mean must be something definitely fixed, the idea of the *important* qualities: or, expressing this in other words, *the qualities on account of which the name is given, and in the absence of which it would be denied.* Our idea of these depends on our knowledge of the things referred to by the name, and will change as that knowledge grows; but the connotation of the term can never be used to signify anything more than what we actually know.

§ 8. We have now to examine the question, whether every term has both connotation and denotation. This discussion has special reference to Mill's views, as set forth in Bk. I. ch. ii. § 5 of his *Logic*.

In § 6 we began by showing that every significant name (*a*) must have a *general meaning*, and (*b*) must refer to some concrete embodiment of that meaning. We then proceeded to speak of the special case of common or class names. It was in special connection with these common names that we introduced the distinction of connotation and denotation (§ 3, *b*); for it is with reference to such names that this distinction is most clear. "Such names," as Professor R. Adamson has said, "mark off a number of individuals (things, persons, cases) as possessing the attributes which constitute the meaning of the name"; the connotation consists of the attributes making up the meaning of the name, and the denotation is the sphere of its application.

The connotation is logically the primary meaning, the denotation is the secondary; for if we wish to refer to objects, otherwise than by pointing with the finger, we must do it by means of the connotation of their name; the connotation determines the denotation. This is fully admitted by Mill; for although he says that the term "signifies the subjects [its denotation] directly, and the attributes indirectly," he does not mean that the fact has any logical significance. It is not always a fact; and when it is so, it is because we have no sufficiently exact ideas corresponding to many of the terms which we use, and so find it easier to think in denotation. Here we have a psychological fact, which is logically a serious defect of thought.

With reference to concrete common names, denotation is strictly relative to connotation. The words "connotation" and "denotation" are in fact correlatives

(§ 5, *ad finem*). But the correlation of the two *appears* to break down in a limit beneath the common name, and in a limit above it. The lower limit is the *proper name*, and the higher limit is the *pure abstract name.* Mill held that in both these cases the correlation does really break down. He maintained that neither abstract names nor proper names have both connotation and denotation; and he described these names as "non-connotative" or merely "denotative."

* Adamson has pointed out that to call abstract names and proper names "denotative" or "non-connotative" involves a *wider* use of "denotation" and "connotation" than is involved in speaking of the denotation or connotation of a concrete common name. It is, in fact, to adopt the vague untechnical use of the word "denote"; and to revive a scholastic use of the word "connote."[1] We could express Mill's view, as Fowler does, without varying the meanings which are given to "connotation" and "denotation" in connection with class names. Thus, adopting Mill's view, Fowler classifies names as follows: (1) those which have both connotation and denotation; (2) those which have connotation only (called by Mill "non-connotative"—*i.e.*, abstract names); (3) those which have denotation only (called by Mill "non-connotative"—*i.e.*, proper names).

It is perfectly true, as Adamson has observed, that in both the limiting cases the *class-reference* of the ordinary common name is absent. The **abstract name** stands for an attribute or group of attributes conceived *in abstracto*—*i.e.*, without reference to any concrete exemplification of them. When thus used, the abstract name cannot have a plural, and may be regarded as a singular term (*see* present chapter, § 2 and § 3, *a*). When it is used as the subject of a proposition, we may regard its connotation and denotation as coinci-

[1] On this historical point, *see* Minto's *Logic*, pp. 46, 47.

dent: "*colour* (*i.e.*, colouredness in general), *extension*, *density*, are properties of bodies." With regard to the **proper name,** Mill's view is in the first place that proper names have "no connotation." He practically limits the meaning of the word "connotation" to the *class-reference* of a common name. With this limitation, it is of course true that a proper name has "no connotation." It stands for a concrete individual without reference to attributes possessed in common with others. But such limitation of the word "connotation" is arbitrary; if the proper name has *meaning*, that meaning *is* its connotation. Now Mill denies that it has any "meaning" or "signification" in the strict sense of these words. He treats the proper name as an "unmeaning mark," given not to signify any qualities but to identify an individual; it *comes to suggest* various qualities to any one who hears it and is acquainted with the person who bears it.

This question does not concern ordinary singular names, in which both of the two sides (connotation and denotation) can be distinguished: "the honourable member who brought forward the present motion." Setting these aside, it is not to be denied that when we hear a proper name mentioned by itself, in detachment from a proposition, then (*a*) it gives us no information as to the qualities or characteristics of the person or place, unless we are acquainted with him or it already; names like Dartmouth, Oxford, which signify particular situations, and personal names which are supposed originally to have signified the occupation of the individual bearing them, have long ceased to have any such meaning. (*b*) And when we know the qualities, &c., of the individual denoted, then, when the proper name is changed, the new name tells us nothing

different from the old:[1] we may contrast this with what is signified by changing the name of a thing from "vegetable" to "animal." (*c*) Also it is the fact that the proper name is, as a rule, not given in order to signify any attributes; in the case of a child, it could not signify attributes which are mostly developed after the name is given. Hence, as remarked above, we are told that the name "comes to suggest" various qualities, but does not "mean" or "signify" any qualities.

The question, then, is—whether the "suggestion" associated with a proper name does or does not correspond to the "meaning" (connotation) of a common name or an ordinary singular name. Mill (and some other writers) maintain that there is no analogy; there is a difference of function so complete as to justify us in saying that proper names have "no signification." Against this, we maintain that the proper name has no *fixed* or *constant* but an *acquired* connotation.[2] When used in a proposition—*i.e.*, when used in the concrete as the designation of a definite individual—the name acquires *meaning* in the strict sense, not merely "suggestions" or "associations." The whole peculiarity of proper names consists not in having no meaning, but in the fact that their use (as the identification of a particular individual) prevents the meaning from becoming general.

The main proof of our position consists in the fact which Professor Bosanquet has pointed out (*Essentials of*

[1] The case of a woman changing her name on marriage seems the only important exception.

[2] It may be held that *all* names have what *was* originally (as language developed from its beginnings) an "acquired connotation." Cp. Bosanquet, *Logic*, vol. i. p. 44; Sigwart, *Logic*, Eng. Trans., vol. i. p. 52.

Logic, p. 92). "The convention of usage which prevents a proper name from becoming general—*i.e.*, from being cut loose and used simply for its meaning—is always on the point of breaking down." This actually takes place when the meaning which a proper name acquired, while it was used as a designation for a particular individual, is made general, and the name is used as a type: "a Don Quixote," "a Daniel,—a second Daniel," "a Solon," "a Crœsus," "a Nero," "a Cæsar Borgia."

Mill's view as to proper names being "unmeaning marks" results from his discussing such names without any reference to their significance *when used in propositions*. The only distinction between (1) a proper name *as applied* and (2) a term like "this great writer," is that in (1) the denotative and in (2) the connotative aspect is more prominent from the point of view of language and expression. The distinction is rhetorical, not logical. And if (as we pointed out above) we may use a well-known name for rhetorical purposes, to signify certain typical qualities, our language gains in connotative character by the use of proper names instead of common names.

As a matter of fact, there are numerous exceptions to the statement which we admitted, that a proper name has no *fixed* meaning. Any name whatever, when used in a proposition, implies an existence of some kind; and if we know it as a proper name, we are justified in taking it to imply either some one's personal existence, or some definite object or locality. The form of the name, *as it stands in the proposition*, is *usually* sufficient to tell us (*a*) whether it implies a person or a place, (*b*) to what nation or country he or it belongs, and (*c*) if a person, whether male or female. And when we consider family names, apart from merely personal or

baptismal names, their partial analogy to common or class names is evident; the name is used of many individuals in the same sense, and it means the attributes in which they resemble one another.[1]

The result of this discussion is that the general conclusion of § 6 remains unshaken; every name has both connotation and denotation,—it has a general or universal meaning, and refers to some embodiment of that meaning. This is frequently expressed by saying that it is a **Universal.**

§ 9. The subject to which we will now pass is closely connected with the relation of terms to their ideas or meanings (the subject of the present chapter), and the relation of the ideas to one another in a judgment (the subject of the two following chapters). What are called the **Laws of Thought** have a reference to both these relations.

The word **law** is not without ambiguity. Most writers on Logic have distinguished two chief meanings of the word. In one sense of the word we speak of **Laws of Nature,** which are general statements of what uniformly happens. A single exception to such a law would make it no longer a law of Nature. In another sense, a law is a precept or rule laid down by some authority,—an injunction or command addressed to persons who are called on to obey it but have it in their power to disobey. This use of the term is exemplified in such phrases as "law of the land," "law of conscience." The authority remains independently of its violation by individuals. When speaking of a **Law of Thought,** we use the term mainly in this second sense. Men constantly fall into errors and confusions in their thinking,

[1] For an interesting application of this to Roman names, see Mr Stock's *Logic*, second edition, p. 45.

and so "disobey" the laws of thought, although as a rule they do not do so consciously or deliberately.

The Laws of Logic, then, set up a *standard* to be followed. They may be compared with the laws of Grammar as regards correct speaking and writing. The science of Ethics also endeavours to formulate a standard, consisting of laws of right conduct which are far from being constantly recognised in life. Hence Logic has been called the Ethics of Thought. The student will already have observed the applicability of this title. In dealing logically with the *concept*, for instance, our main business is not to inquire what kind of Universals are formed in the average mind, as a matter of fact, and what are the processes of thought which lead to their formation; we begin to formulate—and shall formulate more fully in the sequel—an ideal of what the Universal ought to be (*cf.* § 7). This is the characteristic of logical treatment throughout.

In this way we have answered the over-discussed question, whether Logic is a Science or an Art. A *mere* Art would be a body of practical rules, having no scientific connection among themselves; gathered, perhaps, from haphazard experience, or gathered from very various object-matters, as "the art of music." But Logic is first a Science,—a systematic body of doctrine, of "theory," and then a science which aims at distinguishing *correct principles* of thought. Hence many logicians have described it as both a Science and an Art; *e.g.*, Mill, in his *Examination of Sir W. Hamilton's Philosophy*, speaks of Logic as "the art of thinking, which means correct thinking, and the science of the conditions on which correct thinking depends." Logic may be defined as a practical, or better, as a **normative or regulative, science.**

§ 10. In a wide sense, the phrase Laws of Thought means all the general principles or types of Thought (see

chap. I. § 2) which we treat of. In a narrower sense, it signifies certain fundamental principles which lie at the basis of inference.

Since the time of Aristotle, three such principles have been made of fundamental importance. The first of these was not explicitly stated by him. It was subsequently known as the **Law of Identity**, and assumed the form: "a thing is identical with itself"; "A is A." The second principle, afterwards called the **Law of Contradiction**,[1] was thus stated by Aristotle: "the propositions *A is B* and *A is not B* cannot both be true *together*." The third law, now known s the **Law of Excluded Middle**, was formulated by Aristotle thus: "of the two propositions *A is B* and *A is not B*, one must be true and the other false."

§ 11. As it stands, the **Law of Identity**, "A is A," does not give us any information. It may, however, be interpreted so as to make it a genuine principle on which the very life of Thought depends. At this stage we shall mention only its simplest signification.[2]

We have seen that in actual thinking we require terms to identify our thoughts. The Term identifies a "universal meaning" (§ 6). The Law of Identity has an important application to this relation. Let A denote anything thought about, any more or less defined idea which is distinguished from other ideas so far as to be indicated by a single symbol in language, a name or term, M. Then to say that "A is A" means that M must always stand for the same A,—the same for different minds and for one mind at different times.

[1] Sometimes referred to, more appropriately, as the Law of Non-contradiction.

[2] The philosophical aspects of the Law of Identity will be further considered in ch. XI. § 3.

Terms must have fixed meanings, each clear in itself and distinct from others. If the meaning of a term is changed, it should be done deliberately and for a sufficient reason.

§ 12. The **Law of Contradiction** may be interpreted so as to refer (*a*) to the meanings of terms, (*b*) to the consistency of propositions.

(*a*) Just as the principle of Identity secures the identical reference of a term to a meaning, so the principle of Contradiction secures the same result by forbidding a term to be diverted to another meaning in the same discussion or discourse. While we are treating of one subject, we must fix the meanings of our terms, and keep the same meanings.

(*b*) As applied to the consistency of propositions, the principle of non-contradiction declares that the different parts of truth cannot be incompatible with one another.

* We may illustrate this by reference to the manner in which certain types of philosophical doctrine have been maintained.

If we find a thinker maintaining as essential parts of his system the following doctrines: (1) we know, with the highest degree of certainty, that the Reality which lies behind the phenomena of mind and matter is *unknowable*, and (2) we know, with the highest degree of certainty, that it exists, that it is infinite, eternal, the Cause of all things, and manifested in all things: then, by mere comparison of the ideas employed, we see that the system is fundamentally inconsistent. Reality is declared to be altogether unknowable, and also to be knowable in certain important respects. Both statements cannot be true.

If, again, we find it maintained that the "Association of Ideas" is a law of connection among the units of which the mind is composed, which are distinct "sensations"; that, by this law, a present sensation may revive another one with which it was experienced at some former time, we

find the doctrine wrapt in inconsistencies when we ask, "What happened to the second sensation in the interval between its first experience and its revival?" Here the mind is first declared to be only a series of sensations, each of which disappears to give place to the next; then the mind is declared to be such that a sensation when it disappears can leave behind a permanent effect or trace which can come up into consciousness. *Both* these views cannot be true.

If, once more, a scientific man denounces with vigour the assumption of a controlling designing Power at work in the production of certain natural events, and yet allows himself to speak as if "Nature" were a Power acting with a purpose, and is unconsciously influenced by this very idea in his explanation of natural facts, then we may bring the same charge. On the one hand it is maintained that no natural effects are produced by a superhuman designing Power; and on the other hand, that some effects are so produced.

The "inconsistent" doctrine or statement may always be reduced to the one fundamental form, of attempting to make the propositions "A is B" and "A is not B" true *together*. In this form the principle is stated by Aristotle (*Metaphysics*, IV. iii.): "It is impossible that the same predicate should both belong and not belong *to the same thing at the same time and in the same way.*" A thing may have different qualities at different times, as in the changes in a person's character; and it may have a quality in one respect, and not have it in another, as in the celebrated shield that was gold on one side and silver on the other; but these facts do not conflict with the law of Contradiction as Aristotle states it. Aristotle points out that the denial of this principle would be the denial of the very possibility of thinking.

§ 13. The law of **Excluded Middle** also may be interpreted so as to refer either to the meanings of terms or to the consistency of propositions. As applied

to the meanings of terms, it states any given definite meaning either does or does not belong to a given term, in a given context. It is simply a further development of the rule that the meaning of a term must be definite and remain the same in the same discussion or discourse. As applied to the consistency of propositions, the law of excluded middle says that of the two propositions "A is B" and "A is not B," one must be true and the other false. In this form the principle was laid down by Aristotle. Its application is plain in proportion as A and B are exactly defined. If we are in doubt as to where one thing begins and another ends, we are in doubt as to the precise application of our principle. This may happen in cases where we do not find a perfectly definite limit to an event in space or time—*e.g.*, when something is "in the act" of occurring, we seem unable to say, "either it has happened or it has not happened." The sun may be just "rising" without "having risen" or "not having risen." But as soon as we have attached a precise meaning to "rising," in the case of the sun,—*e.g.*, if we make it mean that the actual globe is visible above the true horizon,—then the law of excluded middle is applicable. When we are speaking of natural qualities such as heat, which always have *degrees*, then again we cannot say that a body must either "be hot" or "not be hot" until we know that some *definite* degree of heat is signified by that word. And in the case of the great divisions of Nature, which seem to shade off into one another, as "animal" into "vegetable," and "vegetable" into "inanimate matter," we may be in doubt as to the application of the law of excluded middle to an individual on the borderline of one of these divisions; we may not be able to say either that it is an animal or

that it is not an animal,—it may seem to be something between the two. But this results from our imperfect understanding of what animal life really is; the greater the light which is thrown on this problem, the smaller the extent of the doubtful borderland, of things which seem neither in the class of "animal life" nor outside it.

Sometimes the law of excluded middle has been questioned through a mere confusion. The contrast which the law of thought makes, is between two propositions one of which simply denies or contradicts the other,—between an affirmative and a negative proposition, "This water is hot,—this water is not hot," "This paper is white,—this paper is not white," "This line is longer than that,—this line is not longer than that," "This opinion is simply true [*i.e.*, true without qualification or limitation],—this opinion is simply not true." In each of these pairs of propositions, one and one only must be true; there is no third possibility. But it is not uncommon to apply the law to a pair of propositions which affirm *contrary* predicates of an object, and to say (taking the last of the above examples) that "either this opinion is simply true or it is simply false." Here there may be a third possibility,—it may be a mixture of truth and error. Similarly, between "white" and "black," "hot" and cold," "greater than" and "less than," in each case there are other possibilities. Great care is necessary to avoid confusing **propositions whose predicates are contrary terms** with the **contradictory propositions** which the law of excluded middle has in view.

When we come to deal with the "opposition" of propositions, in the following chapter, the student will find that we deal only with propositions which expressly

say whether all or part of the subject is referred to,—"All S is P," "some S is P," and that, when the subject is thus *quantified*, the contradictory judgments are found to be "All S is P,—some S is not P," and also "No S is P,—Some S is P"; for in each case one and one only of the propositions must be true. In the special case where the subject refers to a single or individual thing,—in other words, when S is a *singular term*,—the contradictory propositions are simply "S is P,—S is not P"; *e.g.*, "This stone is old,—This stone is not old." **Propositions are said to be contradictory when one of them states exactly what is sufficient to deny the other, including no more and no less than what is sufficient to deny it.**

§ 14. Since the time of Leibniz an important principle has been introduced in Logic and placed by the side of the three laws of which we have spoken. It is called the law or principle of **Sufficient Reason**, and is usually stated thus: "For everything there is a sufficient reason why it is so rather than otherwise." In this principle two different laws of thought are brought together, which must be distinguished, and, for the purposes of elementary Logic, carefully separated.

(*a*) The first principle states that for every proposition which is held to be true, there must be reasons for regarding it as true,—arguments which may be brought in support of it. It must be capable of being shown as the **conclusion** from certain **premises**. In other words, every judgment, when questioned, expands into an **inference**. This does not apply to the propositions which state the "laws of thought"; they cannot be proved by argument, from premises to conclusion,—they cannot be, in ~~this sense, *inferred* (ch. I. § 7); for~~ all argument and all inference depends upon them

Such a proposition is called an **Axiom** (ἀξίωμα). The real *ground* of our confidence in the truth of such axioms lies in the fact that if they are not true, there can be no such thing as Thought or Knowledge. An axiom is usually defined as "a proposition general in import and standing in no need of, or indeed incapable of, proof"; that is, of proof by inference from other propositions or from sense-observation.

* The principle that **every judgment justifies itself by expanding into an inference**, is really part of a wider principle,—that all parts of our knowledge, so far as they are true knowledge, are connected together. We know that any statement, once admitted to be true, may have a modifying effect upon any other portion of our knowledge. All the current scientific, theological, and philosophical controversies afford abundant illustrations of this fact; and it is a fact, because **every judgment is at bottom connected with every other one.** We cannot *show* this connection, in many cases; but most of the controversies alluded to consist in the endeavour to discover the connection between different parts of knowledge,—the results of different sciences. It has been said, for instance, that "Man's place in Nature" has been the *cause célèbre* of the nineteenth century. And when we have succeeded in reconciling different results, we find that they mutually support one another.

(*b*) The second principle included under the Law of Sufficient Reason states that for every event in the real world there must be a cause, without which the event could not happen. This is properly described as the Law of Universal Causation; and we shall have to consider it later, along with other principles of Inductive Logic. These also are "Laws of Thought,"—principles

on which knowledge depends, and the trustworthiness of which is to be granted if not only knowledge but thought itself is to be possible.

We have stated the principles of Contradiction and Excluded Middle as they were formulated by Aristotle, who had in view two **contradictory propositions** contrasted with one another. Later logicians stated the laws in the form "a thing cannot be both A and *non-A*," "a thing must be either A or *non-A*."[1] Here, instead of two contradictory propositions, we have a pair of **contradictory terms** opposed to one another. Aristotle did not use the *nomen indefinitum* "non-A." These later statements of the principles are of course true; but they have not the logical significance of Aristotle's statements, for they do not express what formal inconsistency or contradiction is. "Non-A" is a purely indefinite term; and though we call it the contradictory term to A, the relation between these two does not give us the meaning of the logical act of contradiction. Contradiction takes place only between *propositions*; and only when one proposition affirms a predicate and the other simply denies it of the same subject. And of such propositions, both cannot be true, while one must be true and the other false.

* EXERCISE I.[2]

The following are selected questions on the subjects dealt with in this chapter:—

1. What is the logical difference, if any, between Substantives and Adjectives? [L.]

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[1] The following variations are sometimes found: for the Law of Contradiction, "a thing cannot both be and not be"; "a thing cannot be other than itself," "A cannot be non-A." And for the Law of Excluded Middle: "a thing must either be or not be."

[2] For more elementary questions, see chapter iii. *ad finem.*

2. Define briefly the Connotation of a name. Consider carefully any difficulties in determining the connotation of each of the following names: (*a*) Parallelogram; (*b*) Man; (*c*) Mr Jones. [L.]

3. Classify the various kinds of Singular Connotative terms, and examine the question whether any Singular terms are Non-Connotative. [L.]

4. What is a Connotative name? Give examples showing different ways in which the Connotation of a name may be such that the name is understood to be applicable to a single object. Distinguish such names from Proper names. [L.]

5. What different senses have Logicians attached to the expression—*the connotation of a term?* Are there any names which simply *denote* objects without giving any information about the characteristics of those objects? [L.]

6. Are there any terms without Connotation or without Denotation? How far has controversy on this question arisen from the ambiguity of the word "Connotation"? [St A.]

7. Explain concisely the distinction (*a*) between Common and Singular terms, (*b*) between Concrete and Abstract terms, illustrating your answer by a discussion of the logical characteristics of the following terms: The University of London, Logic, Iron, Sound, Fallacy. [L.]

8. Explain the distinction between Concrete and Abstract terms. Does this distinction correspond to that between Substantives and Adjectives? May differences of quantity be recognised in the case of Abstract terms?

9. (*a*) Which of the usual divisions of terms do you consider to be of fundamental significance in logical theory?

(*b*) Is every General term the name of a *class?*

Give reasons for your answer in each case.

10. Give a careful explanation of the Nature of Relative terms. [L.]

11. Distinguish between Positive and Negative names. What ambiguity is there in the case of such a name as "not-white"? [C.]

12. State the three fundamental Laws of Thought, ex-

plain their meaning, and consider how far they are independent of each other. [L.]

13. What have been called the Laws of Thought? Why is it held that these Laws supply only a *negative criterion* of Truth? [G.]

14. State and explain the Law of Sufficient Reason.

15. Criticise the statement that "Connotation is in inverse ratio to Denotation."

16. Examine the view that the species does not differ from the genus in the *number* of its attributes, but only in their relative determinateness.

CHAPTER III.

THE PROPOSITION, THE OPPOSITION OF PROPOSITIONS, AND THE FORMS OF IMMEDIATE INFERENCE.

Part I.—*The Logical Proposition.*

§ 1. GRAMMATICAL sentences may express commands, wishes, questions, exclamations, or assertions. In the last case the sentence makes a statement about something, and must have its principal verb in the indicative mood. Only when it is an assertion can we consider the sentence as expressing truth or falsity. The Proposition is an assertive sentence, a statement which admits of being true or false. But in the *strict* logical sense, the sentence is not a proposition until it is expressed in the form S is P, with a distinct Subject, Predicate, and Copula. The Subject is that about which the statement is made; the Predicate, that which is stated about it. The Copula is not merely a means of connecting S and P as the coupling-gear connects an engine with the carriages which it draws, nor in the judgment, which the proposition expresses, is there a separate thought corresponding to the Copula and coming between the idea of the Subject and that of the Predicate. The Copula simply expresses the mental

act of judgment,—the fact that I think of S and P as really joined together in the way which the proposition expresses.

We may now distinguish the different kinds or propositions.

Propositions of the form S is P are said to be **categorical** or unconditional.[1] They are so called to distinguish them from **conditional propositions**, which predicate P of S "under a condition," that is, provided certain circumstances are supposed or granted. Conditional propositions are of two kinds. They may be (1) **hypothetical** or conjunctive, as "If metals are heated, they expand," where the condition which must be granted is that the metal is heated; or again, "If money is scarce, prices are low," where the condition is that an insufficient quantity of the standard metal is being coined. The general forms of hypothetical propositions are—"If A is B, it is C," as in the first example; and "If A is B, C is D," as in the second. (2) The other class of conditional propositions[2] is the **disjunctive**, as "Man is either immortal or incapable of realising his Ideals," where man being merely mortal is the condition of his Ideals being unrealisable; or again, "Either the Carthaginians were of Semitic origin or the argument from language is of no value in ethnology"—*i.e.*, if the Carthaginians were not of Semitic origin, the argument from language may at any time be untrustworthy. The general forms are, as illustrated in

[1] The *word* "categorical" comes to us from the Greek κατηγορικός, which in Aristotle always means "affirmative." The current modern *meaning*, in which it is contrasted with "conditional," comes from a post-Aristotelian use of the word.

[2] Many Logicians prefer to identify "conditional" with "hypothetical" propositions to the exclusion of disjunctives.

the two examples just given, "A is either B or C," and "either A is B or C is D." Further consideration of conditional propositions may be set aside for the present.

We now come to the question, How many kinds of categorical propositions are there?

Aristotle pointed out (*An. Prior.*, I. 1, *De Int.*, v. vi.) that we may classify them in two distinct ways. When we make an assertion we must either (1) *affirm* something of the subject we speak of, or (2) *deny* something of it. Again the affirmation or denial may be made (*a*) of some one particular thing, or (*b*) of a whole class or kind of things, or (*c*) of a part of such a class, or (*d*) the proposition may be expressed without saying whether the whole or a part is meant. Later logicians called the former division (affirmative and negative) one of quality, and the latter (which is concerned with the distinction of the part and the whole of a class) one of quantity.

According to **quality**, then, propositions are either **affirmative** or **negative**. Aristotle is fond of saying that the affirmative unites or combines, the negative divides or separates. What kind of union, or separation, does the proposition express? The affirmative expresses a union between Subject and Predicate in the sense that the attributes signified by the predicate belong to the subject; thus in the proposition "Fixed stars are self-luminous," the quality of shining by their own light is said to belong to the heavenly bodies called fixed stars. The negative expresses a separation of subject and predicate in the sense that the attributes signified by the predicate do not belong to the subject; "gold is not easily fusible," declares that the quality of being easily fusible does not belong to gold. The typical forms, in Logic, of affirmation and negation are—S is P, S is not P. The student should bear in

mind that in this formal expression of the negative proposition, the word "not" belongs to the copula[1]:—

Subject	*Copula*	*Predicate*
S	is not	P

This fact is of importance, for we shall meet with cases where the *nomen indefinitum* "not-P" appears as predicate in an *affirmative* proposition, so that the word "not" belongs entirely to the predicate, in the formal expression of the statement:—

Subject	*Copula*	*Predicate*
S	is	not-P[2]

Coming now to distinctions of **quantity**, we must examine more closely each of the four classes mentioned above.

(*a*) The affirmation or denial may be made of some one object, so that the subject is a singular term (ch. II. § 3). In this case we have a **singular proposition.** The following are examples,—not, of course, all expressed in strict logical form: "*I* am what I am"; "*He* has blundered"; "*Job* must have committed some secret sin"; "*This statesman* is not dishonest"; "*The Emperor of China* is only in name a ruler." Many of the mediæval writers on Logic excluded singular terms, and hence also singular propositions, from logical treat-

[1] The philosophical aspects of Negation will be further considered in ch. XI. § 5.

[2] For the future we shall form contradictory terms by the use of the Latin word for "not": *e.g.*, "non-A," "non-human." We shall thus distinguish the negative implied in these terms from the negative which belongs to the copula. This convenience (to the beginner) compensates for the awkwardness of occasionally adding "non" to a word of *English* origin.

ment, admitting only common terms, names of classes. Hence, when afterwards singular terms were introduced into Logic, a place had to be found for singular propositions in the accepted classification of propositions—*i.e.*, in one of the two divisions immediately to be mentioned, (*b*) and (*c*). They were ranked with the universals, division (*b*), on the ground that the predicate refers to "the whole of the subject." It is unnatural to treat an individual as a class, but such is the traditional method.

(*b*) The affirmation or denial may be made of every thing of a certain kind or class.[1] In this case we have a **universal proposition**, so called because the predicate is affirmed or denied of every instance of the subject,—the reference is to "the whole of the subject." Thus, in "All planets shine by reflected light," this quality is affirmed of each of the class of "planets," although it is not strictly true as a scientific fact; and in "No men are utterly bad," this quality is denied of each one of the class "human beings." If it is not already in the form "All S is P" or "No S is P," the proposition, if it is really universal, can be expressed in this form without altering its meaning.

(*c*) The affirmation or denial may be made of a part of a certain class. In this case the proposition is said to be **particular**. Its logical form is "Some S is P" or "Some S is not P":—

"Some men are born great"
"Some statesmen are not practical."

The particular proposition, in ordinary language, is an assertion about some quantity between these two

[1] The philosophical aspects of the Universal Judgment will be considered in ch. XI. § 6.

extremes,—that in which the predicate is affirmed of the whole of the subject, and that in which it is denied of the whole—*i.e.*, it means "some only," "only a part."[1] But in its logical form the particular proposition only excludes "none"; it does not exclude the possibility of the reference to "all." In other words, it means "some, and there may or may not be more or all"—*i.e.*, "some at least." The only possible ground for taking "some" in the former, the narrower, sense, in a logical proposition, is our knowledge of its subject-matter, not anything in the formal expression of the proposition: "some metals decompose water,"—here "some" must be interpreted as "some only," for we know from Chemistry that the statement applies only to a particular class of metals. But as far as the logical form of the proposition is concerned, the whole class is not excluded; and we are told nothing as to how much of it, a great or small portion, is included, and nothing as to whether any particular case or group of cases is referred to.

(*d*) The affirmation or denial may be made without explicit reference either to the whole or to a part of the class denoted by the subject. In this case we have an indefinite or **indesignate proposition**, as "Virtue is a condition of happiness," "Pleasure is not a good." Such propositions cannot be dealt with in Logic until their true and precise meaning is made apparent. As Jevons says, "The predicate must be true of the whole or part of the subject, hence the proposition as it stands is clearly incomplete; but if we attempt to remedy this and supply the marks of quantity, we overstep the

[1] In ordinary language this convention is so strict that the word "some" is of itself sufficient to deny "all": "All men are to be bought" may be denied by the simple statement "*some* are to be bought."

boundaries of Logic and assume ourselves to be acquainted with the subject-matter of the science of which the proposition treats." Indefinite propositions, therefore, have no place in Logic, unless they are merely abbreviations, and their real quantity is obvious, as in the following: "Triangles have their three interior angles together equal to two right angles," or "Men are rational," &c.

On the whole, therefore, we have four possible logical forms of the proposition:—

Universal	affirmative	All S is P.
	negative	No S is P.
Particular	affirmative	Some S is P.
	negative	Some S is not P.

The form "All S is P" is denoted by the letter A; "No S is P" by E; "Some S is P" by I; and "Some S is not P" by O. As Dr Keynes has suggested, the propositions may be abbreviated thus: SaP, SeP, SiP, SoP. The letters were chosen because A and I are the first two vowels of *affirmo*, I affirm, and E and O the vowels of *nego*, I deny.

§ 2. Propositions are also classified according to **modality**, into (*a*) **necessary**, as "S must be P"; (*b*) **assertorial**, "S is P"; (*c*) **problematic**, "S may be P." Jevons says, "The presence of any adverb of time, place, manner, degree, &c., or any expression equivalent to an adverb, confers modality on a proposition"; but this is not the ordinary use of the term. Most writers take distinctions of modality in propositions as referring only to the difference between "must be," "is," and "may be." The questions arising out of these distinctions are too difficult to be pursued in an elementary work; but we must add a note on the expression of these propositions in the typical forms A, E, I, O.

(*a*) The assertion of necessity of course forms an A proposition: "An equilateral triangle must be equiangular" means that every example of an equilateral triangle will be found to be equiangular. Similarly the assertion of impossibility forms an E proposition. Necessary truth is also described as "apodictic" or "demonstrative" (ἀποδεικτικός); for the assertion of necessity, though truly represented by the universal affirmative "All S is P," adds to it the implication "that All S is P *can be proved*."

(*b*) The assertorial proposition, which makes a simple unqualified statement as a matter of fact, as "the American Indians are copper-coloured," will fall naturally into one of the four classes. In the example given it is an A proposition.

(*c*) The merely problematical proposition—as "the weather may be fine," "S may be P"—gives us *no information* about S; it only says, "I do not know whether S is P or not." The nearest in meaning to such a judgment, among the four typical forms, is the particular proposition, affirmative or negative. The logical meaning of "some" comes out best when we use the word "may"—*e.g.*, if a person says "Some Irishmen are not Nationalists," he tells us that any chance Irishman *may* not be a Nationalist. **"Some S are not P" means that there is no inseparable connection between S and P; "some S are P" means that there is no incompatibility between S and P.**

We must distinguish the propositions "S is not necessarily P," and "S is necessarily not P"—*i.e.*, it is impossible that S should be P. The latter is an assertion of impossibility, and as we have said forms an E proposition, as in "The circumference of a circle is necessarily not commensurable with its diameter." The

former is merely a denial of necessity, as in "A republic does not necessarily secure good government," or "Old paths are not necessarily the best." The sense of these propositions is that "Some S is not P," "Some republics do not secure good government," "Some old paths are not the best."

The last division of propositions which we need notice here is that of verbal and real, also spoken of as explicative and ampliative, or **analytic** and **synthetic** respectively. This distinction depends on the assumed fixity of definitions, to which we referred before (ch. II. § 7), and it is not applicable unless the fixed definitions of the terms concerned are actually known. The proposition S is P is analytic when P is the definition or part of the definition of S; it is synthetic when P is not part of the definition of S. It is evident that only when we have an accepted definition of the subject, can we tell whether the proposition is synthetic or not. And owing to the very various amounts of knowledge possessed by different minds, a proposition may be analytic to one person, who knows the definition of the subject, and synthetic to another, who does not know it. Again, the growth of knowledge may lead to changes in the definition of a name,—compare, for instance, the "solar system" as it would be defined in the Ptolemaic, in the Copernican, and in the Newtonian theories of astronomy: hence a proposition which is synthetic at one time may be analytic at another. We may make many statements about the solar system which are now analytic, but were not always so.

* The more we know, in the scientific sense of the word "know," of any object, the deeper our definition of it becomes; hence, also, the greater the number of

analytic assertions which can be made about it. We may assume that to a perfect Intelligence, to omniscience, all knowledge must be analytical.[1]

§ 3. We now come to what is one of the most valuable mental disciplines arising out of the study of elementary Logic. It is the exercise of paraphrasing ordinary or poetical or rhetorical assertions, so as to bring them into strict logical form with the least possible sacrifice of meaning. In the forefront of all exercises of this sort should stand the axiom stated by Hamilton: "Before dealing with a Judgment or Reasoning expressed in language, the import of its terms should be fully understood; in other words, Logic postulates to be allowed to state explicitly in language all that is implicitly contained in the thought" (*Lectures on Logic*, vol. iii. p. 114).

We shall first consider compound propositions which may be analysed into two or more simple ones; and subsequently the expression of simple propositions in the strict form. Common speech abounds in condensed and elliptical expressions; and the logical analysis of such expressions into Subject, Predicate, and Copula makes us familiar with what they imply, and strengthens the habit of exact interpretation.

Statements are frequently met with which combine two or more propositions, which have to be distinguished and separately stated in the reduction to logical form. Such **compound propositions** were called by the older logicians **exponible.** The most common instance is the connection of propositions together by simple conjunctions, such as "and," "but," "although," "nevertheless," &c. These are easily analysed.

[1] The philosophical aspects of the distinction between Analytic and Synthetic Judgments will be further considered in ch. XI. § 3.

(1) "France and Germany resolved on war"[1] is equivalent to—

- (a) France resolved on war.
- (b) Germany resolved on war.

(2) "Gold and silver are precious metals"—

- (a) Gold is a precious metal.
- (b) Silver is a precious metal.

(3) "The great is not good, but the good is great"—

- (a) The great is not good because it is great.
- (b) The good is great.

(4) "He is poor but dishonest"—

- (a) He is poor.
- (b) He is dishonest.

(5) "The more the merrier"—

- (a) A given number is enough for some merriment.
- (b) More will produce greater merriment.

(6) "Men who are honest and pious will never fail to be respected, though poor and illiterate; provided they are self-supporting, but not if they are paupers" (Venn).

The whole sense of this can be expressed in two propositions:—

(*a*) Self-supporting men who are honest and pious will be respected;

(*b*) Paupers though they are honest and pious will not be respected.

Two other propositions are emphasised in the original statement; (*c*) is a particular case of (*a*), and (*d*) of (*b*):—

(*c*) Poor and illiterate men who are self-supporting, honest, and pious, will be respected.

(*d*) Poor, illiterate, honest, pious men who are paupers will not be respected.

The analysis of the propositions which are called **exclusive** and **exceptive** is less simple. In **exclusive propositions** the Subject is limited by words like

[1] The word *and* in the Subject occasionally makes it collective, and then the proposition is not compound: "*two and two* make four."

"alone," "only," "none but," "none except," "none who is not": as, "Graduates alone are eligible," "S alone is P." The primary meaning of this assertion is to exclude non-graduates; accordingly the proposition must be expressed—"No non-graduates are eligible," 'No non-S is P."

* Some writers say that exclusive propositions are com pound, and consist of a primary meaning "No non-S is P" with a "secondary implication" that "Some S is P" (in our example, "some graduates are eligible"). This is an error. For (1) this secondary proposition is not a side-suggestion without formal logical dependence on the primary proposition; as we shall see in the sequel, it is an *immediate inference from* the primary proposition; and (2) we cannot make this immediate inference without two assumptions of fact, not warranted by the form of the proposition. There may *be no* S, no persons having the degree in question; and, whether there are any S or not, there may *be no* P; in our example, graduateship is stated to be *one* necessary qualification, but it is not stated to be the *only* one, and there may be no persons possessing all the qualifications. But, on the assumption that there *are* "graduates" *and* "eligible persons," he who says "No non-graduates are eligible," commits himself logically, though, perhaps, unconsciously, to the statement that "Some graduates are eligible." The latter proposition (Some S is P) is an immediate inference by *inversion* from the former (No not-S is P), as we shall shortly explain. Further, the practically important proposition which follows by immediate inference from "No not-S is P" is "All P is S" ("All eligible persons are graduates").

Exceptive propositions cut off the application of the predicate from a portion of the subject by a word like "unless," "except," "but": "No persons are eligible but graduates," "All members voted for the measure except the Irish members." The general form of the exceptive is "No S [or, all S] except X.S is P,"

where X.S signifies the specified part of the subject which is excepted; and its most natural logical form is "No S [or, all S] other than X.S is P," the logical subject being "S other than X.S." We add a few examples of exclusives and exceptives.

(1) "All men, and men alone, are rational."

$\begin{cases} \text{(a) All men are rational.} \\ \text{(b) No beings who are not men are rational.} \end{cases}$

In this example we have two logically independent propositions, of which only the second is exclusive.

(2) "No one can be learned who is not studious and ambitious, and not always then."

This tells us first,

(*a*) None who are not both studious and ambitious can be learned.

Then, that those who *are* both studious and ambitious sometimes do not succeed.

(*b*) Some who are both studious and ambitious cannot be learned.

The original proposition is equivalent to (*a*) and (*b*) taken together.

(3) "All the planets are beyond the earth's orbit except Venus and Mercury." Formally expressed, this becomes "All the planets other than Venus and Mercury are beyond the earth's orbit."

§ 4. We shall now investigate the translation of the simpler propositions into logical form. The student will find the following suggestions of service.

(*a*) If the true subject of the proposition is not obvious at a glance, we have to ask, *of what* is this statement made,—*what* is being spoken about? The answer to this question will bring out the logical subject of the proposition, which is not always the same as the grammatical subject of the sentence.

(*b*) Having found the subject, we next ask, what is stated about it,—what is the assertion made of it? The

answer to this will bring out the logical predicate, and show whether it is affirmed or denied of the subject. The verb must be changed, if necessary, so as to admit of the predication being made by the present tense of the verb *to be*.

(*c*) Then we have to ask whether this predicate is intended to apply to the whole of the subject,—to every instance of it,—or whether the proposition only intends to commit itself to a statement about "some only" or "some at least." In either of these last cases, the proposition is particular; otherwise it is universal.

Some verbal expressions indicating universality may be mentioned. Words such as *All*, *Every* (*Each*), *Any*, *He who* (*Whoever*), *The*, and (sometimes) *A*, when joined to the Subject, signify an A proposition, just as *No*, *None*, signify an E. Similarly *Always*, *Never*, in the predicate, signify A and E respectively. I is indicated by *Some*, *Certain*, *A few*, *Many*, *Most*, &c., or by *Generally*, *Often*, *Sometimes*, standing in the predicate; O by any of these words with a negative. Some signs of quantity are not free from ambiguity; and this is a point requiring special attention. (1) *All* in a negative proposition means *some*, in common language—that is, "some only"; and logicians usually sacrifice part of the meaning of such a proposition by analysing it into O. Thus "All the metals are not denser than water," or "Not all the metals are denser than water," is made equivalent to "Some metals are not denser than water." Similarly, "All cannot receive this saying," is made equivalent to "Some are not able to receive this saying."[1] A proposition of the form "All S are not P" of course might possibly mean "No S are P," but if so it should have stated its meaning without ambiguity (see ex. 8 below). (2) When *Certain* means a definite individual or group which I have in view, it makes the Subject a singular term (sometimes a singular collective): "*A certain man* encountered him"; "*Certain Greek philosophers* were the founders of Logic."

[1] See ex. 15, below.

In the latter statement the reference is to a definite group, whose work as a whole constitutes the foundation of what we know as Logic; the Subject is therefore a singular collective term. (3) The absence of any sign of quantity generally signifies a universal proposition. This applies specially to proverbs and current sayings. But if there is really any doubt on this point the proposition must be made particular.

We add a series of examples, the treatment of which should be carefully noticed by the student.

(1) "Blessed are the merciful."

(*a*) The statement is made about "merciful ones." (*b*) It is affirmed that they are "blessed." (*c*) This predicate is intended to apply to all of the class. Hence the proposition is of the form SaP, "All merciful ones are blessed."

(2) "Democracy ends in despotism."

This proposition makes an assertion about "Democratic governments," affirms that they are things "ending in despotism," and intends this to apply to every instance of democratic government. Hence the form is SaP, "All democratic governments are things ending in despotism."

(3) "Murder will out."

The proposition speaks of "murders," affirms that they are "sooner or later discovered," and intends this to apply to every instance. Hence SaP, "All murders are discovered sooner or later."

(4) "A little knowledge is a dangerous thing."

This is put in logical form simply by attaching a sign of quantity to the Subject. We may assume that the statement is intended to apply to every case of "a little knowledge"; hence the form is SaP.

(5) "Amongst Englishmen a few great generals are found."

Henceforth we shall distinguish the three points in the logical analysis of a statement in the following order—(*a*) What is the statement made about? (*b*) What is asserted about it? (*c*) Does the assertion apply to part or whole? In this example, we have:—

(*a*) Great generals;
(*b*) found amongst Englishmen;
(*c*) affirmed of part of subject;

hence SiP, "Some great generals are found amongst Englishmen.'

(6) "Old things are not *therefore* the best."

This means that old things are not the best merely because they are old; they may be undesirable for other reasons. This last statement, on the other hand, need not apply to *all* "old things."

(*a*) Old things
(*b*) the best simply because they are old;
(*c*) denied of part of the subject.

Hence SoP, "Some old things are not the best . . ."

(7) "One bad general is better than two good ones."

(*a*) one bad general acting alone
(*b*) better than two good ones failing to act together;
(*c*) affirmed of every instance of the subject.

Hence SaP, "In every instance, one bad general . . . is better than two good ones . . ."

(8) "All that act honourably shall not be forgotten."

This cannot be considered ambiguous; it is evidently SeP, "None who act honourably are among those who shall be forgotten."

"Not *all* your endeavours will succeed." Here "all" serves rather to emphasise "endeavours" than to indicate quantity, and the proposition is SeP, "None of your endeavours will succeed."

"All that glitters is not gold." The primary implication of this proposition is, "some things that glitter are not gold."

(9) The logical subject may consist of a name qualified by one or more sentences. In the following, the logical subject includes all the italicised words: "*No one* is free *who is enslaved by his own desires*" (SeP); "all *the officers who are quartered here* are skilled in peaceful pursuits" (SaP).

(10) "Fine feathers do not make fine birds." Here the contrast is between having "fine feathers" and being "a fine bird"; what is denied is that the two facts are necessarily connected (see p. 57).

(*a*) To have fine feathers
(*b*) the sign of being a fine bird;
(*c*) denied of some instances of the subject.

Hence SoP, "To have fine feathers is sometimes not the sign of being a fine bird."

(11) "Some of the English kings have been worthless."

In order to deal with propositions referring to past time, some logicians propose to turn them into propositions of classification, thus: "Some English kings are in the class constituted by the attribute of worthlessness at the given time." But it is not necessary to be so very cumbrous. It is true that "every act of judgment is a present one and expresses a present belief." But in a proposition referring to past or future time, the truth of the proposition lies in its reference to that point of time; and we may express the meaning formally by putting ourselves at that point of time, and therefore using a proposition whose copula is in the present tense: "Some English kings are worthless." Similarly, "all had fled" may be expressed, "all are persons who have fled."

(12) "Half of his answers are wrong."

Here, if "half" is merely indefinite and means "a good many," the proposition is obviously SiP. If we take it as a numerical statement, strictly definite, it has to be treated as a compound proposition, and part of the meaning sacrificed by analysing it into a pair of particular propositions taken together.

$\begin{cases} \text{(a) Some of his answers are wrong.} \\ \text{(b) Some of his answers are not wrong.[1]} \end{cases}$

(13) Similar considerations apply to propositions whose subjects are qualified by *Few*, *Most*. *Few* may be merely indefinite, implying a few cases that have been observed but not really excluding *all*; and similarly *Most* may not exclude *all*. In such cases both words signify I propositions; in this way they are usually taken in Logic, as noted above. But if the words are taken quite strictly, such propositions as *Few S's are P*, *Most S's are P*, "imply that every instance—or at least an extremely large number of instances—of S have been examined, and that in the one case a number

[1] It is worth noting that the phrase "half of his answers" is not so precise as it looks. It is only *abstractly* precise. If it meant "*this* half" it would be *really* precise, and would be a singular (an A) proposition.

less than half (*Few* S's) and in the other case a number greater than half but less than all (*Most* S's) have been found to be P" (Welton). In this strict sense of the words *Few* and *Most* they each imply a pair of particular propositions taken together :

{(*a*) Some S's are P.
{(*b*) Some S's are not P.

Even here we have to sacrifice part of the meaning by eliminating the reference to "more than half" or "less than half."

(14) Examples 12 and 13 introduce us to *numerically definite* propositions. They may appear to be definite without really being so : "two of them were foreigners" may imply that only two were noticed but that others, or all, may have been foreigners. In such a case the proposition is the logical I. If such a proposition is really definite, it implies that all cases have been observed and that two were foreigners, the others not. Then (though even so we sacrifice some of the meaning) it must be expressed by two particular propositions taken together :

{(*a*) Some were foreigners.
{(*b*) Some were not foreigners.

We must remark, however, that by *changing the whole expression of the proposition*, we may analyse it logically with less sacrifice of meaning. Propositions with a *really definite* numerical expression in the subject may be treated as singular propositions with the numerical expression in the predicate : "The proportion of . . . (or number of . . .) is . . ." Thus, in ex. 12 : "the proportion of wrong answers is one half" ; ex. 13, "The proportion of S's that are P is *few* (or *most*) of them" ; ex. 14, "The number of them who are foreigners is two."

(15) Finally, we may notice that propositions of the type mentioned above, page 63, remark (1), may be *fully* expressed if we turn them into a pair of particular propositions. Thus, "All that glitters is not gold" may be treated as equivalent to—

{(*a*) Some glittering things are not gold ;
{(*b*) Some glittering things are gold.

(*a*) is the primary implication, (*b*) the secondary implication, of the original proposition.

It may be asked whether, in turning "All . . . are not . . ." (or "Not all are . . . ") into (*a*) "Some are not . . . " (*b*) "Some are . . . " we break the rule that "some" includes "possibly all"? The answer is, we do not break the rule; but just because "some," in either of the propositions (*a*) or (*b*) *taken by itself*, does not exclude "all," we take the two *together* and by that means exclude "all" and save the meaning of the original proposition, which expressly speaks of *not all*.

Exercise II.

The following are examples illustrating §§ 3 and 4.
Express the following propositions in logical form:

(1) (*a*) The quality of mercy is not strained.
(*b*) Some have greatness thrust upon them.
(*c*) What is not practicable is not desirable.
(*d*) Hypocrisy delights in the most sublime speculations. [St A.]

(2) (*a*) Many were absent.
(*b*) Any excuse will not suffice.
(*c*) All knowledge is but remembrance.
(*d*) St Andrews is the oldest university in Scotland. [St A.]

(3) (*a*) It is never too late to mend.
(*b*) They also serve who only stand and wait.
(*c*) Only ignorant persons hold such opinions.
(*d*) Few books in Logic are easy reading. [St A.]

(4) (*a*) No admittance here except for officials.
(*b*) The old paths are best.
(*c*) Luck has been known to desert a man.
(*d*) Trespassers are not always prosecuted. [St A.]

(5) (*a*) For every wrong there is a legal remedy.
(*b*) Not every advice is a safe one.
(*c*) The object of war is durable peace
(*d*) Improbable events happen almost every day. [St A.]

(6) (*a*) The longest road comes to an end.
(*b*) Only Protestant princes can sit upon the throne of England.

(*c*) Unasked advice is seldom acceptable.
(*d*) Where no oxen are, the crib is clean. [E.]

(7) (*a*) Knowledge is power.
(*b*) Two wrongs do not make a right.
(*c*) Custom blunts sensibility.
(*d*) More haste, less speed.

(8) (*a*) It is only the bold who are lucky.
(*b*) Those who escape are very few.
(*c*) No one is admitted except on business.
(*d*) It cannot be that none will fail. [C.]

(9) (*a*) Nobody undertook these studies but was incapable of pursuing them successfully.
(*b*) Honesty is not always the easiest policy.
(*c*) One man is as good as another.
(*d*) Nothing succeeds like success.

(10) (*a*) Life is change.
(*b*) Probability is the guide of life.
(*c*) Plants are devoid of the power of movement.
(*d*) There is no limit to the amount of meaning which a term may have.

(11) (*a*) To think is to be full of sorrow.
(*b*) There is none righteous, no, not one.
(*c*) No child ever fails to be troublesome if ill-taught and spoilt.
(*d*) No one can be rich and happy unless he is also prudent and temperate, and not always then. [G.]

(12) (*a*) We smile not only when we are pleased.
(*b*) The price of an article does not always rise immediately after it is taxed.
(*c*) Any one may judge of B's conduct who examines the evidence. [L.]

(13) (*a*) Perfection is unattainable.
(*b*) All but he had fled.
(*c*) Sometimes all our efforts fail.
(*d*) Some of our efforts always fail. [L.]

(14) Express in a single proposition of the simplest logical form the sense of each of the following sentences .—
(1) If the sky were to fall, we should catch larks.
2) It never rains but it pours.

(3) Many are called, but few are chosen.
(4) Unless help arrives, we are beaten.
(5) You cannot eat your cake and have it.
(6) Use every man after his deserts, and who should 'scape whipping? [O.]

(15) Express as adequately as you can in a single proposition of the simplest logical form the sense of each of the following sentences:—

(1) A man may smile and smile and be a villain.
(2) Few men think, but all have opinions.
(3) When clouds appear, wise men put on their cloaks.
(4) Oblige her, and she'll hate you while you live.
(5) Angels are bright still, though the brightest fell. [O.]

Part II.—*Opposition of Propositions.*

§ 5. We have now to examine more closely the meaning and use of the four typical forms of the proposition, A, E, I, O.

We have been treating each of these as affirming or denying *certain attributes* of the whole or part of a subject. This implies that the subject S is taken in its full sense, of both extension and intension,—that it means certain objects identified by the possession of certain qualities; and the predicate P is read in intension only,—it signifies certain other qualities which the judgment attaches to the subject. "Potassium is lighter than water": here the subject stands for a real object or kind of objects, whose qualities we are supposed to know sufficiently to identify it; and the judgment predicates another quality of it,—that it will float on water. This is the natural way of regarding most of our judgments,

and this is apparently the natural meaning of "predication." We might therefore express the judgment, as Aristotle often does, in the form "P is predicated of S." We do not usually think of the predicate as a class or an individual thing, unless we are expressly forming "a judgment of classification," as "the whale is a mammal." This proposition means that the class of animals called whales is included in the class called mammals.

Now every term has *two* sides, extension and intension; hence in every proposition we *may* read the predicate in extension also, and think of P as a separate or wider class in which the class S is included. This is merely a possible way of regarding every proposition; but it is the simplest way when we are dealing with propositions in the manner required by those parts of Logic on which we are now entering. Then the proposition **A** expresses the fact that **the thing or class of things denoted by the subject is included in and forms part of the class denoted by the predicate.** Thus (*a*) "all metals are elements" means, on this interpretation, that the class "metals" is included in the wider class "elements," and (*b*) "all equilateral triangles are equiangular" means that the class equilateral triangles is in the class equiangular triangles, and here we know also, from the matter of the proposition, not from its form, that the former class is identical with the latter. These two possibilities always arise in an A proposition. The mathematician Euler (eighteenth century) invented a method of indicating the extent of the denotation of a term by a circle, which is supposed to include all things denoted by the term and nothing else. In this case the proposition A is represented by one of the two following diagrams.

Fig. 1 represents propositions of which (*a*) is a type, and fig. 2 those of which (*b*) is a type, where the classes

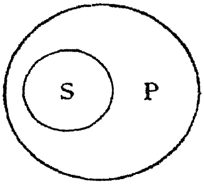

Fig. 1.

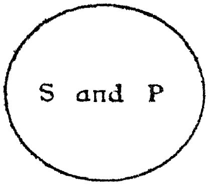

Fig. 2.

S and P coincide. The form of this proposition does not tell us whether they coincide or not; it does not tell us anything of that part of P which is outside S. The subject-matter of any given proposition may show at once which of the two diagrams represents it (usually fig. 1); but the form does not show it; and the student, when operating on an A proposition, is not entitled to assume that either of these diagrams *in preference to the other* represents it.

The proposition **E** expresses the fact that **the class denoted by the subject is altogether outside the class denoted by the predicate.** Thus, "no metals are compounds," means that all the class "metals" is outside of the class "compounds." The proposition is fully represented by the following diagram:—

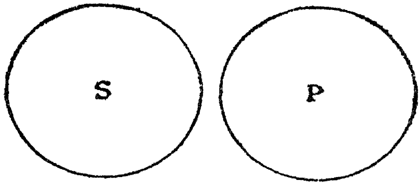

Fig. 3.

Hence this proposition does tell us something about the whole of the predicate as well as the whole of the

subject; if S is wholly outside of P, P must be wholly outside of S.

The proposition I tells us that **some at least of the class S is included in the class P.** There are two principal cases of its possible meaning. (*a*) "Some metals are brittle," means that part of the class "metals" is included in the class "brittle things," but this includes also other things than metals. Hence the diagram is—

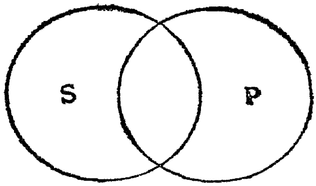

Fig. 4.

Here part of S coincides with part of P. (*b*) "Some Europeans are Frenchmen," means that part of the class "Europeans" *coincides* with the class "Frenchmen" and the diagram is—

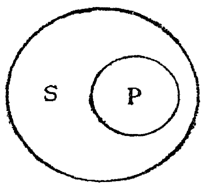

Fig. 5.

where part of S coincides with the whole of P.

We do not know from the form of the proposition whether the predicate signifies the whole or only a part of P. And further, since "some" means "some at

least," we do not know from the form of the proposition whether the whole or part of the subject itself is referred to. In the examples represented by figs. 4 and 5, "some" means only a part, and the propositions are—

(*a*) part of S coincides with part of P,

(*b*) part of S coincides with all of P.

But as far as the mere form of the proposition goes, the two following possibilities are not excluded:

(*c*) all of S coincides with part of P,

(*d*) all of S coincides with all of P.

These are represented by figs. 1 and 2 respectively. Although the student will find from the subject-matter that nearly all propositions, which can be brought to the form I, will be of the type (*a*) or (*b*), the form does not show it; and the form is what we must now attend to.

The proposition **O** tells us that **some at least of the class S fall outside the class P.** Here, again, there are two chief possibilities of meaning, although the distinction does not depend on that of part and whole of P. (*a*) P may be a wider class than S, and S partly outside it, partly within it: "Some metals are not brittle," represented by fig. 4. (*b*) P may be a narrower class than S, and fall entirely within it: "Some Europeans are not Frenchmen," represented by fig. 5. Although any actual instance of an O proposition will be of the same type as one of these examples (*a*) or (*b*), the form of the O proposition does not exclude fig. 3; and, once more, it is the form that we attend to here.

We see, therefore, that the proposition O, like E, tells something of the whole predicate; for if "some S" falls

wholly outside P, P must fall wholly outside *that part* at least of S.

§ 6. A term is said to be **distributed**, when we know merely from the form of the proposition in which it occurs that it refers to every individual of the class. Which terms, then, are known to be distributed in the four propositional forms?

(1) In A, the subject is distributed, as the "all" tells us. But we do not know whether the predicate is taken in its whole extent (as in § 5, fig. 2), or only in part of it (fig. 1); hence the predicate is not distributed.[1]

(2) In E, both subject and predicate are known to be distributed, for the proposition tells us (§ 5, fig. 3) that the whole of S is outside P, and therefore the whole of P must be outside S.

(3) In I, the subject is not known to be distributed as the word "some" tells us; and the predicate is not, for the proposition does not tell us whether it is taken in its whole extent (fig. 5) or in part only (fig. 4).

(4) In O the subject is not known to be distributed, but the predicate is so, for, as we saw (§ 5 *ad finem*), the proposition tells us that the whole of P must fall outside that part of S to which the Subject "some S" refers.

There is no difficulty in remembering the cases in which the Subject is distributed or the reverse, for these are indicated by "all" or "some." As regards the **predicate**, the above table shows that **negatives distribute, affirmatives do not.**

[1] The word "distributed" is always nothing but an abbreviation of the phrase "known from the form of the proposition to be distributed."

§ 7. By the **opposition** of two propositions is meant the extent to which the truth or falsity of one depends on the truth or falsity of the other when they have the same Subject and Predicate. The term "opposition" is used in a technical sense so as to include cases where the statements do not really conflict. It may be defined as the relation of the four propositions to each other, as regards truth or falsity, when they have the same subject and predicate. Now two propositions having the same subject and predicate may differ in both quality and quantity; in quality only; or in quantity only.

(*a*) If they differ in both quantity and quality, then,

(1) one must be universal affirmative, the other particular negative;

or (2) one must be universal negative, the other particular affirmative.

These are the two cases of the most important relation between two propositions. It is called, in both cases, **contradictory opposition.** Of contradictory propositions, **one must be true, and the other false**; in other words, they cannot both be true, and they cannot both be false. For the contradictories are (*a*) SaP, SoP; (*b*) SeP, SiP. If SaP is false, this means that not all the circle S is inside the circle P, therefore, "some at least" of it must be outside P; that is, SoP is true, and *vice versâ.* Similarly we may show that if any one of the four propositions is true, or false, its contradictory is false, or true, accordingly.[1]

[1] Hence contradictory propositions are just sufficient to deny each other. Hence also the contradictory of a *Singular Proposition* is not distinct from its contrary.

(*b*) If the propositions differ in quality only, then,

(1) one must be universal affirmative, the other universal negative;

or (2) one must be particular affirmative, the other particular negative.

(1) In the first case the propositions are called **contraries**, *i.e.*, SaP, SeP. Of contrary propositions, **both cannot be true**, for the whole circle S cannot be at once in the circle P and outside it. But **both may be false**, for the circle S may be partly in the circle P, so that SeP is false, and partly outside it, so that SaP is false. (2) In the second case the propositions are called **sub-contraries**—*i.e.*, SiP, SoP. Of sub-contrary propositions, **both may be true**; for part of the circle S may be in the circle P, so that SiP is true, and part outside it, so that SoP is true. But **both cannot be false**; for if so, the circle S must be all in the circle P, since SoP is false, and at the same time the circle S must be all outside the circle P, since SiP is false.

(*c*) If the propositions differ in quantity only, then,

(1) one must be universal affirmative, the other particular affirmative.

or (2) one must be universal negative, the other particular negative.

In each case the propositions are called **subalterns**—*i.e.*, (1) SaP, SiP; (2) SeP, SoP. Of subaltern propositions, **both may be true**; for the truth of the universal includes the truth of the particular. But if we only know the truth of the particulars—*i.e.*, only know that "at least some S is P," or that "at least some S is not P,"—we do not know whether the respective universals are true or not.

The six relations which we have explained are shown in

a diagram called the **square of opposition**, which would be more accurately called the "square of relation."

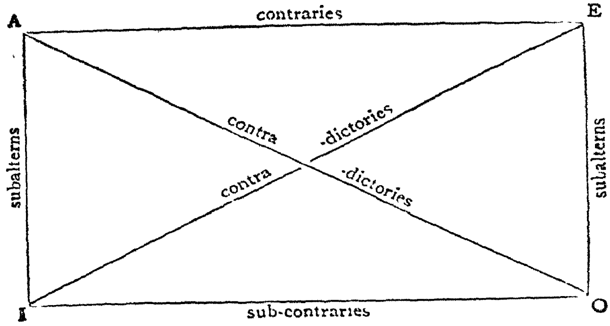

Fig. 6.

The results of this section may be summed up in the following table :—

	A is	E is	I is	O is
If A is true	true	false	true	false.
" E "	false	true	false	true.
" I "	doubtful	false	true	doubtful.
" O "	false	doubtful	doubtful	true.

EXERCISE III.

Give the contradictory of each proposition contained in Exercise II., questions 1 to 13 inclusive.

[Before the contradictory of a proposition can be given, it must of course be expressed in strict logical form.]

The essentials of the doctrine of opposition which we have explained were clearly stated by Aristotle. He says that formally (κατὰ τὴν λέξιν, *An. Prior.*, ii. 15) there are four kinds of opposition :

(*a*) when one asserts of the whole what the other denies of the part (A and O);

(*b*) when one denies of the whole what the other asserts of the part (E and I).

In both these cases the propositions are said to be opposed as contradictories (*ἀντιφατικῶς ἀντικεῖσθαι*). Contradictory propositions admit of no third possibility, and there is no middle way between them. The two other forms of opposition mentioned by Aristotle are:

(*c*) when one proposition affirms of the whole what the other denies of the whole (A and E): in this case they are said to be contraries (*ἐναντίως ἀντικεῖσθαι*), and both may be false.

(*d*) When one affirms of a part what the other denies of a part (I and O). In this case Aristotle says quite truly that the "opposition" is merely verbal.

Part III.—*Immediate Inference.*

§ 8. Immediate Inference is the name given to the process by which, from a single given proposition, we derive another whose truth is implied in the former.[1]

Hence **Opposition is a variety of Immediate Inference**; for from the truth of A we may infer the falsity of E and O, and the truth of **I**: from the truth of E, the falsity of A and I, and the truth of O: from the truth of I, the falsity of E: and from the truth of O, the falsity of A. But the term Immediate Inference is usually restricted to certain formal transformations of which a proposition is capable, and to which Professor Bain has given the name of "equivalent pro-

[1] The process is rather a transformation of the proposition than an addition to our knowledge; but it is more than a merely *verbal* change (see ch. VI. § 1).

positional forms." The name "eductions" has also been proposed.

There are two fundamental processes of eduction: **conversion,** by which we obtain an equivalent proposition in which S and P have changed places; and **obversion,** in which the equivalent has for predicate the contradictory term "non-P" instead of P.[1] All other processes of Immediate Inference, in the proper sense of the term, consist of an **alternate performance of these two elementary operations.** Aristotle recognised only Conversion; for he did not admit the use of the "indefinite name" *non-P* as a Subject or Predicate.

§ 9. The term **conversion,** though sometimes used in a wider sense, is best restricted to signify the process by which from a given proposition we infer another having the subject of the original proposition for its predicate and the predicate of the original proposition for its subject. From a proposition of the type SP we infer an equivalent one of the type PS; no new term, such as S′ (non-S) or P′ (non-P) is introduced.

The rules for conversion follow at once from the meaning of the proposition as we have agreed to accept it. It asserts a relation between two classes. An affirmative proposition states that two classes are wholly or partly coincident (§ 5, figs. 1, 2, 4, 5); a negative proposition, that they are wholly or partly exclusive of each other (figs. 3, 4, 5) In the original proposition, called the **convertend,** this relation is stated from the side of S; in the converted proposition, called the **converse,** the same relation is stated from the side of **P.** Now the relation (of coincidence or exclusion) is

[1] For the future we shall follow a suggestion made by Dr Keynes, and indicate the logical contradictory of any term P by the symbol P′.

the same whether looked at from the side of S or the side of P. If P coincides with S to any extent, to the same extent S must coincide with P; and if P is excluded from S to any extent, S is also excluded to the same extent from P. A glance at the diagrams will make these facts obvious. And as coincidence in the diagram corresponds to affirmation in the proposition, and exclusion to negation, we have the first rule of conversion: **the quality (affirmative or negative) of the original proposition is unchanged in the converse.**

Again, obviously we cannot state in the converse any more than the convertend declares to be known. Apply this principle to the four forms.

(1) "All S is P." When we come to convert this, P and S change places, and P has the sign of quantity instead of S. What quantity must be given to P? This depends on what we know of the quantity of P in the original proposition. Now in an A proposition we do not know, from the form, that P is distributed (§ 6); we only know that some at least of P is referred to. Hence in converting A, we must say "some at least of P is S," or in the logical form, "some P is S." Thus, the converse of "all men are fallible" is "some fallible beings are men." There may be "fallible" beings which are not men; the original proposition tells us nothing as to this.

(2) "Some S is P." Here again we do not know, from the form, whether P is taken in its whole extent, is "distributed," or not; hence we cannot distribute it in the converse, which is "some P is S." The converse of "some men are learned" is "some learned beings are men." Thus A and I have the same converse.

(3) "No S is P." This means that all S is outside P, and therefore all P must be outside S. Both terms

are distributed, and the converse is "no P is S." Converting, "no men are perfect," we get "no perfect beings are men."

(4) "Some S is not P." We saw that this was represented by these diagrams, and that it does not exclude

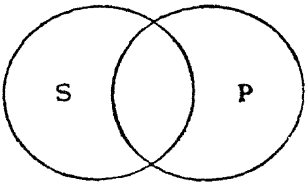

Fig. 7 (*a*).

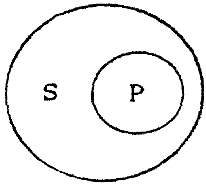

Fig. 7 (*b*).

the diagram for E (§ 5, fig. 3). Now if we transpose S and P, in the proposition O, so that P is quantified and is the subject, and S is unquantified and is the predicate, it will be found that no negative logical proposition of the type P S will satisfy both the above diagrams, to say nothing of the diagram for E. For PoS does not satisfy (*b*), and PeS does not satisfy (*a*). Hence there is no [necessary] formal converse of O. The propositions "some metals are not brittle" and "some brittle things are not metals" are both true *as a matter of fact;* but the latter is not known by mere logical conversion of the former,—it is reached by our knowledge of metals and things which are brittle. There is no logical converse of "some metals are not brittle." Taking examples of another kind,—from "some Europeans are not Frenchmen" we cannot logically infer that "some Frenchmen are not Europeans"; and in this case the attempted converse is not even true as a matter of fact. Similarly, from "some candidates who sit for an examination do not pass it," we cannot infer that "some candidates who pass an examination do not sit for it."

From the four examples just given, we derive the second rule of conversion, **no term must be distributed in the converse which was not known to be distributed in the convertend.**

There are some further aspects of the process of conversion which must not escape the student's attention. In converting I and E we change neither quantity nor quality; the converse of SiP is PiS, and of SeP is PeS. This is called **simple conversion,** to distinguish it from the process which is necessary in converting A. Here, though we do not change the quality, we change the quantity; the converse of SaP is PiS. This is called **conversion by limitation,** the equivalent of the Aristotelian phrase ἀντιστροφὴ κατὰ μέρος (*An. Prior.*, I. 2). The mediæval logicians called it *conversio per accidens.*

The conversion of an A proposition without limitation is a frequent source of fallacy. From "ill-doers are ill-dreaders" (understood as universal) it is easy to slip into the unlimited converse, "ill-dreaders are ill-doers," also understood universally. Similarly, "all beautiful things are agreeable" may be true, but it does not follow that "all agreeable things are beautiful." We may know from the *matter* of the proposition that S and P are coextensive. But the single proposition "all S is P" does not logically express the relation of coincidence or coextension between S and P[1]; to do this, we require the two propositions *together*,

$$\begin{cases} (a) \text{ All S is P.} \\ (b) \text{ All P is S.} \end{cases}$$

The logical converse is thus to be distinguished from the geometrical converse. The geometrical converse is

[1] For another view of this, see Fowler, *Deductive Logic*, ch. ii. § 3 (page 82); and cp. ex. 8 below (page 86).

the simple converse of an A proposition, and it is not logically inferrible from the latter, but has to be proved independently. Thus the geometrical converse of "all equilateral triangles are equiangular" is "all equiangular triangles are equilateral." In every case it will be found that an independent proof is necessary for the geometrical converse. Euclid usually adopts the indirect proof, by *reductio ad absurdum* (as in I. Prop. vi., &c.), in the course of which the truth of the original proposition is appealed to.

We now add some examples illustrating the process of conversion. Every proposition to be converted must first be reduced to strict logical form.

(1) "There is no excellent beauty that hath not some strangeness in the proportion."

(*a*) Excellent beauty
(*b*) [a thing] without strangeness in the proportion;
(*c*) denied of every instance of the subject.[1]

Hence SeP, "No excellent beauty is a thing without strangeness in the proportion."

Converse PeS, "Nothing without strangeness in the proportion is excellent beauty."

(2) "It's a poor centre of a man's actions, himself."

(*a*) A man's self
(*b*) a poor centre of his actions;
(*c*) affirmed of every case of the subject.

Hence SaP, "A man's self in every case is a poor centre of his actions."

Converse PiS, "Something which is a poor centre of a man's actions is himself." Note that "something" is not a singular term.

(3) "Mercy but murders, pardoning [*i.e.*, if it pardons] those that kill."

(*a*) Mercy which pardons those that kill
(*b*) a murderous thing;
(*c*) affirmed of every instance of the subject.

[1] For the proper order of logical analysis see § 4.

Hence SaP, "All mercy which pardons those that kill is murderous."

Converse PiS, "Something murderous is mercy which pardons . . ."

(4) *Non omnis moriar* ["I shall not all die"].

(*a*) Myself;

(*b*) immortality;

(*c*) affirmed of part of the subject.

Hence SiP, "Some part of me is immortal."

Converse PiS, "Something immortal is part of me."

[The original proposition has a secondary implication, "Some part of me is not immortal," which is formally inconvertible unless we express it in the form, "Some part of me is mortal." As we have said, logicians do not usually recognise the secondary implication.[1]]

(5) "'Tis cruelty to load a falling man."

(*a*) To load a falling man;

(*b*) a cruel thing;

(*c*) affirmed of every instance of the subject.

Hence SaP, "In every case to load a falling man is a cruel thing."

Converse PiS, "Something cruel is to load . . ."

(6) "We cannot all command success."

(*a*) We;

(*b*) able to command success;

(*c*) denied of the subject in some cases.

Hence SoP, "Some of us are not able . . ." Formally inconvertible, unless we change it into, "Some of us are unable." [The original proposition has a secondary implication SiP, "Some of us are able to command success," with converse, "Some beings able to command success are ourselves."]

(7) "In man there is nothing great but mind."

This is an exceptive proposition, and formally expressed becomes "Nothing that is not mind is great in man."

Here we have—

(*a*) what is not mind;

(*b*) a thing great in man;

(*c*) denied of the whole subject.

---

[1] Cf. § 4, ex. 15.

Hence SeP, with converse PeS "nothing great in man is other than mind."

(8) "In any case he was not the only one who said so."

Here the phrase "in any case" indicates that the fact that "he said so" is questionable, but that this question is waived; the emphatic assertion is, that "others beside him said so."

(*a*) others beside him;
(*b*) persons who said so;
(*c*) affirmed of part of the subject.

Hence SiP, "Some others beside him are persons who said so," with converse PiS, "Some who said so are others than he."

Singular propositions have been classed as universals, and have to be treated accordingly. "Brutus killed Cæsar": this is converted into, "Some one who killed Cæsar was Brutus"; "St Andrews is an old university," converse, "Some one of the old universities is St Andrews"; "Britain is an island," converse, "Some one of the islands is Britain." If both subject and predicate are singular terms, the proposition may be converted simply: "St Andrews is the oldest university in Scotland," converse, "The oldest university in Scotland is St Andrews" In case of **impersonal propositions** we have, in elementary Logic, simply to introduce a subject; thus "It rains" is in logical form "The atmosphere is letting rain fall," with converse, "Something letting rain fall is the atmosphere."

EXERCISE IV.

Give, where possible, the logical converse of each of the propositions referred to in Ex. III.

§ 10. The process called **obversion** consists in passing from an affirmative proposition to a negative statement of the same truth, and *vice versâ*. The rule is, **change the quality of the proposition and substitute for the predicate its logical contradictory.** Thus:—

Original Propositions.	*Obverses.*
All men are fallible.	No men are "non-fallible."
No men are perfect.	All men are "non-perfect."
Some men are learned.	Some men are not "non-learned."
Some men are not trustworthy.	Some men are "non-trustworthy."

In general terms, we obvert the proposition "S is P" by substituting P′ for P and changing "is" to "is not," or "is not" to "is," as the case may be. Thus :—

A. All S is P,	becomes	E. No S is P′.
E. No S is P,	"	A. All S is P′.
I. Some S is P,	"	O. Some S is not P′
O. Some S is not P,	"	I. Some S is P′.

In obversion it is desirable, to secure neatness in the logical form, to substitute a single term for the contradictory P′, if such a term exists. It will, of course, be one of the *negative terms* of ordinary language. In the four examples given above we may substitute "infallible," "imperfect," "unlearned," "untrustworthy," each of which is general enough in meaning to stand as the pure contradictory of the corresponding positive term. But the error of using a contrary instead of a contradictory must be guarded against—*e.g.*, if the predicate P were "happy," then "unhappy" would not be a true contradictory but a contrary, signifying a definite real quality, which is the opposite of "happy."

That "obversion" produces a really equivalent proposition is evident from the diagrams, if we remember that affirmation corresponds to inclusion, and negation

to exclusion.[1] Thus, if all the circle S is in the circle P, obviously none of S can be outside P—*i.e.*, SaP is equivalent to SeP′; if no S is in P, all S must be outside P—*i.e.*, SeP and SaP′ are equivalent; if some S is in P, then that part of S is not outside P—*i.e.*, SiP and SoP′ are equivalent; if some S is not in P, that part of S is outside P—*i.e.*, SoP is equivalent to SiP.

In obverting propositions, we must try to make the logical forms as neat—or at least as little removed from the common usages of speech—as possible; and to avoid using terms of the form "non-P" when there is a more familiar expression *with the same meaning*. Frequently the phrase "other than P" may be used with advantage. Obversion may produce exceedingly cumbrous and uncouth forms, but with a little care this result may be avoided.

(1) "Some of our muscles act without volition."
(*a*) our muscles;
(*b*) things which act without volition;
(*c*) affirmed of part of the subject.

Hence SiP, "Some of our muscles are things which act without volition." To obvert we substitute "are not" for "are," and take the contradictory of the predicate. Formally, this contradictory is, "not things which act without volition"; and this is exactly equivalent to "things which act with volition." Hence the neatest form of the obverse is, "Some of our muscles are not things which act with volition."

(2) "Every mistake is not a proof of ignorance."
(*a*) mistakes;
(*b*) a proof of ignorance;
(*c*) denied of some of the subject.

Hence SoP, "Some mistakes are not proofs of ignorance." Obvert by substituting "are" for "are not" and taking the contradictory of the predicate, which is "other than proofs

[1] The logical contradictory of a term P is represented by the indefinite region *outside* the circle which stands for P.

of ignorance,"—"some mistakes are other than proofs of ignorance."

[The original proposition has a secondary implication, "some mistakes are proofs of ignorance," with obverse, "some mistakes are not other than proofs of ignorance."]

(3) "No one is free who cannot command himself."

(*a*) those who cannot command themselves;

(*b*) free;

(*c*) denied of the whole of the subject.

Hence SeP, "None of those who cannot command themselves are free." Here the most convenient contradictory of "free" is the negative term "unfree"; and the obverse is "all who cannot command themselves are unfree," SaP′.

(4) "A man's a man."

(*a*) a human being;

(*b*) a being with the capacities and rights of manhood;

(*c*) affirmed of every instance of the subject.

Hence SaP, "All human beings are beings with the capacities and rights of manhood"; obverse SeP′, "no human beings are other than beings having the capacities and rights of manhood."

(5) "Britain is an island."

This is a singular proposition, and therefore SaP. The obverse is SeP′; "Britain is not other than an island."

(6) "Romulus and Remus were twins."

(*a*) Romulus and Remus (a singular collective term);

(*b*) twins;

(*c*) affirmed of the whole subject.

Hence SaP, "Romulus and Remus are twins," with obverse SeP′, "Romulus and Remus are not other than twins." (Cp. § 4, ex. 11.)

Exercise V.

Give the obverse of each of the propositions referred to in Ex. III.

Before passing from this subject we must add a note on the so-called "geometrical obverse." The geometrical obverse of "All S is P" is "No non-S is P," which

is not logically inferrible from the former, and requires independent proof. It is true whenever the classes of figures signified by S and P are coextensive, as in § 5, fig. 2.

§ 11. Other processes, of genuine Immediate Inference, consist in combining Conversion and Obversion. We shall examine two such processes,—Contraposition and Inversion.

Contraposition is the process by which from a given proposition we infer another proposition having the contradictory of the original predicate for its subject, and the original subject for its predicate. In other words, we pass from a proposition of the type S P to another of the type Non-P S, to a proposition giving us direct information about Non-P.

As before indicated, Contraposition is a compound operation, involving the two simple operations already described. To reach the contrapositive the rule is, **first obvert the original proposition, and then convert the proposition thus obtained.** The following table exhibits the steps and indicates the result in the case of the four propositional forms :—

	Original Proposition.	*Obverse.*		*~~Converse of~~ Obverse = Contrapositive.*	
A.	All S is P.	No S is P′.	E.	No P′ is S.	E.
E.	No S is P.	All S is P′.	A.	Some P′ is S.	I.
I.	Some S is P.	Some S is not P′.	O.	None.	
O.	Some S is not P.	Some S is P′.	I.	Some P′ is S.	I.

If the previous real examples be taken, "All men are fallible" yields as its contrapositive "No non-fallible beings are men": "No men are perfect" yields "Some non-perfect beings are men"; "Some men are learned" yields no result, because its obverse is an O proposition and cannot be converted; "Some men are not trustworthy" yields "Some non-trustworthy beings are men."

Jevons describes this method of inference, but apparently supposes that it is only applicable to the A proposition. But he describes precisely the same process as applied to the O proposition, calling it in this case, however, Conversion by Negation. Conversion by Negation is not a variety of Conversion as accurately defined; it is simply another and an undesirable name for Contraposition. And as is seen in the above table, Contraposition is a process applicable not only to the A and the O, but also to the E proposition; in the I proposition alone it yields no result.

The process of obversion may of course be applied to the converse and to the contrapositive of a proposition: the student will find, for example, that the **obverted contrapositive** of SaP is P′aS′, of SeP is P′oS′, and of SoP is P′oS′.

The following are examples of contraposition.

(1) "All that glitters is not gold."

Primary implication, SoP, "Some glittering things are not golden."

Obverse, SiP′, "Some glittering things are non-golden."

Contrapositive, P′iS, "Some non-golden things are glittering things."

[The proposition has a secondary implication, SiP, "Some glittering things are golden."

Obverse, SoP′, "Some glittering things are not other than golden (or, are not non-golden)."

Contrapositive, *none.*]

(2) "Natives alone can stand the climates of Africa."

This is SeP, "None other than natives are able to stand . . ."

Obverse, SaP', "All, other than natives, are unable . . ."

Contrapositive, P'iS, "Some, who are unable . . ., are other than natives."

(3) "He jests at scars who never felt a wound."

This is SaP, "All who never felt a wound are jesters at scars."

Obverse, SeP', "None of those who never felt a wound are other than jesters at scars."

Contrapositive, P'eS, "None, other than jesters at scars, are people who never felt a wound."

EXERCISE VI.

Give, where possible, the contrapositive of each of the propositions referred to in Ex. III.

§ 12. **Inversion** is the name given by Dr Keynes to the process by which, from a given proposition, we infer **an equivalent one having the same predicate but for its subject the contradictory of the original subject.**

In Conversion we have asked, given a proposition SP, what information we can derive from it about P; in Contraposition we have asked, in the same case, what information is derivable about non-P; in Inversion we now proceed to ask what information is derivable, from such a proposition, about non-S.

The processes of obversion and conversion are the only instruments at our command. Starting with the given proposition, we apply them alternately till we either reach the required result (a proposition with non-S in the subject place), or are brought to a standstill by a proposition which cannot be converted. In doing so, we may begin either with Obversion or Conversion. It will be

found that an inverse is obtainable only when the original proposition is universal. From A (all S is P), by applying successively Obversion, Conversion, Obversion, Conversion, Obversion, we obtain O (Some non-S is not P). From E (No S is P), by applying Conversion, Obversion, Conversion, we obtain I (Some non-S is P).

The student should verify these results.

The results of §§ 9 to 12 are summed up in the following table:—

	A.	*E.*	*I.*	*O.*
Original Proposition .	SaP	SeP	SiP	SoP.
Converse . . .	PiS	PeS	PiS	none.
Obverse . . .	SeP′	SaP′	SoP′	SiP′.
Contrapositive . .	P′eS	P′iS	none	P′iS.
Inverse . . .	S′oP	S′iP	none	none.

§ 13. A note may be added on the subjects of—
"Immediate Inference by added determinants,"
"Immediate Inference by complex conception,"
and "Immediate Inference by converse relation."

The first-mentioned process consists in adding the same "determinant" or qualification to the subject and the predicate of the original proposition. If it be true that "S is P," then it follows that "AS is AP"; or, in Jevons's example, if "a comet is a material body," then "a visible comet is a visible material body." Provided that the qualification added to the predicate is in all respects the same as that added to the subject, the truth of the new proposition follows necessarily from the truth of the original, just as the same quantity introduced on both sides of an algebraic equation does not affect the relation of equality. But in dealing with significant terms it is necessary to guard carefully against the

ambiguity of language, as is seen in the two instances given by Jevons :—

"All kings are men," therefore "All incompetent kings are incompetent men."

"A cottage is a building," therefore "A huge cottage is a huge building."

The fallacy is due, in such cases, to the fact that a determinant which is intended to specify the subject (S) alone, is applied in the predicate to the whole of the class (P), of which the subject forms only a part. The determinant is, therefore, not the same in the two cases, inasmuch as its reference or application is different. If the phraseology is so guarded as to maintain the identity of reference, the validity of the inference cannot be challenged, whatever may be thought of its usefulness. The inferred propositions, in the two examples given, would then require to be read, "All incompetent kings are men who are incompetent as kings"; "A huge cottage is a building which is huge for a cottage."

Immediate Inference by Complex Conception is a process essentially similar; it is subject to the same danger from verbal ambiguity, and is valid under the same precautions. The process consists in employing the subject and predicate of the original proposition as parts of a more complex conception—*e.g.*, "A horse is a quadruped," therefore "The head of a horse is the head of a quadruped." But from "All Protestants are Christians," we cannot infer that "A majority of Protestants are a majority of Christians," but only that they constitute a majority of Protestant Christians.

Immediate Inference by converse relation is the name given by Dr Keynes to a process by which, from a statement of the relation in which P stands to Q, we

pass to a statement of the relation in which Q consequently stands to P. Thus, from "P is greater than Q," we infer immediately, "Q is less than P"; from "A is older than B," "B is younger than A"; from "A is the father of B," "B is the child of A"; from "X is equal to Y," "Y is equal to X"; and so on. The two terms of the original proposition are transposed, and **the word by which their relation is expressed is replaced by its correlative.**

EXERCISE VII.

The following questions, of an elementary character, will be found useful for purposes of revision. They refer to the subjects of chapters I. to III.

1. How would you describe the purpose of the Science of Logic?
2. What is meant by saying that Logic deals with the "Form of Thought"?
3. What were the chief contributions to Logic made (*a*) by the Sophists, (*b*) by Socrates?
4. Briefly describe the debt of logical science to Aristotle.
5. Account for the division of Logic into two main branches, "Deduction" and "Induction."
6. What is meant by "Judgment," "Proposition," "Term," "Name," "Subject," "Predicate"?
7. How far is it correct to say that Logic is concerned with Language?
8. What kinds of words may a logical Term consist of?
9. Explain the difference between a Singular and a Common Term.

How may you know a Singular Term when you see it?

10. Explain the difference between a Common and a Collective Name.
11. Distinguish the Collective and Distributive use of the word *all* in the following :—

(*a*) *Non omnis moriar* (*i.e.*, I shall not all die).

(*b*) "All men find their own in all men's good,
And all men join in noble brotherhood."
—(*Tennyson.*)

(*c*) *Non omnia possumus omnes* (*i.e.*, we cannot all do all things). [Jevons.]

12. Which of the following are usually abstract names?—
Act, ingratitude, home, hourly, homeliness, introduction, individuality, truth, true, trueness, yellow, yellowness, childhood, book, blue, intention, reason, rationality, reasonableness. [Jevons.]

13. How are Positive and Negative names distinguished?

14. Carefully explain the different ways in which Contrary and Contradictory names have been distinguished.

15. Distinguish in the following list the terms which are usually (*a*) Singular, (*b*) Common, (*c*) Collective. If a term may belong to more than one class, explain and illustrate its various uses :—
Niagara Falls, gold, chair, a pack of cards, an oak tree, a dancing party, the United States, city, the United States Navy, Brooklyn Bridge, humanity, the centre of the earth. [Creighton.]

16. Give twelve pairs of correlative names.

17. What are "ambiguous words"? Why cannot we make a separate class of "ambiguous terms"?

18. Distinguish between the "connotation" and "denotation" of a name, and mention some synonyms of these words.

19. In what sense may it be said that all terms are general terms? (Ch. II. § 6; only with respect to its *connotation* can we say that *every* term is "general" or "universal"; even a singular term is singular with respect to its denotation rather than its connotation; its connotation *must* be general.)

20. Illustrate the formation of a *concept* (idea of a class).

21. How far is it true that connotation and denotation "vary inversely"?

22. Form a series of at least six terms which may be arranged so as to gradually increase in denotation.

23. What may be said in reply to Mill's contention that Proper Names have "no connotation"?

24. Distinguish some possible different meanings of "connotation" (ch. II. § 7).

25. What is meant by a "Law of Nature" and by a "Law of Thought"?

26. How does a Science differ from an Art? Why is Logic more a Science than an Art?

27. Explain concisely the logical significance of the laws of Identity, Contradiction, and Excluded Middle.

28. Explain and illustrate the terms "Categorical" and "Conditional" as applied to propositions.

29. Name the four kinds of categorical propositions, and their symbols.

Under which classes are singular and indefinite propositions placed, and why?

30. Enumerate the most usual signs of the *quantity* of a proposition (see p. 63). [Jevons.]

31. What is meant by the *distribution* of terms in a proposition?

State precisely what is asserted by the particular affirmative proposition. What forms may the diagrams which represent this proposition assume? [Creighton.]

32. What propositions are true, false, or doubtful, (*a*) when A is false, (*b*) when E is false, (*c*) when I is false, (*d*) when O is false?

33. What quantity would you assign to each of the following propositions?—(*a*) Knowledge is power; (*b*) Nebulæ are material bodies; (*c*) Light is the vibration of an ether; (*d*) Men are more to be trusted than we think; (*e*) the Chinese are industrious. [Jevons.]

34. Why is it desirable in controversy to refute a statement by its contradictory and not by its contrary? (see pp. 45, 76).

35. Express each of the following propositions in logical form, and give, where possible, the obverse, converse, and contrapositive of each:—

(*a*) Fixed stars are self-luminous.
(*b*) Not every mistake is a proof of ignorance.
(*c*) Some of the most valuable books are seldom read.
(*d*) Few are acquainted with themselves.
(*e*) Only the honest are respected
(*f*) No work means no wages.

36. What is *inversion*, and how is it effected? To what kinds of proposition is it inapplicable, and why?

37. Show how to get the Converse of the Contrary of the Contradictory of the proposition "Some crystals are cubes." How is it related to the original proposition? [L.]

* Exercise VIII.

(On the subjects of Ch. III.)

(1) State explicitly which of the following meanings must be assigned to the mark of quantity "some" in the Aristotelian system: *some only; some, perhaps none; some, it may be all or none; some certainly, and it may be all.* Point out the difficulties which arise from an erroneous interpretation of this word. [L.]

(2) Express by means of ordinary categorical propositions the relation between S and P represented by the following diagram.

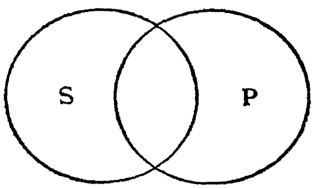

Fig. 8.

(3) All crystals are solids.
Some solids are not crystals.
Some non-crystals are not solids.
No crystals are not solids.
Some solids are crystals.
Some non-solids are not crystals.
All solids are crystals.

Assign the logical relation, if any, between each of these propositions and the first of them. [L.]

(4) Take the proposition "All sciences are useful," and determine precisely what it affirms, what it denies, and what it leaves doubtful, concerning the relations of the terms "science" and "useful thing." [L.]

(5) Give the obverted converse of:—

(*a*) Every truthful man is trusted.

(*b*) No cultivated district is uninhabited.

(*c*) Some British subjects are dishonest.[1]

Give the obverted contrapositive of:—

(*d*) Every poison is capable of destroying life.

(*e*) No idle person is deserving of success.

(*f*) Some unjust laws are not repealed.

Give the obverted inverse of:—

(*g*) Every truthful man is trusted.

(*h*) No unjust act is worthy of praise. [Welton.]

(6) "A St Bernard dog is certainly a dog; but a small St Bernard dog is not a small dog." Comment on this.

(7) What is the logical relation, if any, between each of the following pairs of statements:

(*a*) Heat expands bodies; cold contracts them.

(*b*) "A false balance is an abomination to the Lord; but a just weight is His delight."

(*c*) He that is not against us is for us; he that is not for us is against us.

(8) "To live well is better than to live; hence not to live is better than to live badly." Examine this. [L.]

(9) "Some political organisations ought to be condemned." Can you upon any principle draw the inference "Some political organisations ought to be commended"? [E.]

(10) "Everything which has come into being has a beginning; therefore what has not come into being has not a beginning." Is this a valid Immediate Inference? [St A.]

(11) What can you infer (*a*) from the truth, (*b*) from the falsehood of the proposition "We cannot all do all things" (*non omnia possumus omnes*)?

(12) Is it the same thing to affirm the falsity of the proposition "Some S is P," and to affirm the truth of the proposition "Some S is not P"? Give your reasons. [E.]

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[1] The term "alien" may be taken as the logical contradictory of "British subject."

CHAPTER IV.

THE IMPORT OF PROPOSITIONS AND JUDGMENTS.

§ 1. THE question of the **import of propositions** is this: what kind of relation between subject and predicate do logical propositions express, when stated in one of the four forms A, E, I, O?

Throughout the last chapter we have been dealing with Subject and Predicate as representing classes, which is the simplest way to regard them when studying Opposition, Immediate Inference, and the syllogistic forms to be described in the sequel. There are, however, **four possibilities**,[1] since both Subject and Predicate may be read in Intension or Extension. Take the proposition "Man is mortal." This may be interpreted in four ways—

(1) **Subject in extension, predicate in intension,**
"All the class *men* have the attributes of *mortality*."

(2) **Subject and predicate in extension,**
"The class *man* is included in the class *mortal beings*."

(3) **Subject and predicate in intension,**
"The attributes signified by *humanity* are always accompanied by those of *mortality*."

[1] These four numerically possible cases are arrived at in a purely arithmetical and external way. The fourth case was added by Dr Keynes.

(4) **Subject in intension, predicate in extension,**

"The attributes signified by *humanity* indicate the presence of an object belonging to the class *mortal beings*."

The fourth interpretation is of limited importance. In such a proposition as "Some glittering things are not golden," we have an instance which naturally falls into this division, as it means that the attribute "glittering" does not always indicate the presence of a golden object. Similarly we may interpret "No plants with opposite leaves are orchids." In fact, all such "judgments of identification," in Natural History and elsewhere, are of this type. But it is most unnatural to force our ordinary propositions into this form.

The first three interpretations are of great importance, and we will examine them in turn.

§ 2. The oldest view is the first, according to which the proposition expresses the relation of **subject and attribute**, or, in grammatical terms, of substantive and adjective. The subject of the proposition is read in extension, because it signifies what we call a "real thing" or a group of such;[1] the predicate is read in intension, because it signifies certain qualities which are predicated of the thing. On this interpretation of the proposition, only the subject can have the sign of quantity, "all" or "some," for only the subject refers to a "thing" or "things." Hence this gives the fourfold division of propositions A, E, I, O. This classification fits the diagrams so badly (see ch. III. § 5) because they naturally required the predicate also to be "quantified."

[1] Notice that the subject also implies intension, because it must signify certain qualities by which we identify the thing referred to.

This first interpretation of propositions is called the **predicative view.** The second, which we have already explained (ch. III. § 5), is called the **class view.** Both S and P are regarded as names of classes or groups of individuals, one of which is wholly or partly included in or excluded from the other by the proposition. In order to represent these class relations properly, the predicate should be "quantified" for the same reason as the subject, for both are taken in extension. Representing the classes S and P by circles, we find that the possible relations (of inclusion and exclusion) between them are *five* and five only :—

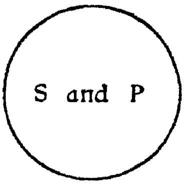

Fig. 9.

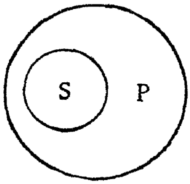

Fig. 10.

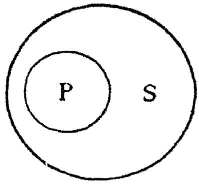

Fig. 11.

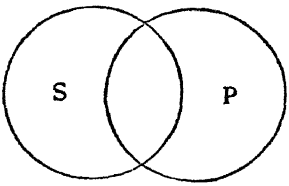

Fig. 12.

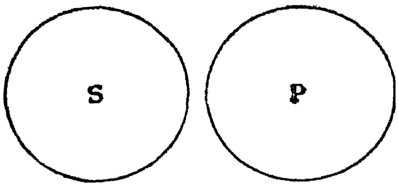

Fig. 13.

Now if we assume that "some" shall be strictly limited to its colloquial meaning of "some only," "some but not all," then each of these diagrams may be expressed fully and without ambiguity by a single proposition, if we quantify the predicate.[1]

(1) "All S is all P" represents fig. 9.
(2) "All S is some P" " fig. 10.

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[1] The doctrine called by Hamilton "Quantification of the Predicate" will be explained and criticised below.

(3) "Some S is all P" represents fig. 11.
(4) "Some S is some P" " fig. 12.
(5) "No S is any P" " fig. 13.

In the ordinary fourfold division the predicate is not quantified, and we are forbidden to treat *some* as expressly excluding *all*. This is the reason why the reconversion of an A proposition leads to a sacrifice of part of what we know :—

(*a*) All S is P.
(*b*) Some P is S, converse of (*a*).
(*c*) Some S is P, converse of (*b*).

In (*b*) the predicate S is in fact distributed, as we know from (*a*), but we cannot indicate this by any sign of quantity. And when converting (*b*), we cannot consider more than the form of the proposition, and this does not warrant us in taking S in its whole extent.

We have seen that the class view is a possible way of regarding any proposition, but that it is not always the natural interpretation; for it is only in what are expressly judgments of classification that we think of the predicate as a class. In most propositions we think of the predicate as adjectival, according to the predicative view. Moreover, no mere class-interpretation of propositions could be entirely true, because **extension and intension cannot be completely separated.** The only way of distinguishing or identifying a class in thought is by some of its qualities, which must therefore enter into the signification of the terms standing as subject and predicate. Hence these terms cannot be taken in extension only; in reading them in extension we must have a reference to intension.

We adopt the class view in treating of Immediate Inference and Syllogism, because that theory has sufficient truth to *work* for the purpose to which it is applied. The whole doctrine of Immediate Infeience and Syllogism may be stated in terms of the first, of the second, and of the third

views of the proposition; but the second view simplifies those doctrines so much that there need be no hesitation in keeping to it.

If, however, it is insisted that the proposition shall be rigidly interpreted *in extension only*, the result is to turn it into a form of words which states nothing. This result is reached in two steps. (*a*) It is not sufficient to say that "All S is some P," unless we specify that the S-part of P alone is meant, for on the class view, as the diagrams show, the copula "is" means "is identical with," "coincides with." In saying that "All men are some mortals," we should specify what "some" is meant; "some" stands for the *human* part of "mortals." Hence, looking simply at the side of extension, we get the form, "All men are men-mortals," not merely "some mortals"; *i.e.*, the form "S is SP." (*b*) In such a proposition, the one side differs from the other only by the addition of P. But if this constitutes a real difference, we must add P to the first side also, in order that the copula may still mean "is identical with" or "coincides with"; that is, we must say SP is SP, "Mortal men are mortal men," which is a proposition telling us nothing. To make the terms S and P signify their extension without their intension, is to make "S is P" into a form of words which says nothing.[1]

The five forms, to which the class view naturally leads, are further removed from the meaning of our ordinary judgments than the traditional four forms, even when the latter are also interpreted by the class view. For in common thought we frequently do not know whether the whole extent of the predicate is to be referred to or not; but the fivefold division supposes us to know in every case whether

[1] The "Identity-theory" of the Judgment will be further considered in ch. XI. § 2.

all or only part of the predicate is referred to. Hence, when adopting the class view, we adapt it to the four forms, as in the previous chapter.

§ 3. The **attributive view** is supported by J. S. Mill. He admits that it is natural to construe the subject in extension and the predicate in intension (as in the predicative interpretation); but he points out, what we have already seen, that the extension of a term, the class denoted by it, can be distinguished only through the attributes. A class is not made by drawing a line round a given number of individuals; it consists of the individuals which are found to have the attributes signified by a given name. When we say "All men are mortal," we do not mean that this attribute is possessed by a particular group of individuals that we have in view; we mean that the attribute is possessed by any individual possessing certain other attributes,—those of "humanity." All this is quite sound. But on this ground Mill holds that in interpreting the proposition we may drop the reference to "things" (the side of *extension*), and regard the proposition as giving evidence only about the "concomitance" of attributes: "Whatever has the attribute humanity has the attribute mortality," or "Mortality always accompanies the attribute humanity." Mill's theory of Science is, that it consists in finding when certain attributes become evidence of certain others; to establish such concomitances is the object of Science.

Propositions, so regarded, must be interpreted thus:

in A, "The attributes signified by S are *always* accompanied by those signified by P";
in E, for "always" substitute "never";
in I, " " "sometimes";
in O, " " "sometimes not."

On this scheme we must observe that though Mill proposes to drop the reference to "things," he is obliged to introduce it again in other words. The words "always," "sometimes," &c., take us at once to instances to which the name is applicable, to the objects in which the intension is *realised*—*i.e.*, to the side of extension. Just as propositions cannot be read in extension merely, without any reference to attributes, so they cannot be read in intension merely, without any reference to objects. In particular, it is not true to our thinking to interpret the subject in intension only. Nevertheless the attributive view is a *possible* way of regarding propositions, for certain purposes.

On the whole, then, we have justified the predicative view as an interpretation of ordinary propositions. "In saying, 'birds are warm-blooded,' we neither think of class within class, nor of attribute with attribute. The word 'warm-blooded' presents to us no conception of a *genus*; it is not a name, but a mere attributive. The word 'bird' expresses to us no *attribute as such*; it is not a mere attributive, but a name. The term in the predicate acts upon the mind by its connotation, or in its comprehension; the term in the subject, by its denotation, or in its extension; and the foregoing sentence has its import in this,—that we refer the attribute 'warm-blood' to the class of objects 'birds.' Hence it is that, while a purely connotative word (an adjective) is all that is required in the predicate, a denotative term is indispensable in the subject. For 'the horse is a quadruped' you can substitute 'the horse is four-footed'; but the attempt to cut down the proposition to a coexistence of attributes does not succeed,—'equine is four-footed.' The mind predicates nothing except about substantive objects of thought; and of

them, in the class of propositions now under consideration, it predicates nothing but attributes" (Martineau, *Essays*, vol. iii. p. 435). But, as Martineau shows, our propositions sometimes express relations which are not attributive in the strict sense, and which cannot be put in that form without much artificial manipulation. He therefore proposes to add to the predicative form of the proposition, as co-ordinate with it, other forms embodying relations of time and space, as, "King John ruled after his brother," or "Fort William lies west of Ben Nevis"; of cause and effect, as, "Friction causes heat"; of resemblance and difference, as, "This doctrine is like that of Herbert Spencer," "That sound is like thunder."

§ 4. The question which we now proceed to raise has been answered by implication in the discussions of §§ 2 and 3; but it is of such importance as to require independent treatment. Is the relation, expressed in the proposition, a relation between words only, or between ideas, or between things?

No one is likely to assert the first. If the proposition were said to express a "relation between two names," all that could be meant is that it expressed a relation between the ideas signified by the names. Every name must stand for some kind of meaning, or it would never be used. But writers who, like Hamilton, take the conceptualist view of Logic—*i.e.*, try to keep Logic within a "world of ideas" without any outlook upon the facts, insist that the proposition asserts a relation between "ideas" only.

Now every proposition expresses a judgment which is an idea of mine, in the sense of being a function of my mind, a mental act of thought. But it is perfectly obvious that what is asserted is not a relation

between *my* idea S and *my* idea P; what is asserted is an objective relation among facts, a relation which does not depend upon my ideas for its existence. The subject-matter of every intelligent proposition belongs to some sphere, region, or "world," so to speak; and the proposition refers to this "world" and assumes its reality. It is not always the "real world" in the ordinary sense, the world of men and things outside us, that our propositions refer to; it may be a mere matter of thought, something "unreal" or even impossible. And the speaker may know that it is an "unreal" world; but as long as it is a systematic world, true judgments concerning any part of it are possible—*e.g.*, "In *Ivanhoe*, the hero does not really marry Rebecca, as Thackeray (when quoting Scott) falsely makes him do." Here we have a reference to a world which is all fictitious, and yet is an *objective system:* "objective," because it is independent and permanent as compared with my fluctuating thoughts about it; "system" because it is a world of *inter-related* parts, so that we may make true or false statements about things in it. This is the test, so to say, of such a world; it must be sufficiently coherent for truth and error to be possible in statements about any part of it. And every rational statement that we make has reference to a "world" of this kind

There are in fact many kinds of "worlds." There is the real world, of common sense and practical life; there is the world of scientific knowledge,—the world described in treatises on Physics, Chemistry, Astronomy, &c.; there are the worlds of philosophical, religious, or ethical theories; the worlds of deliberate romance or fiction; the worlds of individual opinion. The great difference between the first of these, the world which we consider to be "real" *par excellence*, and all the rest, is, that the former comes home to

us in perception and feeling. The other "worlds" come to us as works of thought or works of imagination.

Owing to the great importance of grasping what is meant by the "reference to reality" in a judgment, we will quote Prof. Minto's statement of the same conclusion which we have set forth. "Take a number of propositions: 'The streets are wet'; 'George has blue eyes'; 'The Earth goes round the Sun'; 'Two and two make four.' Obviously, in any of these propositions, there is a reference beyond the conceptions in the speaker's mind. . . . They express beliefs about things and relations among things *in rerum natura:* when any one understands them and gives his assent to them, he never stops to think of the speaker's state of mind, but of what the words represent. When states of mind are spoken of, as when we say that our ideas are confused, or that a man's conception of duty influences his conduct, those states of mind are viewed as objective facts in the world of realities. Even when we speak of things which have, in a sense, no reality, as when we say that a centaur is a combination of man and horse, or that centaurs were fabled to live in the vales of Thessaly, . . . we pass at once to the objective reference of the words [to the world of Greek mythology]."[1]

* The question of the reference to reality expressed by a proposition has been elaborately discussed, in another shape, particularly by Dr Keynes and Dr Venn, who lay the chief emphasis on two questions to be raised about any proposition: In what sense does it imply the existence of its Subject? In what sense does it imply the existence of its Predicate?

[1] The philosophical aspects of the "reference to reality" in Judgment will be further considered in ch. XI. § 4.

These questions constitute the problem known as "the existential import of propositions," as investigated, for example, in Keynes' *Formal Logic* and Venn's *Symbolic Logic*. From the nature of the case, these questions are discussed only with reference to the subjects and predicates as they appear in the forms of the fourfold classification of propositions (A, E, I, O).

* The writers lay great stress on the meaning of "existence," as signifying part of some "world" which is coherent enough for truth and error to be possible in statements referring to it. Such a "world" is called a "universe of discourse,"—a sphere of reference. We have seen the importance of this conception; but the question regarding the separate existence of the subjects and predicates as they appear in the traditional forms of the proposition, is altogether of minor importance. The reason is, that the traditional classification is faulty in several respects, and partly conceals the true nature of Judgment.[1] It is simply a convenient classification to start with, but we cannot keep within its limits. Hence we should not have referred at all to the question of the "existential import" of subjects and predicates, were it not for the fact that this question is intimately connected with the doctrine of Immediate Inference. These processes require this assumption: the logical proposition implies that its subject, its predicate, and their contradictories, are all real in the sphere to which the proposition refers. This assumption is far removed from ordinary usage; but ordinary usage is not in question;[2] the question is only as to what interpretation of the proposition best simplifies our conception of Immediate Inference. The class interpretation does so. If, then, the proposition essentially expresses a relation between classes, it implies that the two classes themselves, and the classes formed by what is outside each of them,—*i.e.*, S, P, non-S, non-P,—are all equally real. That this is required is evident from the fact that Immediate Inference involves changing subjects into predicates and *vice versâ*, affirmatives into negatives and *vice versâ*, and terms into their contradictories.

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[1] To this we shall have to return in our concluding chapter.

[2] An interesting examination of current usage, as regards "existential import," is given by Venn in his *Symbolic Logic*.

* § 5. Certain views of Hamilton as to the import of propositions must be examined on account of their traditional importance.

Hamilton held that every proposition may be read so as to express either of two relations between its subject and predicate—viz., "that the one does or does not constitute a part of the other, either in the quantity of extension, or the quantity of comprehension [intension]." The term which is larger in extension is smaller in comprehension, and *vice versâ*; hence the copula *is* has two meanings. For instance, the proposition "Man is fallible," read in extension, means that the class *man* is included in the class *fallible beings*; read in comprehension, it means that the complex concept *man* includes as part of itself the attribute of *fallibility*. The former of the two interpretations is of course the class view with which we are familiar. The latter is known as the "comprehensive" view of the proposition.

We have found it necessary to assume that the intension of any term is *relatively fixed* (ch. II. § 7); it is expressed in the Definition of the term, giving us an analytic proposition (ch. III. § 2). The "comprehensive view," if taken strictly and without qualification, applies only to propositions where the predicate states the meaning or part of the meaning of the subject-term. In any proposition which *gives us information* about a subject, the idea of the predicate is not simply contained in the idea of the subject.[1]

Hamilton's doctrine of the **Quantification of the Predicate** is a development of the class view of the proposition, but it is an inconsistent development.

[1] Nevertheless the point which Hamilton raises is a very important one; further consideration of its philosophical aspects will be found in ch. XI. § 3.

He adopts the four forms A, E, I, O, which depend on the predicative view, and then doubles them by attaching "some" and "all" to the predicate. This is to abandon the predicative view and treat the predicate as a class; but if we do this (see § 2) we do not get eight forms of the proposition but only five. Hamilton's eight forms are as follows, with the symbols suggested by Dr Thomson:—

A.	All S is some P	*affirmative propositions.*
U.	All S is all P	
I.	Some S is some P	
Y.	Some S is all P	
E.	No S is any P	*negative propositions.*
η.	No S is some P	
O.	Some S is no P	
ω.	Some S is not some P	

The Greek letter η (ē) is employed to denote the proposition formed by making the universal predicate of E particular, and the Greek ω (ō) denotes the proposition similarly formed from O.

Hamilton says that it is a postulate of logic to state explicitly whatever is thought implicitly; and that the predicate is always quantified in thought. If so, Logic should state the point explicitly. Mill and others have maintained that we do not usually think the predicate in quantity at all (cp. § 2 *ad finem*); and it does not seem psychologically true of the ordinary judgment, unless in classificatory sciences or in cases of enumeration, or in propositions introducing "only" or "alone"; "Virtue is the only nobility" = "Virtue is all that is noble." In the main, then, the assumption on which Hamilton's scheme rests is not true to Thought.

Even *formally*, the scheme has obvious defects;

this may best be seen by investigating the meaning of *some*.

(*a*) Assume that *some* means, as in § 2 above, *some only*. Then each affirmative proposition which contains *some* has a negative proposition as its secondary implication. For example, take the proposition "all men are some animals," represented in Fig. 14. It im-

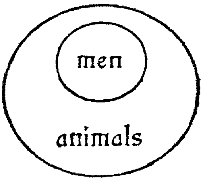


Fig. 14.

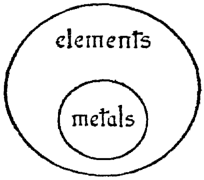


Fig. 15.

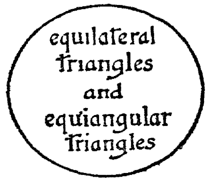


Fig. 16.

plies that there are other animals than *men*—*e.g.*, lions, tigers, &c.; in other words "no men are *some* animals (*i.e.*, lions, tigers, &c.)." That is, Hamilton's A proposition implies η; they are not independent forms. In a similar way we may show that Hamilton's Y proposition —*e.g.*, "some elements are all metals" (fig. 15) implies O, "some elements are no metals." These also are not independent forms.

The proposition ω is peculiarly useless, for it is compatible with each of the five diagrams already given. It is thus compatible with U, unless S and P are the names of an individual (and therefore logically indivisible) object. This seems paradoxical; hence we must show it in detail. Let S and P be both names of classes. The proposition U says that "all S is all P," "all equilateral triangles are all equiangular triangles" (fig. 16). Now "some" means "only a part"; and hence, if we divide the circle which represents the coincident classes into any two separate portions, or

mark off two separate smaller parts within it by smaller circles, we may call one part "*some* equilateral triangles," and the other "*some* equiangular triangles," and it will be true that "*some* equilateral triangles are not *some* equiangular triangles"; that is, U and ω are compatible. That ω is also compatible with each of the diagrams of § 2, figs. 10 to 13, is obvious.

We are therefore reduced to the five forms described in § 2, which, as indicated by the Hamiltonian symbols, are as follows:—

Fig. 9 = U.
Fig. 10 = A or η.
Fig. 11 = Y or O.
Fig. 12 = I or O or η.
Fig. 13 = E.

(*b*) If "some" means "some at least," not excluding "all," then it is obvious that the eight propositions are not independent forms. Detailed proof is unnecessary.

It has been said that in our ordinary thinking we do occasionally quantify the predicate. It is worth while therefore to see which of the new forms U, Y, η, and ω are found in ordinary speech. Thomson, who adopted the Hamiltonian scheme in his *Laws of Thought*, admitted that η and ω are never used, and we have seen that ω is also entirely useless. The *form* η is certainly never used; but a proposition may occur which can be expressed in that form. "Men are not the only rational beings" expresses what is meant by "no men are some rationals." It is a compound proposition, and equivalent to "some men are rational" (I) together with "some rationals are not men" (O). But no proposition ever made could be adequately expressed in the form ω. With regard to U and Y, we may say with Dr Keynes, "it must be admitted that these

propositions are met with in ordinary discourse. We may not indeed find propositions which are actually written in the form *all S is all P;* but we have to all intents and purposes U, wherever there is an unmistakable affirmation that the subject and predicate of a proposition are co-extensive. Thus, all Definitions are practically U propositions [when regarded on the side of extension]; so are all affirmative propositions of which both the subject and the predicate are singular terms." We have already given instances of such propositions, describing them as universal affirmatives which can be converted simply. In ordinary logical form they must be expressed in two propositions: thus, "all S is all P," is equivalent to (*a*) "all S is P," (*b*) "all P is S." As examples of the Y form, exclusive and exceptive propositions are usually given. "The virtuous alone are happy" might be expressed "some of the virtuous are all of the happy," "some S is all P."

In the case of U propositions in geometry, we have really two separate forms, propositions which have to be independently proved: neither of them can be proved from the other (see ch. III. § 9, pp. 83, 84).

The student should bear in mind that Hamilton's scheme of Quantification is open to the objection (see § 2 *ad finem*) which applies to every attempt to read the predicate as a precise quantity. It can give no account of the large class of A propositions where we do not yet know whether P is wider than S or merely coextensive with it. The accompanying diagram (fig. 17) might be adopted to represent such propositions.

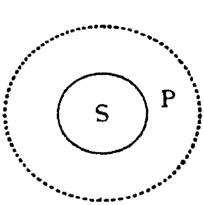


Fig. 17.

* § 6. An interpretation of propositions which is ser

viceable in "Symbolic Logic"—*i.e.*, where propositions are represented by formulæ which can be subjected to algebraic manipulation—has been developed by Boole, Venn, and others. The real reference of the judgment is found in its negative implication; thus, "All x is y" denies the existence of things which are x without being also y; whether there are any x or y is left undetermined; what the proposition does is to empty the class or compartment $x\bar{y}$.[1] Similarly "no x is y" empties the compartment xy; "all y is x" empties $\bar{x}y$; and "everything is either x or y" empties $\bar{x}\bar{y}$. There are only four possible combinations of two terms x and y and their contradictories: xy, $x\bar{y}$, $\bar{x}y$, $\bar{x}\bar{y}$, as in the four propositions which we have examined. The propositions are expressed by making equal to *zero* the class or classes which are ruled out:—

"All x is y" is represented by $x\bar{y} = 0$.

"Everything is either x or y" is represented by $\bar{x}\bar{y} = 0$.

Three terms give eight possible combinations, namely, xyz, $xy\bar{z}$, $x\bar{y}z$, $x\bar{y}\bar{z}$, $\bar{x}y\bar{z}$, $\bar{x}\bar{y}z$, $\bar{x}yz$, $\bar{x}\bar{y}\bar{z}$. Each universal proposition involving x and y and z empties one of these compartments; thus "everything is either x or y or z" empties $\bar{x}\bar{y}\bar{z}$ and is therefore represented by $\bar{x}\bar{y}\bar{z} = 0$. By this method, complex propositions introducing a great number of terms can easily be dealt with, provided they are universal. Special and inconvenient devices have to be employed to represent particular propositions, on this symbolic method. Other methods have been developed by De Morgan, by Jevons, and by various Continental writers. Prof. Minto has observed that "these elaborate systems are not of the

[1] The contradictory of a term x is denoted by $\bar{x}$; and symbols joined together, as xy, denote the class which is both x and y.

slightest use in helping men to reason correctly. The value attached to them is merely an illustration of the 'bias of happy exercise'" (*Logic*, p. 134).

Although the negative interpretation of propositions does not claim to be more than a mere convention, it is less of a convention than we are apt to think; for when we make a universal proposition, All S is P, as the result of enumerating all the instances of S and finding that, "without exception" they are P, the proposition passes its meaning, so to speak, through a double negation. The proposition denies the exception; and in such cases the formula *nemo non* or *nullus non* is the primitive formula, not a circumlocution. As the words "without exception" imply, the primary meaning of the universal affirmative of this kind is "No S is other than P." Nevertheless, to adopt this convention of Symbolic Logic as the ordinary logical doctrine of the interpretation of propositions, as Dr Keynes proposes to do, would be to depart far from ordinary forms and usages.

EXERCISE IX.

The following questions refer to the subjects of the present chapter.

(i) *Elementary*.

(1) State exactly what is meant by the question of the "Import of Propositions," and give *briefly* four possible views on this topic.

(2) What is the value of the interpretation of the proposition which takes Subject in Intension and Predicate in Extension?

(3) What forms result from the class-interpretation of propositions? Is it possible to interpret propositions in Extension to the entire exclusion of Intension? Give your reasons.

(4) Is it possible to interpret the proposition in Intension to the entire exclusion of Extension? Give your reasons. (See § 3.)

(5) "Every proposition expresses a relation among *things*." In what sense is this true?

(ii) *More Advanced.*

(6) State and discuss the different theories as to the Import of a proposition. [O.] Or,—

What different views have been held as to the nature of Predication? [O.]

(7) Explain and discuss carefully the following theories of the judgment:—

(*a*) "Judgment is the comparison of two ideas."

(*b*) "Judgment is the statement of a relation between attributes."

(*c*) "Judgment is the reference of a significant idea to Reality." [St A.]

(8) Bring out the meaning of each of the following accounts of the proposition "All men are mortal," and say which is logically to be preferred:—

(*a*) All men have the attribute mortality.

(*b*) Men are identical with mortal men.

(*c*) Men form part of the class mortals.

(*d*) If a subject has the attributes of a man, it also has the attribute mortality. [L.]

* (9) State the chief theories of the Import of Propositions. On what theory does the adoption of A, E, I, and O, as the fundamental forms, rest? Criticise the additional forms which arise when the quantification of the Predicate is adopted. [C.]

* (10) Explain the precise meaning of the proposition "Some X's are not Y's" (the proposition ω of Thomson). What is its contradictory? Give your opinion of its importance. [L.]

* (11) Examine critically the view that the significance of the proposition "All S is P" is fully and best given in the form "There is no S which is not-P." [L.]

* (12) What do you consider to be the *essential* distinction

between the Subject and Predicate of a Judgment? Apply your answer to the following:—

"From hence thy warrant is thy sword."

"That is exactly what I wanted." [C.]

* (13) What independent propositions may be formed by quantifying the predicate (*a*) when "some" means "some at least," (*b*) when "some" means "some but not all"? Discuss the logical advantages of quantifying the predicate. [L.]

* (14) Explain and examine the terms "Universe of Discourse," "Existential Import of Propositions."

What view or views on the latter subject does the ordinary doctrine of Immediate Inference imply?

CHAPTER V.

THE PREDICABLES, DEFINITION, AND CLASSIFICATION.

Part I.—*The Predicables.*

§ 1. THE Predicables are the various possible relations, in extension and intension, which the predicate of a proposition may bear to its subject, when it makes an affirmation of the subject. In order to understand the doctrine of the "Predicables" in modern Logic, we must have a clear idea of the way in which Aristotle dealt with it. His object was (*Topics*, I. ch. 4, 5, 6) first to classify these relations.

This may be done either inductively (διὰ τῆς ἐπαγωγῆς), by examining every kind of proposition; or deductively (διὰ συλλογισμοῦ), by considering what an affirmative proposition means to say—*i.e.*, the relations (of predicate to subject) which it admits of if it is to be an affirmative proposition at all. Let us adopt the latter plan. The terms standing as Subject and Predicate are either convertible in the Aristotelian sense, without changing the meaning of the proposition, or they are not—*i.e.*, they are or are not *exclusively applicable to the same things.*

(1) If they are convertible this is equivalent to saying that the extension of the two terms is the same. When P thus coincides with S in extension, it may either (a)

entirely agree with *S* in intension, or (*b*) be an inseparable feature of *S*, and *peculiar* to *S*. In the first case (*a*) *P* is the **Definition** (ὅρος) of *S*:—

> "Man is a rational animal."
> "A triangle is a three-sided rectilineal plane figure."

In the second case (*b*) *P* is a **proprium**[1] (ἴδιον) of *S*:—

> "Man has the power of speech."
> "Man is capable of progress in knowledge to an indefinite extent."
> "A triangle has its three interior angles together equal to two right angles."

Aristotle expresses the two possibilities thus: "The definition shows what the Subject *really is*." "The *proprium* does not show what the Subject really is, but is inseparable from it and convertible with it."

(2) If S and P are not convertible, then they do not entirely coincide in extension. Hence P cannot entirely agree in intension with S; it must either (*a*) partially agree or (*b*) entirely disagree.

In the *first* case (*a*) P is part of the definition of S, and is either a "genus" (γένος) or a "differentia"[2] (διαφορά). A **genus** is that which may be predicated of several different kind of things besides the class in question; as Aristotle says, the genus is "contained in the statement of what they really are":—

> "Man is *an animal* (*genus*)."
> "Triangles are *rectilineal plane figures* (*genus*)."

[1] The word "property" has a usage too wide to be given as the translation of ἴδιον.

[2] The word "difference" is too general in meaning to be used as the translation of διαφορά in this technical sense.

That is, the characteristics of "animal" may be affirmed of many different kinds of creatures besides "men"; and similarly, the characteristics of being bounded by straight lines, all in the same plane, belong to many other figures besides "triangles." A **differentia**, again, is a quality or qualities distinguishing one kind of things from other kinds of the same genus:—

"Man is *rational*"
"Triangles are *three-sided*."

In the *second* case (*b*) P is usually named **accident** (Latin *accidens*, after Aristotle's συμβεβηκός),—a quality which may or may not belong to the subject; "some men live for upwards of a century."

It will be found that every proposition must come under one of these four heads. Most of the assertions which we make in common life are cases of the so-called "accidental" predication.

§ 2. We must now consider more fully these four kinds of predication.

(*a*) *Genus and Differentia.*

The concept which is poorer in defined qualities but of wider extension is said to be the concept of a genus; while that which is richer in defined qualities, but narrower in extension, is called the concept of a species (εἶδος). These terms are strictly correlative (ch. II. § 3). The relation of species to genus is that of **subordination.** The relation of different constituent species (of one and the same genus) to each other is that of "co-ordination."

The simplest illustrations of generic and specific concepts may be found in elementary plane geometry—*e.g.*, "a triangle is a three-sided rectilineal figure." A

"rectilineal figure" is a figure bounded by a certain number (not yet defined) of straight lines. This is the concept of a genus (Aristotle's γένος). It is a wider group, including triangles, squares, and other quadrilaterals, pentagons, &c. When we make the number of sides definitely *three*, then we have the concept of the triangle, a three-sided rectilineal figure; this is a species subordinate to the genus, which includes it along with other species. The distinguishing attribute of the species, peculiar to it and distinguishing it from other species of the same genus, is an example of what we called the "differentia" (Aristotle's διαφορά).

For Logic, any pair of classes of which one is subordinate to the other are related as species and genus. But in Natural History, these terms are given a particular place within a hierarchy of divisions and subdivisions: "Kingdom," "Group," "Class," "Order," "Family," "Genus," "Species," "Sub-species" (if necessary) or "Variety." Logically, each of these is genus to the one which follows it.

The relation of "subordination" only holds good between objects *of the same kind*; "yellow" is not the generic concept of "gold," but of the various shades of yellow; although the concept "gold" includes the idea of a colour which is a peculiar shade of yellow.

(*b*) *Proprium.*

Properties which belong to the whole of a class, and are peculiar to it, but do not have any important effect on its other characteristics, are called *propria* (ἴδια). They are inseparable; "the Ethiopian cannot change his skin, nor the leopard his spots."

All such properties are, as it were, a challenge to our Reason, to show that they are connected with the specific concept of the class, and follow from it. They

may follow as "consequent from reason," or as "effect from cause." Examples of the *former* are found in the cases where any characteristic and peculiar property is found to follow from the definition of the figure (*e.g.*, Euclid, Bk. I. 32). Examples of the *latter* will be found in the various explanatory sciences—*e.g.*, when the colouring of certain animals is shown to be protective under Natural Selection. A simpler instance is the fact that "Man is capable of desiring knowledge," which is one of the general *propria* of humanity resulting from the specific property of "rational thought."

(*c*) *Accidens.*

This term is not appropriate, but custom has perpetuated its use. It signifies the concept of a state or condition which does not necessarily belong to the thing. The fact that it is unessential may be recognised in two ways: it may belong to some members of the class and not to others,—"This clover has four leaves"; or it may belong to an individual at one time and not at another,—"The sun is eclipsed," or "Socrates is standing in the Agora."

The first of Aristotle's Predicables, "Definition," is of such importance as to require special treatment below.

§ 3. The account of the Predicables which we have given differs in some important respects from the traditional account. The latter was not derived directly from Aristotle, but from an Introduction to Aristotle's *Categories*, written by Porphyry, who taught Logic in Rome about six centuries after Aristotle's time. This Introduction became accessible to the mediæval logicians in a Latin translation made by Boethius about two centuries after it was first written.

Porphyry explains the "five words," *genus*, *species*,

differentia, *proprium*, and *accidens*, as terms which are used in Definition and Classification, and which it is useful to understand. The mediæval writers supposed that he was giving a classification of possible *predicates*, as such; and great importance was attached to the list. It was considered that every predicate-term must belong to one of these five classes. The essentials of the doctrine which was thus elaborated may be briefly indicated.

Genus, *species*, and *differentia*, were defined as by Aristotle. *Proprium* signified a property not given in the definition of the term but following from it. It may or may not be *peculiar* to the class which the term denotes. In this respect the traditional meaning of *proprium* differs from the Aristotelian meaning (see § 2, *b*). *Accidens* signified a property not following from the definition and not necessarily connected with it. This does not differ from the Aristotelian meaning; but some writers went on to distinguish "separable" and "inseparable" accidents. "Thus the clothes in which a man is dressed form a separable accident, because they can be changed, as can also his position and many other circumstances; but his birthplace, his height, &c., are inseparable accidents, because they can never be changed, although they have no necessary or important relation to his general character." Aristotle would probably have said that this distinction of separable and inseparable accidents applies only to *individuals*, not to *classes;* an "inseparable accident" *of a whole class* he would have called a *proprium*, peculiar either to the given class or to a higher class which contains the given class (*e.g.*, the characteristic of *water*, the power of transmitting pressure equally in all directions, is an Aristotelian *proprium* of the class *fluids* to which water belongs).

As every genus must have at least two species under it, and the species may again be genera to subordinate species, we may arrange terms in a series according to the decreasing extension of the concepts; we may begin with a genus which has no class above it, and hence is called *summum genus;* and we may end with a species which cannot be further subdivided except into individuals, and is therefore called *infima species.* An example has been given (ch. II. § 6), where the *summum genus* is "being (in general)," and the *infima species* is the class "man." Such a series is called a "predicamental line" (*linea predicamentalis*); and the intermediate classes, between the highest and the lowest, are called *subaltern* genera or species. The so-called "Tree of Porphyry," a device of later writers, is based on the "predicamental" series of concepts (see § 10).

Here we may explain the terms "generic (or specific) difference," "generic resemblance," which are frequently used. Generic differences are those which distinguish species belonging to different genera; *e.g.*, "isosceles triangle" is generically different from "square" (the respective genera are, *three-sided* and *four-sided* rectilineal figures): hence they are said to be *heterogeneous* (Greek ἕτερος, other). Generic resemblances are those on account of which different species are referred to one and the same genus: *e.g.*, the mental life of a man and a mouse may be said to be generically alike; so also an isosceles and an equilateral triangle are generically alike; hence they are said to be *homogeneous* (Greek ὁμός, like). On the other hand, by "specific difference" is meant simply the "differentia" as defined above: thus, an isosceles triangle is specifically different from an equilateral triangle; and the mental life of a man is specifically different from that of a mouse. We may further distinguish the "generic property," or that which belongs to the whole of a genus, from the "specific property" which belongs to the whole of a *lowest* species.

Part II.—*Definition.*

§ 4. In defining a term we set forth in words the "universal meaning" which it represents or ought to represent,—the **content of the idea** which the term identifies.[1] The primary, or rather, the practical object of Definition is, "fixing the meaning" of a term for the sake of imparting the idea to another mind.

In common life, the fulness of detail which we find in real things makes it easier to **describe** than to **define.** Description is based on a mental picture, or an immediate perception, of which it gives an account; Definition is based on a *concept.* Description appeals to imagination and memory; Definition to thought. The one, however, passes into the other, and it can hardly be said that there is a difference in kind between the two.

We may roughly distinguish different modes of description, some of which are nearer to definition than others. Furthest from strict definition is the "symbolic description" which is simply artistic vision. It "instinctively seizes the harmonies of the scene before it and frames it into a speaking whole,"—indeed "catches the whole before it fixes upon anything, and carries the entire idea into the interpretation of every part,"—and in passing on the impression, "with a few strokes that seem to have no material in them, will set its picture before you better than you could have found it for yourself."[2] This is the artist's method, as in poetry and eloquence; it makes us know the thing by making us *experience* it, "feel" as we would if it were real for us. The "matter-of-fact" method of description reads its objects piecemeal; by traversing hither and thither and putting together the contents of the field, it seeks to reach the idea of the whole. We may call it "enumerative" description,

[1] On the term "content" see p. 17.

[2] Cf. Martineau, *Types of Ethical Theory*, vol. ii. (2nd ed.) p 159.

as in the naturalist's list of marks for identifying a plant or an animal. Using the terminology of § 3 above, we may say that in Description, inseparable *accidentia* are often used, with or without some of the *propria*, to enable us to recognise the objects denoted by the name.

Aristotle observed (*An. Post.*, i. 8) that definition is the beginning and the end of scientific knowledge. This most important observation suggests two points of view from which Definition may be regarded. In one sense definition is the "beginning of knowledge," inasmuch as we must have clear ideas at least of the objects about which our inquiries are concerned; and the definition with which we begin need not be anything more than methodical description. With regard to these elementary definitions, all that we need ask is, as to the best way of *formulating or expressing them*. This is shown in the traditional **rules of definition** to be given in the next section. Such rules apply to the expression of any definition, elementary or not, *when the definition is already discovered*. But the method of discovering true definitions of what things really are, is not shown by these rules. This is what Aristotle called "definition as the end of knowledge." The process of *arriving at* definitions is the process of science as a whole, the ideal of which is the scientifically defined concept. The discovery of definitions was the business to which Socrates devoted himself, with special reference to the sphere of moral facts. With regard to the comparatively simple task of *formulating* our definition, the chief practical rule is to define *per genus et differentiam*: that is, we *distinguish the object from the class which it most resembles*.

What this implies will easily be seen. In our ordinary descriptions we of course employ general ideas, but we

set them forth in any order, beginning at any point,—so long as we are sure of producing a sufficiently clear and complete picture of what is meant. But in definition we start with that general idea in which the greater part of the features which we wish to indicate is already contained: thus, of the "phœnix," we begin by saying "it is a bird." We refer it at once to a *genus* which is assumed to be familiar to other minds. The nearest genus (*genus proximum*) is referred to because then a simpler specific difference is sufficient to distinguish the object from that genus.

So far, we have treated definition only as a means of marking off an object from others; and this is the etymological meaning of the term (*definitio*, ὅρος, ὁρισμός). The definitions with which we begin an inquiry, in any branch of knowledge, must be of this kind; and whatever breadth and depth may be given to the definition afterwards is not the beginning of the inquiry but the result of it. Obviously it is of great importance, at the beginning of some theoretical or practical discussion about a certain matter, that we should be able to mark it off by characteristics which are precise and easy to find. At first this is of more consequence than any reference to characteristics which are scientifically more profound. In scientific treatises we often find objects referred to by properties which are comparatively unimportant but not easily mistaken.

§ 5. The following rules for the expression and formulation of definitions are based on those given by Aristotle.

(1) The fundamental rule is that **the definition must state the most essential features of the objects to which the term is applicable.**

Aristotle considered that the definition *per genus et differentiam* secured the statement of the essential features. But from the modern point of view this is not so. We have seen that such definition may be nothing more than a preliminary survey of the ground. From the modern point of view, also, Aristotle made

too complete a separation between the "essential" and the "accidental" qualities of objects. For the present we may say that the essential qualities are those without which the thing could not be what it is. A *man*, for instance, by living alone for years on a deserted island, might lose the essential qualities of manhood and become a "wild animal": without them he is not a man. The essential qualities, again, are those from which the largest number of others may be seen to flow as consequences. Thus our distinction is between "essential" and "derived" qualities. And in formulating the rule, we spoke of the "most essential features"; for with the progress of knowledge we may find that some qualities which were supposed to be primary and essential, turn out to be only derivative.

(2) **The term expressing the definition must be simply convertible with the term defined,** neither too wide nor too narrow.

This rule prevents the definition from being too wide —*e.g.*, to define X as AB, when there are some AB which are not X, is too wide, and the definition is not convertible, for it is not true that every AB is X. Examples: "Eloquence is the power of influencing the feelings by speech or writing." Many things, said or written, influence the feelings, but are not eloquent. "Virtue is the capacity for ruling over men." Many who can rule over men are not virtuous. "The cause of anything is the antecedent which it invariably follows" (Hume). But the "invariable antecedent" is not always the cause, though the constant connection of two events may show that they both depend on the same cause (*e.g.*, day and night). A definition which is too narrow may be described as the definition of a higher class by a lower which is included in it, a

genus by a species. Examples: "Wealth consists of money." "Wealth consists of natural products." The student of economics will recognise these errors, each of which is a case of a fatally narrow definition. "Justice is minding one's own business." Even if we put a large interpretation on the term "business," and understand "minding" in a moral sense, the definition is still too narrow. "Grammar is the *art* of speaking and writing correctly." But grammar must consist of more than a set of practical maxims.

(3) **The definition should not be obscure.** Obscurity may arise in various ways :—

(*a*) From the employment of ambiguous expressions ;
(*b*) From the use of metaphorical expressions ;
(*c*) From the use of expressions which are less familiar than the one to be defined (*obscurum per obscurius*) ;
(*d*) From the use of eccentric expressions.

If a statement is made as an epigram, it cannot be criticised as a definition. Assuming that each is intended to be a serious definition, the following examples, of obscurity in defining, may be given :—

"Growth is a *transition* from non-existence to *existence.*" "Life is a continuous adjustment of *internal relations* to *external relations*" (Spencer). "'Sense' is the recognition and maintenance of the *proper* and *fitting relations* in the affairs of ordinary life." "Architecture is *frozen music.*" "Prudence is the *ballast of the moral vessel.*" Such sentences, though technically "obscure" as *definitions*, may be highly suggestive as *metaphors*.

Scientific definitions expressed in the technical language of a particular science are not instances of the fault here referred to. For though the definition is less

familiar than the thing defined, it states what is more important from the scientific point of view. In Aristotelian language, it gives that which in the order of Nature must be known first (γνωριμώτερον φύσει).

(4) **A definition should not use, explicitly or implicitly, the term to be defined.**

An obviously circular definition may be intended to be an epigram: thus, "an archdeacon is one who exercises archidiaconal functions," would have point in the case of a *fainéant* archdeacon. But *implicitly*, this fault is constantly committed: "Justice is giving to each man *his due*." In long and involved scientific discussions, it is very easy to formulate two or three separate definitions which, when taken together, are seen to be merely circular. The same fault may be committed by using the *correlative* of the term defined: "A cause is that which produces *an effect*." Mere repetition of a word does not vitiate a definition; we may define "contrary opposition" as "*opposition* in which, &c.," having already defined "opposition" and being now concerned to define "contrariety."

(5) **The definition should not be negative where it can be positive**; and, as a special instance, **opposites or contraries should not be defined by one another.**

(6) To these rules may be added, what is often a "counsel of perfection," that **the definition should contain nothing superfluous.**

Thus, Euclid's definition of a *square* contains more than is necessary; for it is shown (Euc., i. 46) that if a figure has four equal sides, and *one* of its angles a right angle, the other three angles must also be right angles. Again, when Mill says: "A cause is the assemblage of phenomena, which occurring, some other phenomenon invariably commences, or has its origin," we may

express all this (with the additional advantage of dropping the ambiguous term "phenomenon") in the simple statement: "A cause of an event is that which occurring, the event occurs."

We have said that definition, according to its traditional rules, requires us to define a term by giving the qualities comprised in its intention, *per genus et differentiam*. It is clear, however, that there are certain cases in which this requirement cannot be fulfilled—*i.e.*, there are terms which *in this sense* are "indefinable." The names of simple qualities, like the various elementary sensations, "hot," "red," &c., and mental qualities such as "consciousness," "pleasure," "pain," cannot be defined by an enumeration of their attributes; they are too simple. The same is true of the most general relations of material bodies, such as "time," "space." At the opposite extreme, an individual (person, place, or thing) is indefinable by enumeration of attributes; the countless peculiarities which constitute its individuality cannot be enumerated *per genus et differentiam* or in any other way. An indefinable object is said to be *sui generis*, of its own class, since it cannot be referred to any genus by way of definition.[1]

§ 6. The distinction of "nominal" or "verbal" and "real" definitions was first given by Aristotle. He said (*An. Post.*, ii. 10) that a **nominal definition** gives the current meaning of a term, as when thunder is said to be "a noise in the clouds," or a house "a building in which people live." A verbal definition need not

[1] And in general, when a thing is so peculiar and unlike other things that it cannot easily be brought into one class with them, it is said to be *sui generis;* thus, as Jevons observes, the rings of Saturn are so unlike everything else among the heavenly bodies that they may fairly be called *sui generis*.

have even the implication of real existence added—it may be of things afterwards shown to be impossible—*e.g.*, "perpetual motion," or "squaring the circle." But sometimes this verbal definition has added to it the postulate of real existence or validity, as in the examples given above (cp. also *An. Post.*, i. 1). A **real definition** is the statement of what is essential to the fact in question as a matter of science. In fact, Aristotle's distinction practically coincides with that of definition as the beginning and as the end of knowledge; in this sense we must retain it, but we need not distinguish the two types of definition as "nominal" and "real."

Modern writers usually express the distinction in terms similar to those of Aristotle, "nominal" and "real"; but scarcely two of them explain it alike. If we retain this expression of it, we must remember that *all* definitions define the meanings of terms or names, and so may be called "nominal": while on the other hand, some definitions evidently have a *direct* reference to a real thing,—others again, evidently aim first of all at fixing the meaning of a term, and have only an indirect reference to reality. Even this distinction does not go deep. For as Professor Sidgwick has observed, we never define a term for its own sake merely, but in order to understand the things to which it refers. A mere word, apart from the things for which it stands, has no interest for us. "The truth is,—as most readers of Plato know, only it is a truth difficult to retain and apply,—that what we gain by discussing a definition is often but slightly represented by the superior fitness of the formula which we ultimately adopt; it consists chiefly in the greater clearness and fulness in which the characteristics of the matter to which the formula refers have been brought before the mind in the process

of seeking for it. *While we are apparently aiming at definitions of terms, our attention should be really fixed on distinctions and relations of fact.* These latter are what we are concerned to know, contemplate, and as far as possible arrange and systematise; and in subjects where we cannot present them to the mind in ordinary fulness by the exercise of the organs of sense, there is no way of surveying them so convenient as that of reflecting on our use of common terms" (H. Sidgwick, *Political Economy*, p. 49).

The definitions which we are able to give, in every department of thought and investigation, depend on the general state of knowledge to which we have attained, and even—in the case of words whose meaning refers mainly to practical life—on the general state of civilisation. We should no longer, with Plato, give as a model of definition—"The sun is the brightest of the heavenly bodies which move round the earth"; and we find, again, that words like "school," "house," "monarchy," have to-day meanings very different from those which they bore in the past. For a like reason, to criticise or estimate any definition requires special knowledge of the subject-matter to which it belongs.

When we speak of completely satisfactory definitions of the objects of our experience, we are really asking for the final results of exhaustive scientific inquiry carried to its furthest limits. We are now speaking of that type of definition which is the end of science. Here, "the business of definition is part of the business of discovery"; "discovery and definition go hand in hand"; and nothing is *indefinable* save through our ignorance. We begin by thinking of an object in a loose general way as a whole made up of parts which are familiar. Such an idea may be little more than

a mental picture: but as long as it is precise enough to avoid confusion with other things, we are practically content. But reason suggests a step in advance,—to ascertain the characteristics which the object has in *common* with other species of its genus, and also to *distinguish* it from the other species. Then we are led to inquire into the general law which regulates the connection of its parts,—the "what" of the thing; and the form which this knowledge tends to take is that of the causal conditions—*i.e.*, the real origin of the object, the "how" of the thing. This leads us to connect it with other things. We may therefore say that some definitions are *provisional* and *progressive*, while others are *final*, in the sense that to reach them is the ideal of science.

It has even been held (cp. Aristotle, *An. Post.*, ii. 10) that ideal definition will show the "why" of the thing,—the very reason of its existence; but, short of this, many of the results which we should call "laws of Nature" would have been called "definitions" by the Greeks. Aristotle would have called Newton's Law of Gravitation, or Darwin's theory of Natural Selection, scientific definitions of "Gravitation" and of "Species." As Geometry was in a far more advanced state among the Greeks than any natural science, they took this as their model of "scientific knowledge" (ἐπιστήμη); and, since in Geometry it is easy to sum up results in a brief formula, it was natural to speak of these results as "definitions" rather than "laws." Thus, from this point of view, the whole of the Third Book of Euclid, which deals with properties of circles, is an expanded definition of the circle.

Before leaving this subject, there are some particular types of definition which we must notice. In mathematics, our definitions are not matters to be discovered, or ideals to be reached; they are principles with which

we start. This constitutes the most important practical distinction between mathematical and physical science. In mathematics, we begin by stating the essential characteristics of the objects with which we deal,—hence "Definitions" precede each book of Euclid. In physical science, the essential characteristics of the objects are a matter of gradual discovery. This is why it is possible for mathematical definitions to be of the kind called *Genetic*, showing us indirectly a way in which we may form an idea of the object: "A sphere is a solid figure formed by the revolution of a semicircle about its diameter, which remains fixed." We may also notice the type of fixed definition which results from legal enactments. In Acts of Parliament, for instance, an ordinary term, such as "person," "parent," "owner," "parish," "factory," has a special and precise meaning given to it,—this being artificially made, and constituting a "conventional intension" not capable of growth by advance of knowledge, as in the case of scientific terms. The student should also observe that the same term may be defined in different ways—that is, by reference to different *genera*—according to the point of view from which it is regarded. For instance, "man" constitutes a different subject-matter in Zoology and in Ethics; and a "circle" in Analytical Geometry is regarded as a section of a cone, and not as in Synthetic Geometry (*e.g.*, in Euclid). Finally, in the so-called *Definition by Type*, an individual member of the class is taken as representing the class. This method, which is not really definition at all, may vary greatly in definiteness and value: at one extreme, it is exemplified in merely pointing to a member of the class in question and saying, "I mean something like that"; and at the other extreme, in the elaborate "enumerative

descriptions" (see above, § 4) of individuals *typical* of the various classes in Botany, Zoology, &c.

EXERCISE X.

Questions on §§ 1 to 6.

(i) *Elementary.*

1. What is meant by the terms *genus*, *species*, *differentia*? Give examples.

2. Explain any differences between the use of the terms *proprium* and *accidens* by Porphyry and later writers, and the Aristotelian use of the corresponding terms.

3. Explain the expressions—

sui generis,	homogeneous,
summum genus,	heterogeneous,
infima species,	specific property,
subaltern genus (or species),	generic property.
subordination (among classes),	
co-ordination,	

4. Name a *proprium* and an *accidens* of each of the following classes: Planet, circle, insect, bird, Member of Parliament, British Subject.

5. To which of the Predicables does each of the following predicates belong?—

(*a*) A proper fraction is one whose numerator is less than its denominator.
(*b*) Even Men are animals.
(*c*) Logic is a good mental discipline.
(*d*) Englishmen are keen sportsmen.
(*e*) Certain animals hibernate.
(*f*) The tiger is a predatory animal.
(*g*) Lord Shaftesbury was a great philanthropist.
(*h*) Triangles are three-sided.

6. Give the *genus*, the *differentia*, a *proprium*, and an *accidens*, of (*a*) affirmative proposition, (*b*) rectangle, (*c*) island.

7. In what two ways may we conceive the problem of Definition?

8. Of what does logical Definition, in the narrower sense, consist? How far can we distinguish Description from Definition?

9. Give twelve examples of terms that are "indefinable." In what sense are they "indefinable"?

10. Criticise the following definitions, and show what rules, if any, are violated by them:—

(1) Chance is the cause of fortuitous events.
(2) A power is a force which tends to produce motion.
(3) Tin is a metal lighter than gold.
(4) A gentleman is a man who has no visible means of support.
(5) The body is the emblem or visible garment of the soul.
(6) Man is a vertebrate animal.
(7) Thunderbolts are the winged messengers of the gods.
(8) A moral man is a man who does not lie or steal or live intemperately.
(9) Evolution is to be defined as a continuous change from indefinite incoherent homogeneity to definite coherent heterogeneity of structure and function, through successive differentiations and integrations (Spencer). [Creighton.]

11. Define the following terms by giving the *genus* and *differentia*:—

science,	psychology,
triangle,	gold standard,
republic,	import duty.
monarchy,	

[Creighton.]

(ii) *More Advanced.*

12. Why is it that some names can, and others cannot, be defined? [O.]

13. Why is definition often a question not of words but of things? [St A.]

14. Compare the following terms in respect of their definability: rectangle, motive, brass, tree, table, marriage, theft, feeling, substance. [L.]

15. What qualities are included in the definition of a term? What is meant by saying that our definitions are provisional?

16. What is the propositional form in which a Definition must be stated? How do you know a Definition when you see it? [L.]

17. "Definition is both the beginning and the end and aim of all knowledge." Carefully explain and discuss this statement.

18. Account for the divergence of view on the question, What are the limits of Definition?

Part III.—*Classification.*

§ 7. The close connection between Definition and Classification will now be evident.

Even in the method of "pointing," of showing the denotation in the absence of any serviceable definition, there is a stimulus to mental comparison in order to distinguish the common element which we wish to get at, and make it into a definition. Reference to instances is an inseparable element in the process of defining; and that reference will take the form of an implicit or explicit arrangement into classes according to likenesses and differences. What we find ourselves doing, in attempting to define any object of experience, is first to find a class for it, then to compare representative individuals of the class with it, taking into account also the contrasted classes. If we cannot find a class,—in other words, if the object is like nothing in our previous experience,—we are completely at a loss.

It has been customary to treat what is called "Division" under the head of "Deductive," and "Classification" under that of "Inductive" Logic, as Jevons and Fowler do. But there is no reason for the separation. "Division" tends to

signify the splitting up of a *given* class into sub-classes; "Classification," the systematic arrangement of animals, of plants, of minerals, &c., in Science, for the sake of studying their form, structure, and function. We shall consider the latter process first.

The fundamental rule is that objects are classed together when they resemble one another in a definite quality or group of qualities. But to define a class as an arrangement of objects according to their common qualities, is a definition which errs by being too wide, inasmuch as it would include as "classes" combinations which we never form, and which we should regard as almost absurd. Compare, for example, the two following combinations: (*a*) The classing together of various human beings (negroes, Europeans, Hindoos, &c.) as having in common the attributes of *manhood*; (*b*) the classing together of negroes, coal, and black chalk as being all *black*, *solid*, *extended*, *divisible*, *heavy*. If the concept which is based on classification consisted of *any* collection of common qualities, (*b*) would have to be considered as a "class"; but the mind has not naturally formed such a concept, and never would deliberately form it. On the other hand (*a*) is a type of the concepts which we form both consciously and unconsciously. The difference is that in (*a*) the common qualities on which the stress is laid are those which we have called "essential,"—those **qualities which have a determining influence on the largest number of the others** (see § 5 (1), above).

§ 8. We have seen, then, that the attribute or group of attributes, in virtue of which we form objects into a class, must consist of the common qualities which are essential.

Since these are the characters that carry with them

the greatest number of other characters, we observe that such a classification satisfies the following conditions :—

(*a*) It shall enable the greatest number of general assertions to be made about the class.

(*b*) It shall enable us to infer of any other member a great part of what we know about any one.

(*c*) Its members shall have the greatest number of points of mutual resemblance, and the fewest points of resemblance to members of other groups.

Such a class is said to be **natural.** This term, as used of classes, takes us back to the ancient view, that in Nature there are fixed, permanent kinds or classes of things which can never pass into one another. This idea is now abandoned, although we may retain the term "natural" as applied to *methods of classification.*

The impossibility of drawing any clear dividing line is a fact of universal experience. "To admit of degrees is the character of all natural facts; there are no hard lines in nature. Between the animal and the vegetable kingdoms, for example, where is the line to be drawn? . . . I reply that I do not believe that there is any absolute distinction whatever. External objects and events shade off into one another by imperceptible differences; and, consequently, definitions whose aim it is to classify such objects and events must of necessity be founded on circumstances partaking of this character. . . . It is, therefore, no valid objection to a classification, nor, consequently, to the definition founded upon it, that instances may be found which fall, or seem to fall, *on* our lines of demarcation. This is inevitable in the nature of things. But, this notwithstanding, the classification, and therefore the definition, is a good one if in those instances which do *not* fall on the line, the distinctions marked by the definition are such as it is important to mark, such that the recognition of them will help the inquirer forward towards the desiderated goal" (Cairnes, *Logical Method of Political Economy*, p. 139).

A scientific *system* of classification is the grouping of classes in such an order as will lead to the discovery of their *affinities*,—the relations in which the *real structure*, typical of each class, stands to that of the others. Its result is that the classes thus formed correspond to what appear to be the great divisions of nature. It has also been called *classification by series*. This is illustrated, both on a great scale and a small, in the classifications of natural history—Zoology, Botany, Crystallography, Mineralogy, &c.[1]

The natural classification is not appropriate for all purposes, even in science. We have seen that it takes as a basis the most fundamental properties,—those which have a determining *effect* on the largest number of others. Sometimes "the test of importance in an attribute proposed as a basis of classification is the number of others of which it is an index or invariable accompaniment," while the latter are not its consequences or effects, and may not be in any important respect affected by it. "Thus in Zoology, the squirrel, the rat, and the beaver are classed together as rodents, the difference between their teeth and the teeth of other Mammalia being the basis of division, because the difference in teeth is accompanied by differences in many other properties. So the hedgehog, the shrew-mouse, and the mole, though very unlike in outward appearance and habits, are classed together as Insectivora, the difference in what they feed on being accompanied by a number of other differences" (Minto, *Logic*, p. 98). Again, certain characters in natural objects may

[1] For an elementary treatment of the methods of scientifically *naming* such systems of classes, see Fowler, *Inductive Logic*, ch. ii. § 2, part (2) and (3): "Nomenclature" and "Terminology." For fuller references, see present chapter, § 11 below, *ad finem*.

be comparatively of no importance, but may be invariably present and very easily recognised; in such cases, it is practically convenient, in scientific work, to take these as a basis of division. Such are "characteristic" qualities.

The celebrated Linnæan system of classification in Botany is an example of one which, though made for scientific purposes, is not "natural." Linnæus took as his basis of classification the numbers of the sexual parts of the plants, the pistils and stamens, as a clue to natural affinities. They are indeed an important means of identification; and some of his classes coincide with classes in the "natural" system of division; but his classification is not natural because it goes on the one principle of *number*. The history of botanical classification—on which the student may consult any standard text-book—is the best example of the attainment of a natural system of classification.

It scarcely needs to be said that all natural classification and all classification for scientific purposes, whether natural or not, depend entirely on our knowledge of Nature's processes and objects. The detailed rules of classification depend on the special characteristics of that part of Nature with which the science deals. All that Logic can do is to give a general account of the process which all science employs, in arranging its objects so as to throw as much light as possible on their origin, structure, and affinities.

We have seen that a classification may be "natural," having as a basis the most fundamental or essential qualities, from which the largest number of others are *derived;* or it may have as a basis those characteristics which merely *accompany* the largest number of others. In both cases, the basis of the classification consists of numerous common qualities taken together; and both

may be accounted "natural" classifications, using the term in a slightly wider sense than we did before. But we saw also that even in science, classifications are often made on the basis of a single quality, for the sake of ready identification. Classifications of this kind, **made on the basis of a single attribute**, or very few attributes, are called **artificial**. Usually an artificial classification is made on the basis of one fact only. Examples are easily found: the arrangement of words in a dictionary, the object being to find any word as easily as possible; the arrangement of books in a library, according to size, for economy of space, according to the initial letters of the authors' names, or according to the language in which they are written.

§ 9. We shall now give a formal statement of the rules of a correct "logical division,"—the process of splitting up a given class into sub-classes. These rules are only an expanded statement of the relation of a genus to the subordinate species which compose it.

(*a*) In dividing a genus, the basis of division must be a quality common to the whole extent of the genus; and species must be distinguished according to the different modifications of it which they possess. Hence the basis cannot be a *proprium*, or essential quality of *the genus*, for this would be possessed equally by all the species—*e.g.*, we cannot take "life," "reason," &c., as bases for dividing the genus "man."

(*b*) Each act of division must have one basis only. Violation of this rule leads to "cross division," which practically means that the species overlap. If there is one basis only, the species will be mutually exclusive.

(*c*) The constituent species must be together equal to the genus. In other words, the division must be

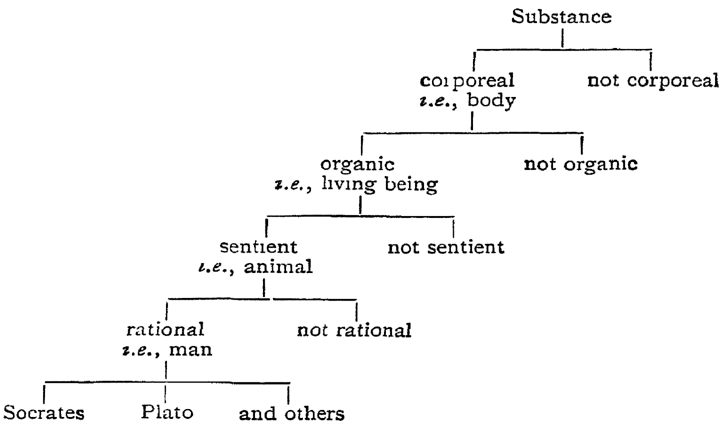
exhaustive. It must not "make a leap"—*i.e.*, leave gaps (*divisio non facit saltum*).

The basis on which the Division is made (see rule *a* above) is called the *principium* or *fundamentum divisionis*. These terms should not be used to signify the basis on which a Classification is made. The relation between the two ideas is seen by referring to the definition of a "natural" class, already given at the beginning of § 8 (rule *c*). It will be seen that a "natural" class is one formed by the *coincidence* of several different *principia* or *fundamenta divisionis*; for each quality common to the whole class, each "point of mutual resemblance" among the members of the class, is a distinct possible basis of *division*.

We will add a few examples; and, first, of processes which *resemble* division. (*a*) "Ireland into Ulster, Munster, Leinster, and Connaught." This is not logical division, but physical "partition,"—the distinction of the various parts of a physical object. A division of "Irishmen" into "Ulster-men," &c., would be correct by the rules. (*b*) "*Mind* into thought, feeling, and will; *body* into extension, resistance, weight," &c. Neither of these is a logical division; both are examples of scientific analysis. (*c*) "*Triangle* into right-angled, acute-angled, obtuse-angled." Correct logical division, exclusive (one basis,—the size of the angles as compared with a right-angle), and exhaustive. (*d*) "*Churches* into Gothic, Episcopal, High, and Low." Here are three bases of division, architecture, government, and dogma; and no account is taken of the *many* different kinds of each.

§ 10. An important traditional method of division, known as **Dichotomy** (called by Aristotle διχοτομία), goes back to Plato. It has been adopted by the mediæval and formal logicians because it appears to provide a theory of division which does not make the process depend entirely on the matter of our knowledge, as classification does (§ 8). But division by dichotomy

is no more independent of our knowledge of the facts than any other kind of classification. This is clearly shown by Aristotle in criticising Plato's view of the process (*An. Prior.*, i. 31). Plato appeared to claim that by this process we might *discover* definitions, or at least *prove* them. Thus it was thought that we could discover what "man" is by taking a suitable *summum genus* to which (we decide) man belongs—*i.e.*, Substance or Being. This we divide into "corporeal" and "not corporeal" being; then, deciding that man belongs to the former class, corporeal substance or body, we divide this into "organic bodies" and bodies "not organic," and decide that man belongs to the former; and so on. Each pair of terms are *contradictories*, and the result may be expressed in a table which was afterwards called the "Tree of Porphyry":—



Aristotle observed, if we did not *already know* the definition required—that man is "corporeal," "organic," &c—we should not know under which of the two contradictory terms to place the term "man"

at each step. And further, only in so far as we know the properties of each thing involved, can we tell whether *any* of the subdivisions is possible or not. Suppose that the term "triangle" is divided thus:—

Triangle

- equilateral
  - *right-angled*
  - not right-angled
- not equilateral

Then we know, from the properties of the triangle, and by no other means, that the last class on the left is impossible. Hence "dichotomy" depends as closely on material knowledge as any other mode of classification.

Has the process of dichotomy any scientific value? It could never be regarded as a scientific form of classification; for if we *know* the sub-classes and divisions included under the *negative* term, it is absurd to indicate them by such a *nomen indefinitum;* and if we do not know them, the negative term is not the idea of a class at all, and we have not made even a purely formal division. The only use of such a method is in occasionally helping us to mark distinctions, as a preliminary to a genuine classification: thus we may find it useful to divide organic beings into sentient and non-sentient, flowers into scented and scentless, fluids into coloured and colourless, &c.

We must add that cases where the negative term is really positive,—"short-hand," so to speak, for one or more positive terms,—do not come under the head of strict dichotomy, for the contrasted terms in each act of division are contraries and not contradictories (ch. II. § 4). Examples of this are: the division of lines into

curved and not-curved (*i.e.*, straight); or the division of men into white and not white (*i.e.*, yellow, red, brown, black). Sometimes, again, when we are arranging objects, as books in a subject-catalogue, and further arrangement becomes impossible, we add a class, "Miscellaneous," which really means "All those *not* in any named class." But we never form a class that can be indicated by a pure contradictory term.

Exercise XI.

Questions on §§ 7 to 10.

(i) *Elementary.*

1. What is Classification?
2. What are the characteristics of a good Classification?
3. What do you understand by the *essential* qualities in classification?
4. What is a *characteristic* quality? Is it always an essential quality?
5. What is an *artificial* classification?
6. State as concisely as possible the rules of logical Division.
7. Examine the following Divisions, and point out which are logical and which are not:—
   (1) Living beings into moral and immoral.
   (2) Men into saints and sinners
   (3) Religions into true and false.
   (4) Man into civilised and black.
   (5) Geometrical figures into rectilinear and non-rectilinear.
   (6) Substances into material and spiritual.
   (7) Metals into white, heavy, and precious.
   (8) Students into those who are idle, those who are athletic, and those who are diligent.
   (9) Books into scientific and non-scientific. [Creighton.]
8. What do you understand by "Division by Dichotomy"?

Give examples, and say what objections are urged against it. Is it a purely formal process? And what is its utility?

(ii) *More Advanced.*

9. Are Definition and Division both necesssary to the full understanding of the meaning of a term? Give reasons for your answer. [O.]

10. State and explain any *general rules* needed for Classification beyond those given for logical Division.

11. Examine critically the distinction of "Natural" and "Artificial" in classification.

12. Show the relation of Classification to logical Division.

13. How far can we distinguish kinds of Classification by reference to the purposes which they serve? [L.]

Part IV.—*The Categories or Predicaments.*

* § 11. We have seen in § 1 that Aristotle makes a fundamental distinction between two kinds of predication: one which tells us what the thing *really is*, another which does not. The former expresses—

(*a*) The definition;

(*b*) Part of the definition,—the genus or *differentia.*

The other kind of predication expresses properties that are "accidental." We may distinguish the two kinds as *essential* and *accidental* predication respectively. Aristotle considers that the latter is improperly called "predication." In the case of essential predication, the predicate necessarily belongs to the subject,—it is *of* the subject; in the case of accidental "predication," the predicate is merely *in* the subject (*Categories*, ch. ii.)

Bearing in mind these distinctions, we proceed to deal with an important question. We know that every judgment is a statement about *facts*,—it affirms (or denies) that something exists in a certain way: "S is

P" affirms that S exists with the qualification P. We may say, in other words, that the judgment predicates *some kind of existence or being* of its subject. Can we *classify* these "kinds of existence" which can be predicated in judgments? This is the question which Aristotle answers in his theory of the Categories. In the first place, we want a general term for the *subjects* of our judgments. The primary subject of judgment—in other words, that which our knowledge first takes hold of or attacks—is the concrete or real "thing" of ordinary experience. These individual *things* or groups of things which meet us perpetually in the course of experience may be called "primary substances," or "primary realities"—*πρῶται οὐσίαι*. These are always subjects, not predicates—*i.e.*, they are what we think about and form judgments of. We wish, then, to classify the modes or forms of being which may be predicated of them.

Consider the typical case of essential predication—that is to say, Definition. Here the subject is a "primary reality," and the predicate consists of a *genus*, with an added qualification distinguishing the thing, the subject, from that genus. Let us call the genus a "secondary substance (or reality)," *δευτέρα οὐσία*. A secondary substance is, therefore, any class, higher or lower, in which a primary substance is included. We have now distinguished two aspects, or two forms, of the first and most fundamental of the "Categories"—"substance" or *οὐσία*; and we note that in every case the primary substance and the secondary substance are essentially related.

Coming now to the predication of what is "accidental," we have to notice that this is possible with both forms of "substance," primary and secondary,—

each may have accidental qualifications *in* it. Aristotle considered that these real qualities or kinds of existence which are predicable of the "substance," fall into nine classes. We give the Greek, Latin, and English words:—

ποσόν	*quantitas*	quantity.
ποιόν	*qualitas*	quality.
πρός τι	*relatio*	relation.
ποῦ	*ubi*	place.
πότε	*quando*	time.
κεῖσθαι	*situs*	posture
ἔχειν	*habitus*	having.
ποιεῖν	*actio*	doing.
πάσχειν	*passio*	suffering.

For example, if the "primary substance," the subject of discourse, is Socrates, we may say of him, taking Aristotle's illustrations of the categories in the order given, that he "is five feet five (in height)," "is scholarly," "is bigger," "was in the Lyceum," "yesterday," that he "reclines," "has shoes on," "cuts," "is cut" (*Categories*, ch. iv.) Two or three of the words are used in a narrower sense than their English renderings suggest. "Relation" consists chiefly of comparatives of adjectives and of ideas which are strictly *correlative*. "Posture" does not mean *position* in the sense of *place* but "attitude." "Having" signifies *condition*—*e.g.*, "armed," "sandalled."

The categories correspond closely with a possible arrangement of the grammatical "parts of speech," substantive and adjective, verb and adverb. Thus, "substance," in its secondary form, is expressed by the Common Noun; "quantity," "quality," and "relation" by the Adjective; "condition," "doing," "suf

fering," by the Verb; "place" and "time" by the Adverb. Nevertheless the categories are not merely grammatical; they represent the various kinds of predicable existence. They should properly be called "Predicables," but long usage has fixed the application of this term to the logical relations explained above (§ 1).

* EXERCISE XII.

We add some general questions on the subjects dealt with in this chapter.

(1) What difficulties attend the process of defining the names of material substances, of sensations and emotions, and how may they be overcome? Illustrate your answer by examples. [O.]

(2) Show that Division belongs to Applied Logic [or, Material Logic], and can have no place in a purely formal system. [O.]

(3) How far are the rules of logical Division of use in actual science? [L.] Or,

What is Scientific Classification? What are the chief difficulties that attend it? [O.]

(4) In what respects is Aristotle's classification of the Predicables superior to the ordinary one? How may we suppose that each was arrived at? [O.] Or,

Criticise (1) the Predicables and (2) the Categories (or Predicaments) as examples of classification. [O.]

(5) "The Categories originally belong to grammar rather than to Logic." How may they be given an intelligible place in a system of Logic? [O.]

(6) Explain the meaning and mutual relation of the terms Predicate, Predicable, Predicament.

* REFERENCES FOR READING.

References for further study of the topics dealt with in this book will be given at the end of our concluding chapter, except in the case of two or three special subjects, for which references are given as we proceed. Among these are the

subjects of our present chapter. For Definition, Division, and Classification: Minto, *Logic*, pp. 82 ff.; Mill, *Logic*, Bk. I., ch. vii., viii., Bk. IV., ch. vii., viii.; Venn, *Empirical Logic*, ch. xi, xii., xiii.; Sigwart, *Logic* (Eng. Trans.), vol. i., §§ 42 to 44. For logical questions regarding Language, (*a*) on "Thought and Language": Lloyd Morgan, *Psychology for Teachers*, ch. viii.; Stout, *Manual of Psychology*, Bk. IV., ch. v.; Lloyd Morgan, *Comparative Psychology*, ch. xv.; (*b*) on Scientific Language: Mill, *Logic*, Bk. IV., ch. iv., v., vi.; Venn, *Empirical Logic*, ch. vi., xxii.; Whewell, *Novum Organum Renovatum*, Bk. IV. The general treatment of "Words" given by Locke, *Essay*, Bk. III., is by no means antiquated, especially when read in connection with Leibniz, *Nouveaux Essais*, Part III. Sound guidance to the modern treatment of the same topic will be found in Prof. James Ward's article on "Psychology," pp. 75 to 78.

* NOTE.

"Real Kinds," and "Essence."

We have referred to the ancient view that in Nature there are fixed, permanent kinds or classes of things which can never pass into one another: and hence a classification which corresponded to these divisions was called "natural," for it was taken to be a recognition of ready-made kinds or classes, given to us in Nature. This view prevailed in ancient science, and was supported by J. S. Mill. The "natural kinds" or "real kinds" were held to be separated from one another by a practically infinite number of differences—in other words, they are *at bottom* different and separate. Hence arose the importance attached to the scheme of predicables given by Porphyry, and to such arrangements as the Porphyrian tree. The natural kinds were supposed to have been fixed at the beginning of things; "human beings," for instance, constituted a "natural kind" in this sense. Hence when we conformed our concepts to the distinct kinds which Nature shows us, any arrangement of the concepts, such as the Porphyrian tree, had a scientific significance,—it dealt directly with relations

of real things; and when seeking for *summa genera*, we were really investigating the fundamental differences in Nature.

It will be advantageous to have a clear answer to the question—How much of this theory is still tenable?

The rigid notion of natural kinds as mutually exclusive —or, as the Greeks would have said, of εἴδη, species, as mutually exclusive—arose like other peculiarities of Greek Logic, because Geometry, *as then understood*, was taken as the type and model of genuine Science. In Greek Geometry, in Euclid, for instance, divisions or classes like *circle*, *polygon*, or *figure*, *line*, were rigidly cut off from one another; there was no conceivable passage from polygon to circle, from ellipse to circle, from figure to line. But according to modern Geometry, a circle may be conceived as an ellipse whose foci coincide, or as a polygon with an infinite number of sides; similarly, by conceiving of a triangle in which the difference between two sides and the third is infinitesimal, so that one angle$=180°$ and the other two$=0°$, we reach the straight line. Hence there may be a *geometrical evolution* of one figure out of another; but the possibility of this does not take away the meaning of the "real kinds" of figure indicated by the names circle, polygon, &c.

The same consideration applies *mutatis mutandis* to "real kinds" in Nature, with the important difference that the *transition forms* actually exist in large numbers. The real kinds run into one another; between them there are margins of debateable ground, as it were,—objects which appear to constitute a transition from one kind to another. Still, there are natural divisions, marked off by typical differences which are obvious and clear; and in this sense we can maintain that real kinds exist in Nature. The theory of Evolution teaches that many, if not all of them, have descended from a common stock, and forbids us to regard the divisions between them as permanent; but it has not taken away the meaning of "real kinds." It has given them a relative instead of an absolute stability

It is an interesting fact that the "natural" classifications, in Botany and Zoology, were worked out before the Evolution

theory was generally accepted; and Evolution has given them a fuller meaning. A natural classification is now a genealogical tree; and the words "kind," "affinity," "genus," "family," are no longer mere metaphorical expressions.

Closely connected with the foregoing subject is the question, what do we mean by "essence" and "essential" qualities or characteristics, by the "nature" of a thing, or by what the thing "really is"? These various phrases may be taken as synonymous. When referring to Aristotle, we said that the "essential" qualities are those without which the thing could not be what it is; and subsequently we said that the essential qualities are those which have a determining influence over the largest number of others. From the point of view of modern knowledge, the former statement is less satisfactory than the latter. The latter emphasises our departure from the doctrine of the fixity of classes; and it may be briefly expressed thus: *a thing is what it does.* What it *is*, is shown by the way it responds when *acted on* by any thing or person. To say that a thing has a nature or essence at all, means simply that it is capable of definite modes of behaviour in response to what is done to it. Thus, let us consider some substance which is being *used* by man for his own purposes. However *plastic* it is to his designs, whatever transformations he makes it undergo, there remains something which he cannot alter, and which seems indeed to dictate the limits within which his transforming power over the substance shall extend. This is the truth which underlies the ancient doctrine of fixity. There is a "nature" of the thing, not separable from the changeable qualities (as Aristotle supposed the "essence" to be from the "accidents"), but *revealed in* the changeable qualities as a law controlling their changes in action. Hence to understand things we must make an extensive study of their behaviour, and if possible *make* them act,—experiment with them. By such means we gradually learn certain characteristic practical ways of behaviour on the part of things. These "ways of behaviour" are what the "attributes" or "qualities" spoken of in Logic, really mean; and they are comprehended, more or less, in our *concepts.*

CHAPTER VI.

MEDIATE INFERENCE AND THE ARISTOTELIAN SYLLOGISM.

§ 1. WE have dealt with the forms of Immediate Inference, in which from a single proposition we derived another, stating the same relation between S and P, but from a different point of view, as it were—in Conversion, for instance, we find what the given proposition tells us of the relation of P to S; in Obversion, of S to not-P, and so forth.

The question has been raised whether these changes in a given proposition have a right to be called **Inference.** We defined Inference (ch. I. § 7) as a process in which from given facts, or given propositions, we pass to a **new proposition** distinct from them—*i.e.*, to a new fact or truth. This does not mean an *absolutely new* proposition. Such a proposition would be *unconnected* with the premises—*i.e.*, would be absolutely discontinuous with previous knowledge. It would be a contradiction in terms to say that such a proposition was *inferred* at all. But the conclusion of an inference states a relation which is not stated in any one proposition among those which form the premises. Now in Immediate Inference we do not pass to a proposition which is "new" even in this second sense of the word; for the conclusion states no new relation. On the other hand, in Immediate Inference we have not

merely a *verbal* change—*i.e.*, the same relation stated in different words. We have another side or aspect of the original fact stated. On the class-view of propositions, this is evident; and it appears to be equally true on any other interpretation. We begin with a given relation between two classes or spheres, as "All S is P"—

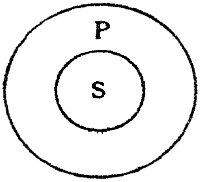


Fig. 18.

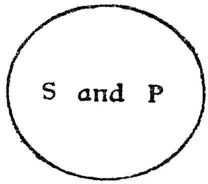


Fig. 19.

The diagrams make it visibly evident that the relation of S and P here spoken of has several aspects, of which the given proposition states only one—viz., that all S is included in P. Another aspect is, that some at least of P is included in S (the converse); another that no S is outside of P (the obverse); another, that nothing outside of P is in S (the contrapositive); another, that some at least of what is outside S is outside P (the inverse). Hence, in Immediate Inference, we have not the same relation between S and P restated (a merely verbal change); and we have not a new relation between S and P stated (a complete inference); but we have another aspect of the original relation stated.

Immediate Inference is not a trivial matter. It is of real practical importance. In the interpretation of legal documents, rules, &c., the real implications of the statements made will be much more evident if we remember these elementary logical processes. In ordinary thought we are constantly making mistakes through neglect of them; this is seen especially in the tendency to convert

A propositions simply, and to give wrong interpretations to exclusive or exceptive propositions,—to take "only S are P," for instance, as though it implied that "All S are P."

We now come to **Mediate**, as distinct from Immediate, Inference. We shall begin by defining the process in its simplest form. We must have two propositions which are not equivalent, from which we derive a third proposition that could not be obtained from either of the others taken alone. The two given propositions are the **premises**, the third is the **conclusion**. It is evident that we do not necessarily derive a conclusion from the combination of *any* pair of propositions whatever—*e.g.*, "All men are fallible" and "All metals are elements." These statements have nothing in common. But the following combinations will yield conclusions :—

Premises	All men are fallible. All kings are men.
Conclusion	All kings are fallible.
Premises	All metals are elements. Gold is a metal.
Conclusion	Gold is an element.

In order that two propositions may result in a conclusion they must have something in common; and this means that when expressed in logical form they must have a **common term**, otherwise there is no link of connection between them.

The typical example of mediate inference in what for the present we must regard as its simplest form is, therefore :—

Premises	All M is P. All S is M.
Conclusion	All S is P.

The relation expressed in the conclusion, between the terms S and P, is obtained because S and P are compared in turn with the same term M. Thus their relation to each other is found *by means of* this comparison; for this reason the process is called "mediate inference," and the conclusion is said to be "mediated." An argument of this type was called by Aristotle a **syllogism** (συλλογισμός, a "thinking together"—*i.e.*, thinking two propositions together). Syllogism may be defined as Jevons has done, almost in the words of Aristotle: "The act of thought by which from two given propositions we proceed to a third proposition, the truth of which necessarily follows from the truth of these given propositions."

§ 2. In the typical syllogism, the object of the reasoning is to decide something about a particular case. In order to do this, we look for a general rule which is accepted, and under which the case comes. The rule is stated in one premise (the first premise in both the foregoing examples); the particular case is brought under it in the other. Aristotle maintained that all true reasoning can be expressed in this form, and in particular that the syllogism is the appropriate form for scientific reasoning. But he had also a practical aim in working out the doctrine of the syllogism; to teach the art of reasoning,—the means of presenting propositions in such a light as to compel assent to them. The Sophists had attempted this; but in order to gain acceptance of a proposition, they relied on mere persuasion, on "rule-of-thumb" methods, on questionable rhetorical devices or verbal tricks. The syllogism of Aristotle is essentially a process of strict demonstration, which establishes some fact of statement by connecting it with a general principle, a rule

or law which is admitted. As we have seen, they are connected by having a common term. The truth of the premises must be granted; the doctrine of the syllogism does not give us any means of examining that question; it shows us how to estimate their *inter-dependence* when they are accepted as reliable. It affords a method of testing given arguments; for when we have expressed the statements in logical form and compared them according to syllogistic rules, we see at once whether they are really connected in the way which the argument asserts, or not.

We have now to investigate the possible different forms of syllogistic argument.

The syllogism is composed of logical propositions, which can only have four forms, A, E, I, O. We have to find the different ways in which these may be combined so as to lead to correct conclusions, and to show that *no other* combinations yield correct conclusions.

Suppose that we have to prove a universal affirmative conclusion, "All S is P"; how may this most compendiously be done? It is required to prove something of a whole class,—to prove that the quality P is possessed by a whole class S. Is P admitted to be a quality of any higher class to which S undoubtedly belongs? Suppose that M is admitted to be such a class—*i.e.*, that the qualities of M are predicated of all S, and that the quality P is predicated of all M. Then it follows at once that the quality P must be predicated of all S:—

$$\left\{\begin{array}{l}\text{P is predicated of all M.}\\ \text{M is predicated of all S.}\end{array}\right.$$
$$\therefore \text{P is predicated of all S.}$$

This statement of the syllogism is based on the predicative view of propositions, and is usually adopted by Aristotle. Expressed according to the Class view, the argument is:—

$$\begin{cases} \text{All of M is in P.} \\ \text{All of S is in M.} \end{cases}$$
$$\therefore \text{All of S is in P.}$$

We have here three A propositions; hence this form of syllogism is referred to as AAA. As we shall see, this is the only way in which an A proposition can be syllogistically proved. We shall denote the syllogism thus:—

$$\begin{array}{l} \text{MaP,} \\ \underline{\text{SaM;}} \\ \therefore \text{SaP.} \end{array}$$

As already indicated, MaP, SaM, are the **premises**, and SaP the **conclusion**. We shall (apart from an occasional exceptional case) always use S to denote the subject-term of the conclusion (hence also the matter about which the conclusion is to be proved); and, for clearness, we shall draw a line between the premises and the conclusion. The term M, which is common to the two premises, is called the **middle term** (τὸ μέσον, the mean). For one reason, it is the *means* by which the two propositions are connected, or the other two terms compared. The other two terms, S and P, are the **extremes** (ἄκρα). Comparing the

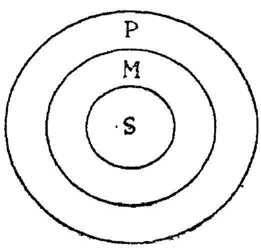


Fig. 20.

extent of the terms S, M, P, in our given syllogism AAA, we see that the extent of S is less than that of M, and the extent of M less than that of P; for the argument states that S is *in* M, and M *in* P. Hence in the syllogism AAA, S is called the **minor term** (τὸ ἔλαττον, τὸ ἔσχατον), and P the **major term** (τὸ μεῖζον, τὸ πρῶτον); and we have another reason for calling M the "middle" term.[1] The relation of the three terms is evident in fig 20, which represents the most usual form of the syllogism AAA.

The conclusion was often called the "problem" (*πρόβλημα*, *quæstio*)—*i.e.*, the question in dispute. What the conclusion is to be, is usually known beforehand; the subject of it is always known, and usually also what we desire to prove of the subject. Now the terms which are in their extent minor and major in the syllogism AAA, stand respectively as the subject and predicate of the conclusion. Hence by analogy Aristotle speaks of the subject of the conclusion in *any* syllogism as the "minor term," and the predicate as the "major term," whatever their relative extent may be. Hence when speaking of syllogisms in general, we shall always mean by the **minor term** the **subject**, and by the **major term** the **predicate**, of the conclusion. This is the only proper definition of the names in question. This being understood, the premise which contains the major term is called the **major premise**, the premise which contains the minor term is called the **minor premise**. It must be carefully remembered that **whether the major premise or the minor stands first, is logic-**

[1] The case, which *might* occur in the syllogism AAA, of two or all of the terms S, M, P, being co-extensive, is set aside, for the purposes of this definition.

ally indifferent. The two following syllogisms are the same:—

(1) MaP,	(2) SaM,
SaM;	MaP;
∴ SaP.	∴ SaP.

It is, however, an almost invariable custom to place the major premise first, as in (1), and in each of the previous examples. All difficulty over the right use of the names "major" and "minor" disappears when we remember that **we start from the conclusion,** which is the question at issue.

* In Aristotle's treatment, propositions are usually formulated according to the predicative view, expressly and explicitly:—

A is predicated of B,
B is predicated of Γ.

This expression, with the predicate before the subject, is the natural one according to the Greek idiom, but not in Latin or English. In Greek we should naturally say τὸ Α παντὶ τῷ Β ὑπάρχει, or τὸ Α κατὰ παντὸς τοῦ Β κατηγορεῖται; but in Latin or English, *omnis B est A*, all B is A. And when the propositions are written as Aristotle expresses them, and also with the major premise first, then the major term is the first term, and the minor term the last: "A is predicated of B, B is predicated of Γ." Hence in Aristotle πρῶτον and ἔσχατον, *first* and *last*, are far more prominent expressions than μεῖζον and ἔλαττον, *major* and *minor*, which only apply to what is with him the rarer "extension" or "class" interpretation.

§ 3. The conditions on which the **formal validity** of a syllogism depends, have for long been drawn up in a group of rules, known as the Rules or Canons of the Syllogism. The most convenient arrangement gives us eight rules.

I. Relating to the structure of the syllogism :—

(1) A syllogism must contain three, and only three, terms.

(2) A syllogism must contain three, and only three, propositions.

II. Relating to *quantity* :—

(3) The middle term must be distributed in one, at least, of the premises.

(4) No term must be distributed in the conclusion unless it was distributed in the premise which contains it.

III. Relating to *quality* :—

(5) From two negative premises there can be no conclusion. In other words: One, at least, of the premises must be affirmative.

(6) If one premise is negative, the conclusion must be negative, and if the conclusion is negative, one premise must be so.

IV. *Corollaries* :—

(7) From two particular premises, there can be no conclusion.

(8) If one premise be particular, the conclusion must be particular.

(9)

The first two rules tell us what a syllogism is. It consists of the "synthesis" (combination or union) of two propositions by means of a common term, and the statement of the result in a third proposition. Hence (1) there must be three propositions only. If there are more than three, we have more than one syllogism; if less than three, we have no syllogism, but either an Immediate Inference or a mere assertion, giving a statement as a reason for itself: "I know it because I know it."

Also (2) there must be three terms only, for the two premises have a common term. If there are less than three terms, we have no syllogism; if there are more, we have either no syllogism or more than one: usually no syllogism, because the premises have no link of connection, and contain four different terms between them. Such mistakes are possible because of the ambiguity of language. If any term is used ambiguously, it is really *two terms;* hence the syllogism containing it has at least four terms, and is not a true syllogism at all, though at first sight it may appear to be one. If there is ambiguity, it is most likely to occur in the middle term; **ambiguous middle** is the most common breach of rule 1.

Some good examples are given by Jevons. "If we argue that 'all metals are elements and brass is metal, therefore it is an element,' we should be using the middle term *metal* in two different senses, in one of which it means the pure simple substances known to chemists as metals, and in the other a mixture of metals commonly called metal in the arts, but known to chemists by the name alloy. In many examples which may be found in logical books the ambiguity of the middle term is exceedingly obvious, but the reader should always be prepared to meet with cases in which exceedingly subtle and difficult ambiguities occur. Thus it might be argued that 'what is right should be enforced by law, and that charity is right and should therefore be enforced by the law.' Here it is evident that *right* is applied in one case to what the conscience approves, and in another case to what public opinion holds to be necessary for the good of society." We add one or two further examples of "ambiguous middle" which the student may examine for himself. "All criminal actions ought to be punished by law; prosecutions for theft are criminal actions, and therefore ought to be punished by law" (De Morgan). "Every good law should be obeyed; the law of gravitation is a good law, and there-

fore should be obeyed" (Creighton). "Partisans are not to be trusted; the supporters of the government are partisans, and therefore are not to be trusted."

For like reasons, if the subject, or the predicate, of the conclusion is used in a different sense there from that which it bears in its premise, the inference is invalid.

The violation of the third rule is called the fallacy of **undistributed middle.** The rule states that the *whole extent* of the middle term must be referred to universally in one premise, if not in both. For if the middle term is not compared in its whole extent with one at least of the extremes, we may be referring to one part of it in one premise, and quite another part of it in the other; hence there is no real middle term at all, but practically four terms.

Consider the premises, "All rash men are confident; all brave men are confident." These propositions tell us nothing about the relation of "the rash" to "the brave"; they only tell us that the rash are a part of the class of "confident persons," and the brave are also a part, as fig. 21 shows. The premises allow of the circles "rash" and "brave" being placed anywhere within the circle "confident," either overlapping or outside of each other. Jevons adds an example in which all the propositions are true, while the argument has an undistributed middle. "The two propositions, 'All Frenchmen are Europeans; all Parisians are Europeans,' do not enable us to infer that all Parisians are Frenchmen. For though we know, of course, that all Parisians are included among Frenchmen, the premises would allow of their being placed anywhere within the circle of Europeans."

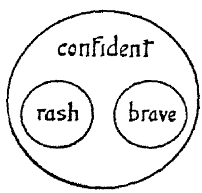


Fig. 21.

The fourth is a double rule. (*a*) The minor term

must not be distributed in the conclusion unless it is distributed in the premise in which it occurs; the breach of this rule is called an **illicit process of the minor.** (*b*) The major term must not be distributed in the conclusion unless it is distributed in the premise in which it occurs; the breach of this rule is called an **illicit process of the major.** The proof of the rules consists in seeing that "if an assertion is not made about the whole of a term in the premises, it cannot be made about the whole of that term in the conclusion without going beyond what has been given." The conclusion must be no more definite than the premises warrant.

We take again an example given by Jevons. "If we were to argue that 'because many nations are capable of self-government, and that nations capable of self-government should not receive laws from a despotic government, therefore no nation should receive laws from a despotic government,[1] we should be clearly exceeding the contents of our premises. The minor term, *many nations*, was undistributed in the minor premise, and must not be made universal in the conclusion. The premises do not warrant a statement concerning anything but the *many nations capable of self-government*." An illicit process of the minor is generally easy to detect; in the case of the major, it is much less apparent. The following example, given by Professor Creighton, might pass for a correct syllogism, especially as the conclusion may be accepted as true: "All rational beings are responsible for their actions; brutes are not

[1] The form of the syllogism, as stated by Jevons, is this :—

$$\begin{array}{l} \text{SiM,} \\ \underline{\text{MeP};} \\ \therefore \text{SeP,} \end{array}$$

with the minor premise first. It is evident that S is distributed in the conclusion and not in the minor premise.

rational beings; therefore brutes are not responsible for their actions." The form is—

$$\begin{array}{l} \text{MaP,} \\ \underline{\text{SeM};} \\ \therefore \text{SeP.} \end{array}$$

Here the major term P—*i.e.*, "beings responsible for their actions"—is distributed in the conclusion, but was not distributed when it appeared as the predicate of an A proposition in the major premise. Hence we have an illicit major. The major premise only tells us that "rational beings" are *some at least* of "beings responsible for their actions." As far as this proposition is concerned, there may be responsible beings who are not rational. Hence the exclusion of brutes from the class "rational beings" does not necessarily exclude them from the class "responsible beings."

The rule forbids us to take more of a term in the conclusion than is referred to in the premise; but it does not forbid us to take less. There is no illicit process when a term is *distributed in the premise* and undistributed in the conclusion; as in the following: "All M is P, all S is M; ∴ some S is P."

The fifth rule states that one premise, at least, must be affirmative; or, which is the same thing in different words, from two negative premises there can be no conclusion. A negative major premise is equivalent to a denial of any connection between the major term and the middle, a negative minor premise is equivalent to a denial of any connection between the minor term and the middle. Hence there is no means of comparing the major and minor terms: there is no middle term, and the condition of a valid syllogism does not exist.

Jevons, in his *Elementary Lessons in Logic*, has given the following explanation of the case,—not of uncommon

occurrence,—where from two apparently negative premises we obtain a valid conclusion. "It must not, however, be supposed that the mere occurrence of a negative particle ("not" or "no") in a proposition renders it negative in the manner contemplated by this rule. Thus the argument—

'What is not compound is an element,
Gold is not compound;
Therefore gold is an element,'

contains negatives in both premises, but is nevertheless valid, because the negative in both cases affects the middle term, which is really the negative term 'not-compound.'" Now this explanation applies to an example which Jevons himself gives, in his *Principles of Science*, as a case where two negative premises give a valid conclusion. The example is—

"Whatever is not metallic is not capable of powerful magnetic influence,
Carbon is not metallic;
Therefore carbon is not capable of powerful magnetic influence."

This argument *appears* to consist of three *E* propositions. But the same explanation holds; the middle is the negative term "not-metallic," and the syllogism is really of the form EAE.

The sixth rule says that if one premise is negative, the conclusion must be negative, and *vice versâ*. For, if one premise is negative, the other must be affirmative (by rule 5). The affirmative premise asserts some amount of coincidence between *one extreme* and the middle term,—that all or part of it is *in* the middle term; the negative premise says that all or part of *the other extreme* is outside the middle term. Hence the only conclusion can be, that all or part of this second extreme is outside the area of coincidence of the first extreme and the middle term. This is a negative conclusion. Further, a negative conclusion implies a negative premise. For it asserts that one

extreme is wholly or partly outside the other; and this result is reached by comparing both extremes with the middle term. Hence one of the extremes must be wholly or partly outside the middle term,—that is, one of the premises must be negative.

The seventh rule says that from two particular premises there is no conclusion. This may be deduced from the preceding rules. The only particular propositions are I and O; and as each of them may be either major or minor premise, there are four possible cases, II, IO, OI, and OO. (*a*) Of these, OO is excluded by rule 5. (*b*) In II, *no term* is distributed, hence rule 3 is broken. (*c*) In IO and OI, *only one term* is distributed, namely the predicate of O. If this is not the middle term, rule 3 is broken. If it is the middle term, then neither the minor nor the major term is distributed. But the conclusion must be negative (rule 6), and therefore its predicate (the major term) is distributed. And as the major term was not distributed in its premise, we have a breach of rule 4.

The eighth rule says that if one premise is particular, the conclusion must be particular. The proof of this lies in seeing that one universal and one particular premise can only distribute enough terms to warrant a particular conclusion by the previous rules. There are eight combinations possible: AI and IA, AO and OA, EI and IE, EO and OE. (*a*) The last pair are excluded by rule 5. (*b*) In AI and IA, *only one term* is distributed (the subject of A); this must therefore be the middle term (rule 3). That is to say, the minor term is not distributed in its premise. Therefore it must not be distributed in the conclusion (rule 4); that is, the conclusion must be particular. (*c*) In AO and OA, and in EI and IE, *two terms* are distributed

(the subject of A and the predicate of O; or the subject and predicate of E). One of these must be the middle term (rule 3); hence there is *only one* of the extremes distributed in the premises. Now one premise is negative, therefore the conclusion is negative (rule 6), and the major term (its predicate) is distributed; hence the other extreme, which is the minor term, the subject of the conclusion, cannot be distributed. The conclusion therefore must be particular. *No conclusion* is possible from the premises IE (see § 4, below).

The following are examples of questions which may be solved by direct application of the above rules:—

(1) Prove that when the minor term is predicate in its premise, the conclusion cannot be A. [L.]

It is required to show that the conclusion is either negative, or, if affirmative, is not A. Now it is given that the minor term is predicate in its premise. It must be either distributed or undistributed. If the minor term is distributed in its premise, this premise is negative, and therefore the conclusion is negative (rule 6). If the minor term is undistributed in its premise, it is undistributed in the conclusion (rule 4)—*i.e.*, the conclusion is particular.

(2) If the major term of a syllogism be predicate in the major premise, what do we know about the minor premise? [L.]

The major term must be either distributed or undistributed in the major premise. If distributed, the major premise is negative, and therefore *the minor is affirmative* (rule 5). If undistributed, it is undistributed also in the conclusion (rule 4); and as it is the predicate of the conclusion, the conclusion must be affirmative; therefore both premises are affirmative[1]—*i.e.*, *the minor is affirmative.*

§ 4. Syllogisms may be divided into three classes,

[1] For, if one premise were negative, the conclusion must be negative (rule 6), which it is not.

called figures (σχήματα), according to the position of the middle term.

In the **first figure** the middle term is the subject of one premise and predicate of the other; the general form is—

$$\begin{array}{rll} & M & P, \\ & S & M; \\ \hline \therefore & S & P. \end{array}$$

We leave the quantity and quality of the propositions undetermined, as we have only to indicate the position of M as compared with that of S and P. In this arrangement of the terms, M has a *middle position:* this appears more clearly when the premises are written as Aristotle usually writes them, "P is predicated of M, M is predicated of S." This was Aristotle's reason for the name "middle term." All the syllogisms given in §§ 1, 2 of this chapter are of the first figure.

In the **second figure** the middle term is predicate **in** both premises :—

$$\begin{array}{rll} & P & M, \\ & S & M; \\ \hline \therefore & S & P. \end{array}$$

In the **third figure** the middle term is subject in both premises :—

$$\begin{array}{rll} & M & P, \\ & M & S; \\ \hline \therefore & S & P. \end{array}$$

This was Aristotle's principle of division, and is very simple: in fig. i. M is middle (its proper position); in fig. ii. it is predicate in both premises; in fig. iii. subject in both. Aristotle did not require to make a distinction between the major and minor *premises*. This distinction was made by later logicians, and was taken to be of great importance by the mediæval writers on the

subject. Hence Aristotle's first figure was divided intc two parts, one of which was afterwards made into a separate "fourth figure." In this case we must distinguish between the major and minor premises in distinguishing the figures. In fig. i. the middle term is subject in the major premise and predicate in the minor :—

$$\begin{array}{rll} & M & P, \\ & S & M; \\ \hline \therefore & S & P. \end{array}$$

In fig. iv. the middle term is predicate in the major and subject in the minor :—

$$\begin{array}{rll} & P & M, \\ & M & S; \\ \hline \therefore & S & P. \end{array}$$

It is not desirable in elementary Logic to depart from the traditional recognition of the fourth as an independent figure.

Besides the division into four figures, syllogisms are divided into classes according to the quantity and quality of the premises. These classes are called **moods** (*modi*, τρόποι τῶν σχημάτων). Now there are two premises, and each premise must be A, E, I, or O. Hence the greatest possible number of moods will be the number of permutations[1] of these four letters, two at a time. There are, in all, sixteen such permutations :—

AA	EA	IA	OA
AE	EE	IE	OE
AI	EI	II	OI
AO	EO	IO	OO

[1] The name permutation is used in its strict mathematical sense, according to which AE and EA, for example, are different "peimutations."

In this table AA means, of course, that both premises are universal affirmative; IA, that the major is particular affirmative, the minor universal affirmative; and so on. In each case the first of the two letters denotes the major, and the second the minor, premise.

We cannot take for granted that all of these are valid—that is, lead to correct conclusions—in each or any figure. The valid moods will have to be found in another way. Aristotle discovered the valid moods by testing one by one the possible cases in each figure. But the principal methods by which he examined or tested them were afterwards formally drawn up, and known as the Rules or Canons of the Syllogism, as explained in the previous section.

There are, then, sixteen possible moods to examine. Seven of these lead to no valid conclusions, in any figure, by the rules: EE, EO, OO, OE are excluded by the rule against two negatives, and IO, II, OI by that against two particulars. This leaves us with nine possible moods—AA, AE, AI, AO, EA, EI, IA, IE, OA. We have to examine each of these in each of the four figures, testing the results by the rules.

But it may be further proved from the general rules of the Syllogism alone that IE can yield no conclusion in any figure:—

If possible, let there be a conclusion: then it must be negative.
And every negative proposition distributes its predicate (the major term);
But the major premise I distributes neither subject nor predicate;
Therefore there would be an Illicit Major.

We are thus left with eight moods, and we shall ex-

amine each of these in each of the four figures, testing the results by the rules.

§ 5. The form of the first figure is :—

$$\begin{array}{ll} M & P, \\ S & M; \\ \hline \end{array}$$
$$\therefore\ S \quad P.$$

The major premise stands first, according to the usual custom; P, the predicate of the conclusion, being the major term, and S, the subject of the conclusion, the minor.

(1) The mood AA in fig. i. is :—

All M is P.
All S is M.

When the distribution or non-distribution of each term is considered, it is easily seen that the only conclusions about S, valid by all the rules, are :—

(*a*) All S is P.
(*b*) Some S is P.

The second of these is called a "weakened conclusion," because it infers less than the premises warrant; for the term S is distributed in its premise and undistributed in the conclusion. But this is not technically a logical fault.

(2) The mood AE in fig. i. would be :—

All M is P,
No S is M,

from which there is no valid conclusion about S; for if there were, it must be a negative conclusion, distributing its predicate P, and thus giving an Illicit Major.

(3) The mood AI in fig. i. is :—

All M is P,
Some S is M,

from which the only valid conclusion about S is :—

Some S is P.

(4) The mood AO in fig. i. would be :—

All M is P,
Some S is not M,

from which there is no conclusion, for the same reason as in (2)—it would lead to an Illicit Major.

(5) The mood EA in fig. i. is :—

No M is P,
All S is M,

from which the only valid conclusions about S are :—

(*a*) No S is P,
(*b*) Some S is not P,

the second being the weakened conclusion.

(6) The mood EI in fig. i. is :—

No M is P,
Some S is M,

from which the only valid conclusion about S is :—

Some S is not P.

(7) The mood IA in fig. i. would be :—

Some M is P,
All S is M,

from which there is no conclusion, for the premises involve an Undistributed Middle.

(8) The mood OA in fig. i. would be :—

Some M is not P,
All S is M,

from which there is no conclusion, because of the Undistributed Middle.

We have thus found six moods[1] in fig. i., giving valid conclusions about S, of which two are "weakened moods" (*i.e.*, have weakened conclusions).[2] We shall name each of them by the symbols of its three propositions. They are :—

AAA, EAE, AII, EIO,

together with the two weakened moods :—

AAI, EAO.

By similar applications of the rules the student will be able to arrive at the following results. In the **second figure**, where the middle is predicate in both premises, the valid moods, including two weakened moods, are these: EAE (together with the corresponding weakened mood EAO), AEE (and AEO), EIO, AOO. In the **third figure**, where the middle is subject in both premises, the valid moods are these: AAI, IAI, AII, EAO, OAO, EIO. In the **fourth figure**, where the middle is predicate in the major premise and subject in the minor, the valid moods, including one weakened mood, are these: AAI, AEE (and AEO), IAI, EAO, EIO.

* It is an error to say that Aristotle overlooked the fourth figure; but he paid no attention to it beyond recognising its

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[1] Henceforth we shall use the term "valid *mood*" to signify the two premises *plus* their valid conclusion.

[2] Also called "subaltern moods."

possibility. He considered it an awkward and useless variety of the first figure. His pupils, Theophrastus and Eudemus, worked out its five principal moods and added them as "indirect moods" to fig. i. Some writers have supposed that Claudius Galenus was the first to make these moods into a separate "fourth figure"; hence the fourth has been called the "Galenian figure."

The formation of the five moods as a subordinate variety of fig. i. may be based on suggestions made by Aristotle, *An. Prior.*, i. 7 and ii. 1. We take the eight possible combinations of premises which we have examined in fig. i., namely:—

MaP SaM	MaP SeM	MaP SiM	MaP SoM
MeP SaM	MeP SiM	MiP SaM	MoP SaM

and ask, not (as before) which of them yield valid conclusions *about S*, but which of them yield valid conclusions *about P?* This involves making the old major premise into a new minor, and *vice versâ*, so that the middle term becomes predicate of the major premise and subject of the minor. Testing the new moods by the rules, as before, we find that they lead to valid conclusions in four cases, giving the moods indicated above as belonging to the "fourth figure," except EIO, which (as we shall see, § 9) may be derived from EAO. The student should verify this result. The same process may be gone through in the second and third figures; but it will be found that no *new moods* are thus obtained.

§ 6. Aristotle held that there is one canon, rule, or axiom,[1] to which all true reasoning conforms, either directly and visibly in its very expression, or, if not, in such a way that it can be expressed in direct conformity with the rule.

The canon is a concise statement of what mediate

[1] On the meaning of "Axiom," see above, pp. 45, 46.

inference or syllogism really is. Syllogistic inference is the application of a general principle (affirmative or negative) to some particular case or cases or to a whole class of cases. In the syllogism which expresses the true nature of reasoning, the general principle is the major premise; the assertion that something falls under it is the minor premise. Such a syllogism shows the rule of reasoning by the way in which it must naturally be expressed, and hence was called by Aristotle a **perfect** syllogism. In a perfect syllogism, the major premise must be universal (affirmative or negative), for it states the general principle which is to be applied, and therefore it *naturally* comes *first;* the minor premise must be affirmative (and may be universal), for it states that a given case comes under this principle. Hence all the syllogisms of the first figure, and no others, are perfect; for they alone conform to the special rules :—

(*a*) The major premise must be universal.

(*b*) The minor premise must be affirmative. These are called the **special rules or canons of the first figure,** to distinguish them from the general rules (§ 3) which are applicable to all the figures.

The usual statement of the Aristotelian canon is called the *dictum de omni et de nullo.* It has come down to us from the mediæval logicians: **Whatever is predicated, affirmatively or negatively, of all of a class, must be predicated, affirmatively or negatively, of everything contained under that class.** The affirmative predication of the class is *de omni*, the negative *de nullo.* The application of this to the rules of the first figure is obvious. The major premise makes a statement about all of a class, so that it must be universal, and may be negative; the minor asserts that

a given case comes under that class, so that it must be affirmative; and, in accordance with the *dictum*, the conclusion makes the original statement of the given case.

* There are different views of the logical significance of the *dictum;* and these differences depend entirely on the different views of predication [ch. IV. §§ 2, 3] which are adopted. This has been most clearly pointed out by Martineau.[1] "Were the *dictum* of Aristotle shaped into perfect conformity with the *class* theory [ch. IV. § 2] it would be expressed thus: Whatever is found in a contained class is in the containing. When, instead of this, it is said, Whatever is *predicated of* a class is predicated of the individuals or species within it, the expression is of a mixed kind. It begins, in its description of the major premise ('whatever is predicated of a class'), without committing itself to any particular theory of predication; but immediately, in its description of the minor premise (the individuals or species within it), it adopts the doctrine of subject within predicate as class within class." Accordingly, critics of the *dictum* deal with it as if pledged to the *class* theory of predication, and regard its authority as destroyed when this doctrine is refuted. This view is expressly taken by Mill (*Logic*, Bk. II., ch. ii. § 3), who declares that if we consider the *dictum de omni* as the foundation of the syllogism, we are committed to that inadequate account of the meaning of a proposition which supposes it to refer something to (or exclude something from) a *class*, and are therefore committed to the following interpretation of the typical syllogism —*i.e.*, "All of the class M is in the class P, all of the class S is in the class M, therefore all of the class S is in the class P." On this ground[2] Mill rejects the *dictum*, and proposes instead of it two alternative but equivalent axioms which he expresses thus (limiting ourselves, for brevity's sake, to the *affirmative* forms): "Things (attributes) which

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[1] *Theory of Reasoning*, Essays, vol. iii. pp. 431-434.

[2] And also because he regards the *dictum* as an "identical proposition," giving us no information.

coexist with the same, coexist with one another," "Whatever is a mark of any mark, is a mark of that which this last is a mark of." These lead to the following interpretations of the syllogism: "The attributes signified by M always coexist with (or are accompanied by) those signified by P; the attributes S always coexist with the attributes M; therefore the attributes S always coexist with the attributes P"; or again: "the attributes M are a mark of the attributes P; the attributes S are a mark of the attributes M; therefore the attributes S are a mark of the attributes P."

* On the foregoing theory Martineau justly observes: "All that is here achieved is, avowedly, the substitution of the maxim of comprehension for the maxim of extension; and the author imagines that by doing this he cancels the *dictum*. His own final rule is nothing but a translation of Kant's 'Supreme Rule of the Syllogism,' *nota notæ est etiam nota rei ipsius*.[1] Kant himself, after enunciating this rule, immediately proceeds to show how the *dictum* arises from it as a direct corollary. And subsequent writers have very properly given both the scholastic and the Kantian maxims as two representations of the same truth [in relation to the *comprehension* of the terms, *nota notæ est etiam nota rei; repugnans notæ repugnat etiam rei;* in relation to the *extension* of the terms, *quidquid de omnibus valet, valet etiam de quibusdam et singulis; quidquid de nullo valet, nec de quibusdam nec de singulis valet*]. Aristotle himself is not in the least pledged to the one form of the axiom more than the other. His clearest and most concise expression of it is perfectly neutral: 'Whatever is said of the predicate [M] is said of the subject [S].'[2] Indeed, the two modes of statement are adverted to by Aristotle, and are expressly declared to be equivalent: 'To say that one thing is completely included in another, and to say that this other is universally a predicate of the one, *amount to just the same thing*.'"[3] This is not the only case in which confident criticisms made by later and ill-informed writers are found to have been anticipated and answered by Aristotle himself.

[1] Kant's short treatise on "The Four Syllogistic Figures," § 2 (Abbott's Translation).

[2] *Categories*, § 5.

[3] *An. Prior.* i. § 1.

The first figure is of the greatest importance both in science and practical life. Whenever we apply previous knowledge to a given case, we employ one of the moods of this figure,—although no syllogism, and even no distinct propositions, may be consciously before our minds. Sometimes even an ordinary "judgment of perception," or *recognition* of an object, may be analysed in this form. Crusoe's footprint affords an example; the minor premise, being our perception of the general qualities of the particular fact, may be placed first:

> This mark in the sand is a mark having such and such qualities of size, shape, &c.;
>
> Every mark having these qualities is the imprint of a man's foot;
>
> Therefore this mark is the imprint of a man's foot.

The process by which the conclusion is reached "passes in a flash" through the mind, in such cases; but none the less it is a true syllogistic argument in fig. i.

The four moods of the first figure are known by the names:—

Barbara, Celarent, Darii, Ferio.

These names contain the vowels of the respective moods in their proper order; and the student will shortly see this by no means exhausts their "connotation." Our present object, however, is to discuss the special characteristics of the four moods.

The mood *Barbara* is so familiar and constant a mode of thought that its importance usually escapes attention. But we might know beforehand the large part that it must play in science; for science seeks for results which are *laws*—*i.e.*, statements true universally about certain kinds of fact. Every time we explain a fact by the law, *i.e.*, find a new complete application

of the law, we make a syllogism in *Barbara*—not *formally*, explicitly, or in expression, for this would make the reasoning long and tedious; but implicitly at every step we reason in this form.[1]

The following are examples of this mood, regarded as the application of a law—

When a material substance is heated, it expands;
Glass is a material substance;
∴ Glass expands when heated.

Hence we may explain the liability of thick glass to crack more easily than thin glass, when heated:—

Hotter substances expand more than those that are less heated;
When thick glass is heated, the surface is (at first) hotter than the interior;
Hence the surface expands more than the interior.

In all such reasoning, the major premise (the general law) is supposed to be known independently of this particular case to which we apply it. In the following example, Newton discovered the major premise by mathematical calculation:—

Whenever one body revolves round another which attracts with a force decreasing as the square of the distance increases, it will describe an orbit of which Kepler's Laws are true.
The planets are bodies holding this relation to the sun;
Therefore the planets describe orbits of which Kepler's Laws are true.

In *Grammar*, every application of a grammatical rule to the construction of a sentence is a syllogism in *Barbara*. In *Ethics*, all appeals to accepted moral rules in judging particular acts are syllogisms in fig. i.; and if the result is affirmative, the mood is the same fundamental one. In *Law*, the procedure is equally syllogistic. "The whole aim

[1] For a very full and sound account of the meaning of the several syllogistic figures, with numerous examples, see Ueberweg, *Logic*, §§ 101, 110-117.

of legal procedure is to determine whether a particular case does or does not fall under a given general rule. Thus, in a criminal trial, the law which has been violated furnishes the major premise, and the examination of the acts of the accused supplies the minor premise." In *Economics*, the whole "Deductive Method" is an application of general rules to cases coming under them, and therefore consists in a continual use of the mood *Barbara*. In *History*, explanation by general laws is resorted to whenever possible. Our knowledge of human nature individual and social supplies various major premises, of what men and nations will do under given circumstances; and having found historical examples of these circumstances, we explain what occurred by reasonings in *Barbara*. In *Medicine*, any case of *diagnosis* is a syllogism in this mood. Certain bodily conditions are known to be the symptoms of a certain disease; this is a case of those conditions; therefore this is a case of that disease.

When speaking, in the present section, of the valid moods of figure i., we ignored the "weakened moods," AAI and EAO, mentioned in § 5. This is because, though technically valid, they are practically superfluous. They have been given the names *Barbari* and *Celaront*, respectively. They are sometimes called "subaltern moods," for the conclusion of *Barbari* can be inferred by "subalternation" (ch. III. § 7, p. 77) from that of *Barbara*, and the conclusion of *Celaront* in a similar way from that of *Celarent*.

Of the remaining moods of fig. i., we may notice that *Celarent* can prove only a universal negative—that nothing in a given class has certain stated qualities:—

Nothing that increases taxation can be long popular:
All wars increase taxation;
∴ No wars can be long popular.

This mood is of less importance than *Barbara*, for we

can only clear the ground, not directly advance knowledge, by proving what things *are not*:—

> Nothing involuntary can be cured by punishment:
> Stupidity is involuntary;
> ∴ Stupidity cannot be cured by punishment.

But no syllogism in *Celarent* could tell us how stupidity *may* be cured.

§ 7. The second figure proves only negatives. Its valid moods are:—

Cesare, Camestres, Festino, Baroco.

It is useful in establishing distinctions between things. We prove a distinction between S and P by pointing out that P has an attribute M which S has not (in the moods *Camestres* or *Baroco*); or that P has not an attribute M which S has (in the moods *Cesare* or *Festino*).[1]

The following is an example of *Camestres*.

Before the planet Neptune was discovered:—

> The sun and all the planets belonging to our solar system must completely determine the orbit of Uranus;
> The sun and the known planets do not do this;
> ∴ The sun and the known planets are not the whole of our solar system.

For *Baroco*, we may give:—

> All true theories are self-consistent;
> Some scientific theories are not self-consistent;
> ∴ Some scientific theories are not true

Again:—

> All moral acts are done from a praiseworthy motive;
> Some acts that are legal are not done from a praiseworthy motive;
> ∴ Some acts that are legal are not moral.

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[1] On *Camestres* and *Cesare* in connection with the hypothetical syllogism, see ch. VII. § 3.

For *Cesare*, we give two examples from Aristotle (*Ethics*, ii. 4) which are excellent illustrations of the value of figure ii. in establishing distinctions :—

The feelings (πάθη) are not objects of moral judgment;
The virtues (ἀρεταί) are objects of moral judgment;
∴ The virtues are not feelings.

Again :—

The passions are not the result of conscious choice;
The virtues are the result of conscious choice;
∴ The virtues are not passions.

The following is a good example of *Festino* :—

Forces in Nature, working by strictly mechanical laws, cannot produce organic beings capable of growth and reproduction ;
Some forces in Nature have produced such beings ;
∴ Some forces in Nature do not work by strictly mechanical laws.

We must add that *Cesare* and *Camestres* have "weakened moods," EAO and AEO respectively, sometimes called *Cesaro* and *Camestros*.

The student must notice that in all the syllogisms of figs. i. and ii., the premises state exactly enough, no more and no less than enough, to warrant the conclusion. That means that the middle term is distributed only once in each syllogism, and neither of the extremes is distributed in the premises without being distributed in the conclusion. Syllogisms of which this is true have been called *fundamental syllogisms*.

§ 8. The third figure proves only "particular" propositions. Its valid moods are :—

Darapti, *Disamis*, *Datisi*, *Felapton*, *Bocardo*, *Ferison*.

The moods with an I conclusion are useful in proving a rule by positive instances ; those with an O conclusion, in proving *exceptions* to a rule. A frequent use of the

former is to *disprove* sweeping denials (or assertions of incompatibility).

The mood *Darapti* contains more than enough to warrant its "particular" conclusion. The following is an example :—

All whales are mammals;
All whales are water-creatures;
∴ Some water-creatures are mammals.

The syllogism establishes an instance of the fact that some mammals live in the water. The argument is perfectly valid from every point of view; but the middle term is distributed twice. A syllogism like *Darapti*, which contains more than enough to prove the conclusion, is called a "strengthened syllogism."

The mood *Darapti* is specially appropriate when the middle is a singular term, and then no other mood will prove the conclusion. In this connection we must again emphasise the fact that a proposition making an affirmation about a singular subject (ch. II. § 3) is ranked as universal, as an *A* proposition (ch. III. § 1, p. 54). The nature of the argument is the same if the middle is a *collective* singular term. If, then, we require an instance of the rule that poetic genius and scientific ability are compatible, we may argue :—

Goethe was a man of poetic genius;
Goethe was a man of scientific ability;
∴ Some men of scientific ability are men of poetic genius.

For another example we may give :—

Potassium floats on water;
Potassium is a metal;
∴ Some metals float on water,

which is an instance of the fact that metallic qualities do not exclude the degree of lightness necessary for floating.

With regard to the remaining moods of this figure, the student should be able to show for himself how *Disamis* may be derived from *Darapti*, and *Bocardo* from *Felapton*, by applying "subalternation" to the major premise, and *Datisi* from *Darapti*, by taking the "subaltern" of the minor premise.

§ 9. The awkwardness of the fourth figure is due to the fact that a term which is naturally subject is taken as predicate in the conclusion. Thus, if we have these premises—

{ All roses are plants,
All plants need air,

we should naturally expect the conclusion to be about "roses"—*i.e.*, we should naturally regard the syllogism as one in *Barbara*, fig. i., the conclusion being—

All roses need air.

But in the fourth figure the conclusion unexpectedly makes the statement about "things needing air"—

{ All roses are plants,
All plants need air;
Some things needing air are roses.

This is the mood AAI in fig. iv., called *Bramantip*. It is entirely superfluous, as well as unnatural, for the conclusion, if desired, can be obtained by merely converting the conclusion in *Barbara*. The same remark applies to the moods AEE and IAI in fig. iv., called *Camenes* and *Dimaris* respectively,—in which the conclusion, when we think naturally, is drawn in *Celarent* and *Darii* respectively; and if the conclusion of the fourth figure is required, it is obtained by conversion.

The two remaining moods of fig. iv.—EAO and EIO, called *Fesapo* and *Fresison* respectively—fall less readily into the form of fig. i. If we convert the major of *Fesapo* simply, and the minor *per accidens*, we have a pair of premises from which the conclusion of *Fesapo* follows, in *Ferio* of fig. i.; also, from *Fesapo* we may derive *Fresison* by taking the "subaltern" of the minor premise.

The following mechanical device for remembering the names of the valid moods (excluding weakened moods) in the four figures, by fitting them into Latin hexameters, comes down to us (with a few slight changes) from the mediæval logicians :—

> Barbara, Celarent, Darii, Ferioque, prioris;
> Cesare, Camestres, Festino, Baroco, secundæ;
> Tertia Darapti, Disamis, Datisi, Felapton,
> Bocardo, Ferison, habet; quarta insuper addit
> Bramantip, Camenes, Dimaris, Fesapo, Fresison.[1]

§ 10. We may thus sum up the reasons why the first figure is, as Aristotle held, superior to the others :—

(*a*) It alone complies directly with the Canon of Reasoning; hence its scientific value, as illustrated above.

(*b*) It will prove each of the conclusions A, E, I, and O, and is the only mood in which A can be proved.

(*c*) In the principal mood of this figure, the relative *extension* of the major, middle, and minor terms corresponds to the relative order of their names.

(*d*) The subject in the conclusion is also subject in its premise, and the predicate in the conclusion is predicate in its premise.

The most fundamental of these considerations is of course the first, which rests on an assumption of what true reasoning is. On this ground also, we were able to prove the special rules of the first figure. They are really a repetition of the Canon of Reasoning itself.

[1] These lines are usually referred to as the "Mnemonic Lines" or "Mnemonic Verses."

These special rules may be proved also from the general rules of the syllogism.

Proof of the Special Rules of Fig. i.

Rule 1. *The minor premise must be affirmative.*

The form for fig. i. is :—

$$\begin{array}{ll} & \text{M} \quad \text{P}, \\ & \text{S} \quad \text{M}; \\ \hline \therefore & \text{S} \quad \text{P}. \end{array}$$

If possible let the minor premise be negative. Then the major must be affirmative, and P is undistributed there; and also the conclusion must be negative, and P is distributed there. Hence if the minor premise is negative we have an Illicit Major. Therefore the minor must be affirmative.

Rule 2. *The major premise must be universal.*

Since the minor premise is affirmative, the middle term is not distributed there. Hence it must be distributed in the major premise; and as it is subject there, this premise must be universal.

Exercise XIII.

Prove, from the General Rules of the syllogism, the following Special Rules for the second and third figures respectively.

Fig. ii.

1. One premise must be negative.
2. The conclusion must be negative.
3. The major premise must be universal.

Fig. iii.

1. The minor premise must be affirmative.
2. The conclusion must be particular.

It is also possible to deduce every one of the General

If. one pr: is neg, the major must be uni
If. the minor is neg. the major must be [illegible]
If. the minor is aff. the Con: must be partic

Rules of the syllogism from the *dictum de omni et de nullo.*

Aristotle called figs. ii. and iii. the "imperfect figures," as they have not the cogent and conclusive character of fig. i. They may be made independent by constructing canons or *dicta* applicable directly to them, as the *dictum* of Aristotle is applicable to fig. i. This has been done—*e.g.*, by Lambert (Ueberweg, *Logic*, § 103). But these maxims, it may be affirmed, have not the clear, distinct, and self-evident character of the Aristotelian *dictum.*

Aristotle himself exhibited the cogency of the moods in the imperfect figures by means of the first figure. The process is called **Reduction**; and its general nature may be stated thus: **Transform the premises of the imperfect syllogism in such a way that its conclusion may be drawn from them in one of the valid moods of the first figure.** The transformation of the premises is effected (*a*) by one of the processes of immediate inference, applied to one or both of the premises; (*b*) by transposition of the premises, if necessary, in order to keep the major premise first.

The names given to the various moods in the imperfect figures are not only the means of indicating, by their three vowels, the quantity and quality of the major premise, the minor, and the conclusion: some of the intermediate consonants indicate the processes by which reduction is effected. The significant consonants are *s*, *p*, *m*, and *c*; and also the initial letters of the names, B, C, D, F.

(1) *s*, except when it is the last letter of the name, indicates that the proposition denoted by the

preceding vowel is to be converted simply (*conversio Simplex*).

(2) *p*, except when it is the last letter of the name, indicates that the proposition denoted by the preceding vowel is to be converted *Per accidens* (*in Particularem propositionem*).

(3) *m*, indicates that the premises of the imperfect syllogism are to be transposed, the major becoming the minor, and *vice versâ* (*Metathesis sive Mutatio præmissarum*).

By these means we shall have changed the premises of the imperfect syllogism into two equivalent but *new* premises, from which a valid conclusion may be drawn in fig. i. The initial letter B, C, D, or F, of the name of the imperfect syllogism, shows the mood in fig. i. in which the new premises give a valid conclusion. If there is an *s* or *p* at the end of the name of the imperfect syllogism, it means that the new syllogism in fig. i. does not give a conclusion *identical* with that of the imperfect syllogism, but one from which the latter can be derived by conversion, simple or *per accidens*.

(4) *c*, indicates that the mood must be reduced by a distinct process called indirect reduction, to be explained shortly. The process was formerly called "*Conversio syllogismi*," *or* "*ductio per Contradictoriam propositionem sive per impossibile*." Hence it is an error to substitute a *k* for this *c*, as Jevons and Fowler do.

Example: Reduce *Camestres*. This is in fig. ii.:—

All P is M,
No S is M.
∴ No S is P.

The first *s* in the name indicates that the original minor premise is to be converted simply; the *m* indicates that the original premises are to be transposed. The *C* indicates that from the new pair of premises, thus obtained, we are to draw the conclusion in *Celarent*, fig. i.; and the second

s indicates that if we convert this conclusion in *Celarent* simply, we shall get our original conclusion.

Convert the original minor, and transpose :—

No M is S,
All P is M,

from which in *Celarent* the conclusion is,

No P is S,

from which again by simple conversion,

No S is P,

which is the original conclusion.

The process of Reduction in the case of the fourth figure has already been illustrated (§ 9).

This operation, of direct application of Immediate Inference and transposition, is called **direct reduction.** By this means we are also said to reduce *ostensively* (δεικτικῶς). Aristotle did not admit any Immediate Inference except *conversion ;* and under this limitation we cannot reduce *Baroco* and *Bocardo* directly. Accordingly they are reduced by a distinct process known as reduction *per impossibile* (διὰ τοῦ ἀδυνάτου) or **indirect reduction :** assume the falsity of the conclusion (*i.e.*, the truth of its contradictory) ; take this contradictory with one of the original premises, as the two premises of a new syllogism in *Barbara*,[1] the conclusion of which will be incompatible with the other premise of the original syllogism. Hence either the original conclusion is true or one of the original premises false ; and, since in Deductive Logic the premises are always assumed to be true, we can only accept the former alternative.

[1] *Barbara*, being in the first figure, is known to be valid.

Examples: (*a*) Reduce *Baroco per impossibile*:—

$$\begin{cases}\text{All P is M.}\\ \text{Some S is not M.}\end{cases}$$
$\therefore$ Some S is not P.

If this conclusion is false, its contradictory must be true; that is:—

All S is P.

Make this the minor of a new syllogism with the original major:—

$$\begin{cases}\text{All P is M,}\\ \text{All S is P,}\end{cases}$$

from which the conclusion in *Barbara* is,

All S is M,

which contradicts the original minor. Therefore All S is M is false, and if so one of its premises must be false. This can only be the assumed premise All S is P; and if this is false, Some S is not P, the original conclusion, is true.

(*b*) Reduce *Bocardo per impossibile*:—

$$\begin{cases}\text{Some M is not P.}\\ \text{All M is S.}\end{cases}$$
$\therefore$ Some S is not P.

Take the contradictory of this conclusion with the original minor and draw a conclusion from them in *Barbara*:—

$$\begin{cases}\text{All S is P.}\\ \text{All M is S.}\end{cases}$$
$\therefore$ All M is P.

This new conclusion must be false, for it contradicts the original major; hence its assumed premise All S is P is false—*i.e.*, the original conclusion, Some S is not P, is true.

(*c*) The process of indirect reduction may be applied to any of the imperfect moods. Aristotle when mentioning the process applies it to *Darapti*:—

$$\begin{cases}\text{All M is P.}\\ \text{All M is S.}\end{cases}$$
$\therefore$ Some S is P.

The new syllogism formed by the contradictory of this conclusion, with the same minor, gives a new conclusion in *Celarent*:—

$$\begin{cases} \text{No S is P.} \\ \text{All M is S.} \end{cases}$$
$$\therefore \text{No M is P.}$$

This conclusion is the contrary of the original major. One of them must be false, and it can only be this conclusion. Therefore its assumed premise is false—*i.e.*, the original conclusion is true.

(*d*) By the employment of obversion, *Baroco* and *Bocardo* may be reduced directly. (1) *Baroco* may be reduced to *Ferio* by contrapositing the major premise and obverting the minor. (2) *Bocardo* may be reduced to *Darii* by contrapositing the original major, transposing the premises, and taking the obverted converse of the new conclusion.

It must be borne in mind that the term Reduction has no meaning except on the Aristotelian view of the inferiority of the other figures to the first; and to "reduce" a mood must always mean "reduce it to fig. i." It is possible to transform some of the imperfect moods into other imperfect moods; but this is a mere exercise in mechanical manipulation, and should not be called "reduction."

§ 11. When one of the premises of a logical syllogism is omitted in the verbal expression of it, we have what in modern text-books is called an **enthymeme** (*syllogismus decurtatus*). This is the form in which syllogistic arguments are commonly met with. The missing premise is supplied in thought; hence the enthymeme has the same characteristics as the completely expressed syllogism. Most commonly the premise which is omitted but understood is the major, and then the enthymeme is said to be of the *first order*;

sometimes, the minor premise is omitted, when it is of the *second order;* rarely, the conclusion is omitted, when it is of the *third order.* The omission of the conclusion is less a logical than a rhetorical device, to "insinuate" or "suggest" what is to be proved; it is a "figure of speech."

The syllogism which when fully expressed is stated as follows: "All religious wars are fought out with the greatest pertinacity and bitterness; the Thirty Years' War was a religious war; hence its length and bitterness"—may be expressed "enthymematically" in the three forms:—

First order: "The Thirty Years' War was long and bitter; for it was a religious war."

Second order: "The Thirty Years' War was long and bitter, for all religious wars are so."

Third order: "All religious wars are long and bitter; and the Thirty Years' War was a religious war.

Understood thus, an enthymeme is a *formally valid* syllogism with one premise (or the conclusion) not expressed. This use of the term has largely prevailed since Hamilton wrote. But the term is much more serviceable when understood to mean a "condensed" syllogism whether formally valid or not. Jevons has pointed out that even a single proposition may have a syllogistic force if it clearly suggests a second premise which thus enables a conclusion to be drawn. "The expression of Horne Tooke, 'Men who have no rights cannot justly complain of any wrongs,' seems to be a case in point; for there are few people who have not felt wronged at some time or other, and they would therefore be likely to argue, whether upon true or false premises, as follows:—

Men who have no rights cannot justly complain of any wrongs;
We can justly complain;
Therefore we are not men who have no rights.

In other words, 'we have rights.'"

And Professor Minto has also observed that the arguments of common life are often less explicit than the Hamiltonian enthymeme. "A general principle is vaguely hinted

at; a subject is referred to a class the attributes of which are supposed to be definitely known. Thus:—

He was too ambitious to be scrupulous in his choice of means.

He was too impulsive not to have made many blunders.

Each of these sentences contains a conclusion and an enthymematic argument in support of it. The hearer is understood to have in his mind a definite idea of the degree of ambition at which a man ceases to be scrupulous, or the degree of impulsiveness that is incompatible with accuracy."

* The **Aristotelian Enthymeme**[1] (ἐνθύμημα) is not necessarily an elliptically expressed syllogism; it is an argument which aims only at establishing a result as *probable*,—as more than *possible*, but not *certain* to happen,—so far as our premises tell us. This extremely important and frequent form of reasoning will be discussed when we come to treat of Induction. Because Aristotle and his commentators spoke of the enthymeme as an "incomplete syllogism,"—meaning a syllogism or argument which does not furnish complete *proof*,—later logicians supposed that he meant "incomplete," in the sense of being not fully expressed in words. Hence the modern doctrine of the enthymeme is simply a notice of the ways in which, in ordinary reasoning, we move on from one fact or statement to another without stopping to *state* all the steps definitely and explicitly. This is why fallacies are so often *hidden*; an argument is based upon some unexpressed assumption which will not bear examination.

§ 12. Syllogisms may be combined, in various ways, into "chains of reasoning." A common form is that in which the conclusion of one syllogism furnishes one of the premises of the next:—

[1] For derivation of this word, see below, ch. VIII. § 3.

$$\begin{cases}\text{All M is P,}\\ \text{All S is M;}\end{cases}$$
therefore All S is P:
but All R is S;
therefore All R is P.

Here we have two syllogisms in *Barbara*, the conclusion of the first forming the major premise of the second. The syllogism whose conclusion furnishes one of the premises is called the **Prosyllogism**; the syllogism which borrows one of its premises from a previous conclusion is called the **Episyllogism**. There may, of course, be three or more syllogisms combined in this way. When we pass steadily from one syllogism to another, making each conclusion as soon as it is established the premise of a new syllogism, we are said to adopt a **synthetic** or **progressive** method, building up our results as we go along. If we state the episyllogism first, and then the prosyllogism, we are said to adopt an **analytic** or **regressive** method.[1] In this case the prosyllogism is often condensed into an enthymeme, which stands as one of the premises of the episyllogism: "No man is infallible, for no man is omniscient; Aristotle was a man, therefore Aristotle was not infallible." A syllogism of this kind, in which one (or both) of the premises is expanded by the addition of a reason, is called by modern logicians an **Epicheirema** (ἐπιχείρημα, *aggressio*,—a term used by

[1] The terms *analysis* and *synthesis* are not used only with reference to arguments expressed in a series of formal syllogisms: *any* process of inference in which we work out the consequences of the premises from which we start, or argue from causes to effects, is called a synthetic process; and when we work back to find the grounds on which our premises rest or to argue from effects to causes, the process is called analytic.

Aristotle in a different sense). In the example given, the full prosyllogism is: "All infallible beings are omniscient; no men are omniscient, therefore no men are infallible."

A chain of prosyllogisms and episyllogisms, in which all the conclusions, except the last, are omitted in expression, was called by post-Aristotelian logicians a **Sorites** (σωρείτης, *acervus*). According to the order in which the premises follow one another, it is usual to distinguish the **Aristotelian** and the **Goclenian**[1] Sorites. The Aristotelian form is: A is B, B is C, C is D, D is E, hence A is E. It progresses from terms of narrower to those of wider extent; and (in addition to the conclusions) the minor premise of every syllogism except the first is not expressed. The Goclenian form is: D is E, C is D, B is C, A is B; hence A is E. It progresses from terms of wider to those of narrower extent —*i.e.*, E, D, C, B, A; and the major premise of every syllogism except the first is omitted.

For the sake of clearness we add an analysis of the two forms.

Aristotelian Sorites.	*Goclenian Sorites.*
A is B,	C is D,
B is C,	B is C,
C is D; ∴ A is D.	A is B; ∴ A is D.
Analysis.	*Analysis.*
(1) A is B (minor).	(1) C is D (major).
B is C (major).	B is C (minor).
A is C (conclusion).	B is D (conclusion).
(2) A is C (minor).	(2) B is D (major).
C is D (major).	A is B (minor).
A is D (conclusion).	A is D (conclusion).

[1] The "Goclenian" form is so called because it was suggested by a German logician of the sixteenth century, Goclenius.

In both forms the procedure is synthetic or progressive. In these examples the syllogisms are all in fig. i. Dr Keynes has shown that Sorites are possible in which each syllogism is of the second figure, and also in which each syllogism is of the third figure; but these are only mechanical curiosities. For the Aristotelian Sorites the following rules may be given:—

(1) The first premise alone can be particular;
(2) The last premise alone can be negative;

for if any of the intermediate premises were either negative or particular, the chain of connection would be broken. The Goclenian Sorites proceeds in the reverse order:—

(1) The first premise alone can be negative;
(2) The last premise alone can be particular.

In the examples given all the premises were universal and affirmative. The student should construct examples in which the first or last premise is particular or negative, according to the rules.

Aristotle refers to arguments of this kind (*An. Pr.*, Bk. I. 25); but by the Greeks the name σωρείτης was given to a particular kind of fallacy (*e.g.*, *Ueberweg*, § 125).

§ 13. The student who has grasped the general principle of each of the first three figures, will have no difficulty in turning ordinary or colloquial reasonings into syllogistic form, and so testing their validity. To do this is a valuable exercise in accuracy of thought. For instance, if an argument aims at proving or disproving some attribute of a thing, by applying a general rule or principle, or by bringing it under a higher class: then the *dictum* of Aristotle is directly applicable, and the figure is the first. If the argument aims at a negative conclusion, separating two things by reasoning from the fact that an attribute which is characteristic of one is absent in the other; the figure is the second. If the argument aims at establishing a rule,—a general or partly

general statement,—by an *instance;* or if it endeavours to deny such a rule by means of a *negative instance:* then the figure is the third.

In order to express the argument strictly in the form of mood and figure, it is usually necessary to make changes in the given expression of it, supplying any premise which may be understood but not expressed, according to Hamilton's postulate, that what is implicit in thought may be made explicit in language (ch. III. § 4). It is a mistake to say, as Jevons does, that such changes "are of an extra-logical character, and belong more properly to the science of language"; for they are changes made in order that the words may express the true logical relations of the thoughts.

Jevons quotes from the Port Royal Logic two examples of arguments which, he says, "cannot be proved by the rules of the syllogism," and yet are perfectly valid. The examples are: (*a*) "The sun is a thing insensible; the Persians worship the sun; therefore the Persians worship a thing insensible." (*b*) "The Divine Law commands us to honour kings; Louis XIV. is a king; therefore the Divine Law commands us to honour Louis XIV." Now if we were limited to making *merely grammatical* changes in these arguments, it would be difficult if not impossible to express them as strict syllogisms. But it should have been evident that they can be so expressed. The first of them adduces an *instance* in support of the general statement that Persians are worshippers of a thing insensible, hence it is of the type of fig. iii.; the second is an application of a general principle, and hence is of the type of fig. i. The arguments may be expressed syllogistically in *Darapti* and *Barbara* respectively:—

(*a*) The sun is an object of Persian worship;
The sun is a thing insensible;
Therefore something insensible is an object of Persian worship.

(*b*) Kings are to be honoured by command of the Divine Law;
Louis XIV. is a king;
Therefore Louis XIV. is to be honoured by command of the Divine Law.[1]

We add a few more examples illustrating this logical transformation of the ordinary expressions of reasoning.

"He must be a Buddhist, for all Buddhists hold these opinions."

Here the unexpressed minor premise evidently is, "he holds these opinions":—

All Buddhists are persons holding these opinions;
He is a person holding these opinions;
Therefore he is a Buddhist.

This is the mood AAA in fig. ii., and is formally invalid, as it involves an undistributed middle. This is an example of a fallacy which frequently arises through arguing, in the second figure, from resemblances. Any one may hold opinions resembling some Buddhist doctrines without being a Buddhist. Inductively such arguments are of great importance, and the conditions under which we may rely on them will be discussed in the sequel; but they are *formally* fallacious. If the original argument had been as follows: "He must be a Buddhist, for *none other than* Buddhists hold these opinions," it would have been valid in *Cesare*, fig. ii., leading to the conclusion, "He is none other than a Buddhist," or (by obversion) he is a Buddhist.

The following is a common rhetorical form of argument: "Why be ashamed of a mistake? All men are fallible." The *question* is equivalent to the statement that "no mistakes are things to be ashamed of"; this is evidently the conclusion. The given premise, "all men are fallible,"

[1] Example (*a*) might also be taken as an instance of what Jevons calls "immediate inference by complex conception" (see above, ch. III. § 13) followed by *Barbara:* "The sun is a thing insensible, therefore worshippers of the sun are worshippers of a thing insensible; the Persians are worshippers of the sun, therefore the Persians are worshippers of a thing insensible." As regards (*b*), the valid conclusion is that the French subjects of Louis XIV. were bound to honour him as an official, not necessarily as a man.

must be restated so as to connect it with the conclusion, thus: "a mistake is what all men are liable to." This contains the subject of the conclusion, and is therefore the minor premise; it is universal, for it means to refer to every instance of "a mistake." The syllogism then becomes:—

What all men are liable to is not a thing to be ashamed of,
A mistake is what all men are liable to;
Therefore no mistakes are things to be ashamed of. This is valid in *Celarent*, fig. i.

EXERCISE XIV.

Questions on Chapter VI.

(i) *Elementary*.

1. Distinguish Mediate and Immediate Inference. Is Immediate Inference merely a verbal change in the proposition? What is its practical utility?

2. What is Mediate Inference, and why so called? What is its simplest form? Give the etymological meaning of the term Syllogism.

3. Explain the method of argument characteristic of the Aristotelian syllogism (§ 2). How is it exemplified in the following arguments?

(*a*) Silver is a good conductor of electricity, for such are all the metals.

(*b*) Comets cannot be without weight, for they are composed of matter, which is not without weight.

4. In the following syllogisms, point out *in this order* the conclusion, the middle term, the major term, the minor term, the major premise, the minor premise:—

(*a*) All men are fallible, and all kings are men; hence all kings are fallible.

(*b*) All roses are plants, and all plants need air, so that all roses need air.

(*c*) Natives of South Africa are capable of education, for they are men, and all men are capable of education.

5. State the six principal and two subordinate rules of the syllogism.

6. Explain carefully what is meant by "ambiguous middle," "undistributed middle," "illicit process of the major" and "of the minor."

7. In what cases may two *apparently* negative premises lead to a valid conclusion?

8. Name all the rules of the syllogism which are broken by each of the following moods :—

AIA, EEI, IEA, IOI, IIA, AEI. [Jevons.]

9. Distinguish the four Figures of the syllogism.

10. Write out the sixteen possible different moods (combinations of premises A, E, I, or O, two at a time) and strike out those that are directly excluded by the rules.

11. Prove from the rules alone, irrespective of figure, that IE can yield no valid conclusion.

12. In what figures do the following premises yield a valid conclusion?—

AA, AI, EA, OA. [Jevons.]

13. Name the figure and mood of each of the following valid syllogisms :—

(*a*) Some M is S,
No P is M,
∴ Some S is not P.

(*b*) All S is M,
No M is P,
∴ No S is P.

14. Draw valid conclusions from each of the following pairs of premises, and name the figure and mood in each case :—

(*a*) All planets are heavenly bodies;
No planets are self-luminous.

(*b*) Nothing that can be cured by punishment is involuntary;
Stupidity is involuntary.

(The premises (*b*) can be taken in two ways, giving two possible valid conclusions.)

15. Show, with examples, that false premises may give true conclusions.

16. Supply premises to the following conclusions :—

(*a*) Some logicians are not good reasoners.
(*b*) The rings of Saturn are material bodies.
(*c*) Party government exists in every democracy.
(*d*) All fixed stars obey the law of gravitation. [Jevons.]

17. State the *dictum de omni et de nullo*, and prove from it the "special rules of the first figure." Prove them also from the general rules.

18. What are the special uses of each of the first three figures?

19. What is Reduction? Why did Aristotle consider it necessary?

20. Explain and illustrate "Indirect Reduction."

21. State and explain the "Mnemonic Lines."

22. Explain the terms: Enthymeme, Prosyllogism, Episyllogism, Epicheirema, Sorites.

What are the rules to which a Sorites must conform?

23. State each of the following arguments in logical (*i.e.*, syllogistic) form, so far as necessary, and examine its validity :—

(1) Every book is liable to error, and every book is a human production; therefore all human productions are liable to error.

(2) No persons destitute of imagination are true poets; some persons destitute of imagination are good logicians; therefore some true poets are not good logicians.

(3) All tyrants deserve to die; Cæsar is not a tyrant, therefore he does not deserve to die.

(4) Some mathematicians are logicians; no logicians are unacquainted with the works of Aristotle; therefore some mathematicians are not unacquainted with the works of Aristotle.

(5) Socrates was self-restrained, for all wise men are so.

(6) Personal deformity is an affliction of nature, and no disgrace is an affliction of nature; therefore personal deformity is not disgrace.

(7) No idle person can be a successful writer of history; therefore Hume, Macaulay, Hallam, and Grote, must have been industrious.

(8) Bacon was a great lawyer and statesman; and as he was also a philosopher, we may infer that any philosopher may be a great lawyer and statesman.

(9) Every candid man acknowledges merit in a rival; every learned man does not do so; therefore every learned man is not candid.

(10) Warm countries alone produce wines; Spain is a warm country, therefore Spain produces wines. [Fowler, Jevons.]

24. Describe the logical character of each of the following arguments, and expand each into a series of syllogisms :—

(1) Whatever tends to withdraw the mind from pursuits of a low nature, deserves to be promoted: classical learning does this, since it gives us a taste for intellectual enjoyments; therefore it deserves to be promoted. [Jevons.]

(2) All unnecessary duties on imports are impolitic, as they impede the trade of the country; the American protective duties are unnecessary, as they support industries which are quite able to stand alone; therefore the American protective tariff is impolitic. [Welton.]

(3) An avaricious man is one who desires more than he possesses; a man who desires more than he possesses is discontented; a discontented man is unhappy; therefore an avaricious man is unhappy.

(ii) *More Advanced.*

25. Enumerate and describe briefly what you regard as the distinct varieties of Immediate Inference. Discuss the right of these forms to be regarded as modes of Inference. [L.]

26. Upon what principle have the names major, middle, and minor been applied to the terms of a syllogism? How far are these names generally applicable? [O.]

27. If the major premise is a particular negative, determine the mood and figure.

28. Show that there are only four ways of proving (syllogistically) a universal negative.

29. If the major term be universal in the premises and particular in the conclusion, determine the mood and figure, it being understood that the syllogism is one of those mentioned in the mnemonic lines.

30. Prove that in every figure, if the minor premise is negative, the major must be universal. [O]

31. What can be determined respecting a syllogism under each of the following conditions?—

(*a*) That only one term is distributed, and that only once;

(*b*) That only one term is distributed, and that twice;

(*c*) That two terms only are distributed, each only once;

(*d*) That two terms only are distributed, each twice. [L.]

32. Give what you consider the best statement of the fundamental axiom of the syllogism, and discuss the logical character of the axiom.

33. Which figure is most convenient (1) for overthrowing an adversary's conclusion; (2) for establishing a negative conclusion; (3) for proving a universal truth? Give your reasons. [O.]

34. Why cannot a particular negative stand as a premise in the first, as a major in the second, as a minor in the third, or as a premise in the fourth figure? [C.]

35. What moods are good in the first figure and faulty in the second, and *vice versâ*? Why are they excluded in one figure and not in the other?

* 36. Ought the Fourth to be treated as an independent figure, or only as a variety of the First?

* 37. Critically compare the statements of the *dictum de omni et de nullo* given by Aristotle, by the mediæval logicians, by Kant, and by J. S. Mill.

* 38. In what does the peculiarity of the Enthymeme consist? In what sense did Aristotle use the term Enthymeme? What is the derivation of the word? [O.]

39. State the following arguments in complete logical form, and examine their validity:—

(1) We know that the policy was mistaken, for otherwise it would not have failed.

(2) Only members of the society took part in the discussion. You must have done so, for you are a member.

(3) Some statesmen are also authors; for such are Gladstone, Beaconsfield, Balfour, and others.

(4) Every true patriot is disinterested; few men are disinterested, therefore few men are true patriots.

(5) If he did not steal the goods, why did he hide them, as no thief fails to do?

(6) Haste makes waste and waste makes want, therefore a man never loses by delay.

(7) No fallacy is a legitimate argument; any legitimate argument may fail to win assent; therefore no fallacy fails to win assent.

(8) He must know a great deal, for he says so little.

(9) His liberality might have been inferred from his ambition, for the ambitious are never sparing of their money.

(10) A little knowledge is a dangerous thing, hence I had better not try to learn Logic.

(11) You need not expect him, for he is too busy to come.

(12) This book must have been read, because the pages are cut.

(13) He could not face bullets on the field of battle, and is therefore a coward.

(14) Professor Stout in his *Manual of Psychology* says that some idiots have very remarkable powers of memory. I ought to feel thankful, therefore, that my memory is a very bad one.

(15) "Have you a strawberry-mark on your left shoulder?" "No." "Then you are my long-lost brother!" (How far would the answer "yes" improve the argument?)

(16) An office-clerk may be a man of letters, for Charles Lamb was both.

(17) Some of the books had evidently not been used, for the pages had not even been cut.

(18) This document cannot be genuine, or it would have been referred to by the supposed author's contemporaries.

(19) Unless the weather is wet, the open-air meeting will be held. It is not raining, so I suppose they will hold it.

(20) The style of Sir Philip Francis has certain peculiarities which appear in the *Letters of Junius;* therefore he was probably the author of them.

(21) It sometimes happens that worthy pursuits do not conduce to material gain, for certainly philosophical studies deserve to be pursued, and yet they often bring no pecuniary reward.

(22) He must have committed the murder, for he was the only person present with the deceased at the time.

(23) Only the ignorant despise knowledge; hence this man cannot be ignorant, for he praises it.

(24) Whatever is given on the evidence of sense is a fact; the existence of God, therefore, is not a fact, for it is not evident to sense.

(25) Seeing that abundance of work is a sure sign of industrial prosperity, it follows that fire and hurricane benefit industry, because they undoubtedly create work.

(26) I will have no more doctors; I see that all of those who have died this winter have had doctors.

(27) None but the wise are good, and none but the good are happy, therefore none but the wise are happy.

(28) What is the use of all this teaching? Every day you hear of fraud and forgery by some one who might have led an innocent life if he had never learnt to read and write.

(29) Nations are justified in revolting when badly governed, for every people has a right to good government.

(30) Death cannot be an evil, for it is universal.

(31) Protection from punishment is due to the innocent; therefore, as you maintain that this person ought not to be punished, it appears that you are convinced of his innocence.

(32) He must be suffering from some disease, for he failed to pass the Medical Examination when he wished to enter the army.

(33) Had there been satisfactory proof of the prisoner's guilt, beyond doubt it would have been produced; but as there was no such evidence brought forward, he was innocent.

(34) It is impossible to maintain that the virtuous alone are happy, and at the same time that selfishness is compatible with happiness, but incompatible with virtue.

(35) A little knowledge is not a dangerous thing unless we imagine it to be greater than it really is.

* (36) The planet Mars resembles the earth in possessing atmosphere, water, and moderate temperatures; it is therefore inhabited.

* 37) M is the only possible cause of P; hence if we find P occurring, we may be sure of the presence of M.

* (38) The *esse* of a Sensation is *percipi;* all objects are groups of sensations; hence, for all objects, *esse* is *percipi.* [St A., E., G., L.]

CHAPTER VII.

CONDITIONAL ARGUMENTS AND THE VALIDITY OF THE SYLLOGISM.

§ 1. ALL the syllogisms hitherto examined have consisted of categorical propositions.

We have seen (ch. III. § 1) that, in addition to categorical propositions, there are conditional propositions in which P is predicated of S under a condition. Of these there are two kinds:—

(*a*) Hypothetical or conjunctive:—

If S is P it is Q.
If S is P, Q is R.

(*b*) Disjunctive:—

S is either P or Q.
Either S is P or Q is R.

In a disjunctive proposition there may, of course, be more than two alternatives. In a hypothetical proposition the condition is introduced by "if," or an equivalent phrase—*e.g.*, "suppose that," "granted or provided that," "allowing that," "whenever," "wherever."

The part of the hypothetical proposition which states

the condition or supposition is called the *antecedent;* the other (the *result* of the supposition) is called the *consequent.* The proposition is in fact an application of the principle of Sufficient Reason. It has two usual forms:—

(1) If S is P it is Q. This asserts that a relation between two concepts P and Q holds universally, without qualification, so that whenever P is predicated, it follows that Q must be. The simplest examples are from Mathematics: "If a triangle is equilateral it is equiangular." It is the natural form for scientific laws or principles: "If the planet Venus does not rotate upon her axis, but always turns one face to the sun and the other to the outer cold, Venus is incapable of supporting life."

(2) If S is P, Q is R. This asserts a connection between two judgments, such that if one is true the other follows. "If a triangle is rectangular, the square on the hypotenuse is equal to the sum of the squares on the other two sides." "If the force of gravity on the planet Mars is too small to prevent water-vapour from escaping into space, there is no life on Mars." "If organic life is possible on a planet, oxygen must be present in the atmosphere or dissolved in water."

The student will see later—what these examples make evident—that the two forms of the hypothetical proposition are at bottom the same.

The essence of the hypothetical proposition is the *relation of dependence* which it expresses, and which holds *between its antecedent and its consequent.* Hence all real hypotheticals are universal. Such a proposition as "if a figure is a parallelogram it is sometimes a square" is not a real hypothetical. It is not on the figure's being a parallelogram that its being a square depends.

§ 2. Conditional arguments consist of—

(1) hypothetical syllogisms,
(2) disjunctive syllogisms,
(3) dilemmas, consisting of hypothetical in combination with disjunctive premises.

Hypothetical syllogisms are sometimes said to be either (*a*) pure, in which both premises are hypothetical, (*b*) mixed, in which the major premise is hypothetical and the minor categorical; but "pure" hypothetical syllogisms are of no importance.

When a "hypothetical syllogism" is spoken of, a *mixed* hypothetical syllogism is usually meant. It consists of a hypothetical major and a categorical minor. Syllogisms of this kind are of great importance; for they give the natural form for expressing the application of a general principle to a particular case.

The hypothetical proposition is always taken as the major premise, for it asserts that a relation of Reason and Consequence, between two concepts or judgments, holds universally as a matter of theory; and the minor premise applies it to a matter of fact. The principle of the hypothetical syllogism is that of the Aristotelian first figure, expressed in the general Canon of Reasoning (ch. VI. § 6).

The minor premise may affirm or deny the antecedent or consequent of the major; hence there are four numerically possible forms:—

(*a*) If S is P, Q is R;
S is not P;
no conclusion.

(*b*) If S is P, Q is R;
Q is R;
no conclusion.

(*c*) If S is P, Q is R
S is P;
∴ Q is R.

(*d*) If S is P, Q is R;
Q is not R;
∴ S is not P.

There is no conclusion in (*a*) and (*b*); if we deny the antecedent, we cannot therefore deny the consequent, for the latter may be true for other reasons; and if we affirm the consequent, we cannot therefore affirm the antecedent, for the consequent may result from other reasons.

We will now give concrete examples of each of the four cases.

(*a*) "If the study of Logic furnished the mind with a multitude of useful facts, like other sciences, it would deserve to be cultivated; but it does not furnish the mind with a multitude of useful facts; therefore it does not deserve cultivation." [Jevons.]

This conclusion does not follow from the premises; for the acquiring of a multitude of useful facts is not the only ground on which the study of a science can be recommended. To correct and exercise the powers of judgment and reasoning may be regarded, for example, as a sufficient justification of logical study.

(*b*) "If a man's character is avaricious, he will refuse to give money for useful purposes; this man refuses money for such purposes; therefore this man's character is avaricious."

But we are not entitled to infer this from the premises; for there may be many good reasons why he refuses, although his character is not avaricious.

(*c*) "If oxygen and nitrogen exist in Mars, life is possible there; these elements do exist in that planet, hence life is possible there."

Though the minor premise is not an established fact, this argument is formally valid. To affirm the antecedent is to declare that the condition exists, and this justifies the affirmation of the consequent.

(*d*) "If life is possible on Mars, the planet has warmth sufficient for protoplasmic metabolism; but the planet has not warmth sufficient, and therefore life is not possible on it."

The minor premise again goes beyond our present knowledge; but the argument is formally valid. To deny the

consequent is to declare its non-existence; and this justifies us in denying that the condition (stated in the antecedent) exists.

Hence the rule for hypothetical syllogisms is this: **Either affirm the antecedent, or deny the consequent.** In the former case, as in (*c*), we have a **constructive** hypothetical syllogism; in the latter, as in (*d*), a **destructive** hypothetical syllogism. These are sometimes spoken of as the *modus ponens* and *modus tollens* respectively.

§ 3. We have seen that a hypothetical proposition expresses a relation between two concepts or two judgments. When expressed in the hypothetical form the proposition invites us to attend more to the relation between the concepts employed than to any special instances. But if we attend chiefly to the particular instances, actual and possible, to which the proposition may be conceived to apply, then we may express the proposition in a categorical form, the universal affirmative A. Thus, take the proposition, "If S is P it is Q." Looked at on the side of extension,—in other words, looking at the instances of its application,—this proposition means that wherever there is a case of S being P, it is also Q. Here we may express the hypothetical proposition in the form "All S which is P is Q," or "All SP is Q."

For example, the propositions "If iron is impure, it is brittle," and "All impure iron is brittle," express the two aspects of intension and extension respectively. Other examples are: "If a substance becomes gaseous, it absorbs heat,—all substances in becoming gaseous absorb heat"; "If a substance is a metal it is a good conductor of heat and electricity,—all metals are good conductors," &c. This change is sometimes called the "reduction" of hypothetical

propositions to the categorical form. But this is an inaccurate use of the term "reduction"; the two forms of the judgment are not identical; they emphasise the two different aspects of the meaning,—intension and extension.

Hypothetical syllogisms may consequently also be expressed in categorical forms:—

(*a*) *Modus ponens*—

"If life is full of distraction, it is exhausting;
Modern life is full of distraction;
Therefore modern life is exhausting."

This becomes a regular syllogism in *Barbara*.

(*b*) *Modus tollens*—

"If Aristotle is right, slavery is a justifiable social institution;
But slavery is not this;
Therefore Aristotle is not right."

This becomes a regular syllogism in *Camestres*, fig. ii., the chief importance of which mood consists in its representing the extremely common mode of argument which is exemplified in the "destructive" hypothetical syllogism. If the consequent of the major premise in the hypothetical syllogism is negative, it is denied by an affirmative (A), and the mood is *Cesare*:—

"If S is P, Q is not R; Q is R; ∴ S is not P."

"No case of S being P is a case of Q being R;
This is a case of Q being R;
Therefore this is not a case of S being P."

The student will find that, when the hypothetical is expressed as a categorical syllogism, the fallacy of affirming the consequent appears as Undistributed Middle; and the fallacy of denying the antecedent appears as Illicit Major.

The real structure of the hypothetical syllogism is now evident:—

(*a*) The major premise affirms only that the relation of Reason and Consequence holds between two judgments or concepts. It does not expressly refer to instances where the relation actually occurs; and about any particular instance it tells us nothing at all. We

may know that "*if* A is B, *then* C is D," without knowing that "A *is* B, *therefore* C is D." To say that "if the barometer falls, the weather will be bad," is not the same thing as to say that "the barometer is falling, and so the weather will be bad." But when, independently of the major, we know the truth of the minor, "A is B," "the barometer is falling," then we may assert the conclusion. And we cannot assert it unless *both* premises are conceded; that is to say, the inference is *mediate.*

(*b*) Similarly, from the minor premise alone, A is B, we cannot draw the conclusion C is D, unless the relation of Reason and Consequence is admitted to hold between them—*i.e.*, unless the major premise is conceded as well as the minor.

We must notice, before leaving the subject of the hypothetical proposition, that *all* such propositions can be brought into the form "if S is M it is P." Usually there is no difficulty in doing so. But occasionally the hypothetical with four terms, "if S is M, P is R," conceals the unity of the judgment which it expresses,—by giving no obvious point of union between S and P. The empty symbolic statement, with the four letters, always does this; but it may happen when the judgment is expressed in significant words.

The following examples will illustrate what we have said. In each case we give (*a*) the form with four terms, (*b*) the fundamental form with three terms.

(1) (*a*) If the report is true, what you say is untrue. (*b*) If the report is true, it proves the untruth of what you say.

(2) (*a*) If two parts of hydrogen combine with one part of oxygen, water is formed. (*b*) If the combination of two parts of hydrogen with one part of oxygen takes place, it (*i.e.*, the combination) forms water.

(3) (*a*) If some agreement is not speedily arrived at between employers and workmen, the trade of the country will be ruined. (*b*) If trade continue to be injured by this strike, it will soon be ruined.

Sometimes, in the four-term form, "if S is M, P is R," the point of union between S and P consists in P being a species of the genus S: "if savages are cruel, the Patagonians are cruel"; or S and P may be co-ordinate species under a common genus: "if virtue is voluntary, vice is voluntary."

Similar considerations show that the two forms of the disjunctive proposition, "S is either P or Q," and "either S is P or Q is R," are at bottom the same.

§ 4. The **disjunctive syllogism** has a disjunctive major premise and a categorical minor and conclusion. The major is, "S is either P or Q," and there are four possible minors, "S is P" or "S is Q" (both A propositions), or "S is not P" or "S is not Q" (both E propositions).

Before we can settle the question—Which of these lead to valid conclusions?—we must be clear as to another point. When we say "S is either P or Q," do we mean that it cannot be both—that the alternatives are mutually exclusive? To answer this it is necessary to distinguish between what we often do mean in ordinary thinking, speaking, and writing, and what we ought to mean according to the requirements of Logic. As a matter of fact, frequently we do mean the alternatives to be exclusive, but not always. Take the following instances: "All the men in this college either boat or play cricket"; "A good book is valued either for the usefulness of its contents or the excellence of its style"; "Either the witness is perjured, or the prisoner is guilty." In all these propositions, the meaning is merely that if one alternative *does not* hold, then the other does hold. In such cases we do not want to deny

that both the alternatives may be true. But for logical purposes there is no doubt that the alternatives ought to be mutually exclusive; and this is *necessary* if such statements are to have any scientific value.

We cannot make an exclusive disjunction about anything unless we have a considerable amount of knowledge about it. Even to say such a thing as this, "You must either pay a fine or go to prison," implies knowledge of the legal bearings of the circumstances as a whole; "A line must be either straight or curved," implies geometrical knowledge of the meaning of straight and curved, and the relation between the two concepts; "This tree is either an oak or an ash," implies some knowledge of both these varieties, and a comparison of that knowledge with the given instance. It is a fundamental error to suppose that the disjunctive judgment expresses mere ignorance as to which of two predicates belongs to a given object. We shall have to return to this extremely important point.

Let us suppose, however, that the disjunction is not exclusive, and proceed to ascertain which of the four possible minor premises give valid conclusions.

(1) S is either P or Q;
S is P.
No conclusion, because S may be also Q.

(2) S is either P or Q;
S is Q.
No conclusion, because S may be also P.

(3) S is either P or Q;
S is not P;
∴ S is Q.

(4) S is either P or Q;
S is not Q;
∴ S is P.

Thus when the alternatives are not exclusive, we may resolve the disjunctive proposition into a pair of hypotheticals :—

(*a*) If S is not P it is Q ;
(*b*) If S is not Q it is P.

If the alternatives are mutually exclusive, as for logical and scientific purposes they ought to be, we get four instead of two hypotheticals—viz., beside (*a*) and (*b*) already mentioned :—

(*c*) If S is P it is not Q ;
(*d*) If S is Q it is not P ;

and then there are conclusions in (1) and (2) above as well as in (3) and (4). In (1) we can draw the conclusion S is not Q ; and in (2) the conclusion S is not P. These two conditional syllogisms are sometimes said to belong to the *modus ponendo tollens*, the mood which denies by affirming ; and the other two, (3) and (4), to the *modus tollendo ponens*, the mood which affirms by denying.

§ 5. A **dilemma** is a syllogism with one premise disjunctive and the other hypothetical.

In practical life we are said to be in a dilemma when we have only two courses open to us, and both will have unpleasant consequences. So, in Logic, the dilemma shuts us up to a choice between two admissions.

The structure of the dilemma will be apparent from the following rules and examples.

(1) The **major premise** is a hypothetical proposition :—

(*a*) with more than one antecedent ;
(*b*) or with more than one consequent ;
(*c*) or with more than one of both, so as to be two hypotheticals combined.

(2) The **minor premise** is a disjunctive proposition.

(3) The **conclusion** is either a categorical or a disjunctive proposition, according as the hypothetical major has only one antecedent (or consequent) or more than one. The dilemma is said to be simple or complex according as its conclusion is categorical or disjunctive.

(4) The essentials of the dilemma are the **plurality of antecedents or of consequents** in the major, and the **disjunctive minor.**

Hence there are four possible forms of the dilemma:—

(1) *Simple Constructive.*
If A is B or if C is D, E is F;
Either A is B or C is D;
∴ E is F.

(2) *Simple Destructive.*
If A is B, C is D *and* E is F;
Either C is not D or E is not F;
∴ A is not B.

(3) *Complex Constructive.*
If A is B, C is D; and if E is F, G is H;
Either A is B or E is F;
∴ Either C is D or G is H;

(4) *Complex Destructive.*
If A is B, C is D; and if E is F, G is H;
Either C is not D or G is not H;
∴ Either A is not B or E is not F.

We have stated the dilemmas in their longest possible form. Usually there are less than six terms in the simple, and less than eight in the complex, dilemmas, as the following examples will show:—

(1) *Simple Constructive.*

"If she sinks or if she swims there will be an end to her;
But she must either sink or swim;
Therefore there will be an end to her."

"If a science furnishes useful facts, or if the study of it exercises the reasoning powers, it is worthy of being cultivated;
But either a science furnishes useful facts, or its study exercises the reasoning powers;
Therefore it is worthy of being cultivated."

(2) *Simple Destructive.*

"If he goes to town he must pay for his railway ticket and his hotel bill;
But either he is unable to pay his hotel bill, or to pay his railway ticket;
Therefore he cannot go to town.

(3) *Complex Constructive.*

This is a very common form.

"If he stays in the room he will be burnt to death, and if he jumps out of the window he will break his neck;
But he must either stay in the room or jump out of the window;
Therefore he must either be burnt to death or break his neck."

In this case the dilemma is an analysis of a practical situation. Professor Minto gives as the standard example the fallacious dilemma in which the custodians of the Alexandrian Library are said to have been put by Caliph Omar in 640 A.D. "If your books are in conformity with the Koran, they are superfluous; and if they are at variance with it, they are pernicious."

(4) *Complex Destructive.*

Dilemmas of this type are less common.

"If he were clever, he would see his mistake, and if he were candid, he would acknowledge it;
Either he does not see his mistake or he will not acknowledge it;
Therefore either he is not clever or is not candid."
[Stock.]

Jevons says, "The destructive dilemma is always complex, because it could otherwise be resolved into two unconnected destructive hypothetical syllogisms"; but this does not appear to hold of the simple destructive dilemma whose major premise is stated as above.

The dilemma has the reputation of being fallacious. Thus Jevons says, "Dilemmatic arguments are more often fallacious than not." If it is properly constructed, the dilemma is absolutely correct; but many fallacies have been put into this form. Fallacy may arise from a faulty major or a faulty minor premise. In the major premise the antecedent, or the consequent, may be false *in fact*, or the asserted connection between them may be false. In the minor premise—where the fallacy usually lies—the antecedent of the major may be denied or the consequent affirmed; or the alternatives may not be exclusive or not exhaustive. This last is the most common source of hidden fallacy in the dilemma, as Jevons has well shown.

"It is seldom possible to find instances where two alternatives exhaust all the possible cases, unless indeed one of them be the simple negative of the other in accordance with the law of excluded middle. Thus if we were to argue that 'if a pupil is fond of learning, he needs no stimulus, and that if he dislikes learning, no stimulus will be of any avail; but as he is either fond of learning or dislikes it, a stimulus is either needless or of no avail,' we evidently assume improperly the disjunctive minor premise. Fondness and dislike are not the only two possible alternatives, for there may be some who are neither fond of learning nor dislike it, and to these a stimulus in the shape of rewards may be desirable. Almost anything can be proved if we are allowed thus to pick out two of the possible alterna-

-tives which are in our favour, and argue from these alone"

~~The most famous illustration~~ of these observations is the ancient fallacy known as *Ignava Ratio*, the "lazy argument": "If it be fated that you recover from your present disease, you will recover, whether you call in a doctor or not; again, if it be fated that you do not recover from your present disease, you will not recover, whether you call in a doctor or not: but one or other of these contradictories is fated, and therefore it can be of no service to call in a doctor." Here the minor premise assumes that "fate does not act through doctors,"—that the calling in of a doctor is not a link in the "fated" series of events.

In the dilemma with respect to the Alexandrian Library, Caliph Omar tacitly assumed in the minor premise that the doctrines of the Koran are not merely sound, but contain all that is really worth knowing. Or, to put it otherwise, he ignores the possibility that the books may contain useful matter on which the Koran does not touch. In other words, the alternatives given in the minor premise are not exhaustive.

A faulty constructive dilemma may be "rebutted" by a dilemma which appears equally cogent, and appears to prove an opposite conclusion. As an example we may take a story which has come down to us of an Athenian mother who urged her son not to enter on public life, on the following grounds:—

"If you say what is just, men will hate you; and if you say what is unjust, the Gods will hate you.
You must say one or the other;
Therefore you will be hated."

The son replied that he ought to enter on public life, giving the following reasons:—

"If I say what is just, the Gods will love me; and if I say what is unjust, men will love me;
I must say one or the other;
Therefore I shall be loved."

In most of such cases the two dilemmas are equally fallacious. And the "rebutting" is only apparent, for

the two conclusions are compatible; they are merely proved by using the fallacy, so to speak, in two opposite ways.

The usual way of rebutting a faulty dilemma will be seen from the following instances:—

If A is B, C is D, and if E is F, G is H;
Either A is B or E is F;
Therefore either C is D or G is H.

Transpose the two consequents in the major premise, changing each to its negative:—

If A is B, G is not H, and if E is F, C is not D;
Either A is B or E is F;
Therefore either G is not H or C is not D.

The student should notice that a hypothetical syllogism with a disjunctive antecedent or consequent must not be mistaken for a dilemma. For example: "Whether geometry be regarded as a mental discipline or as a practical science, it deserves to be studied; but geometry may be regarded as both a mental discipline and a practical science; therefore it deserves to be studied."

§ 6. The question has been raised, Whether there is any real inference in the syllogism,—whether the conclusion gives us any *new* truth?

We must reply that the conclusion of an inference can never be entirely "new,"—*i.e.*, absolutely unconnected with the premises; for if so, it would not follow from these premises. The conclusion is contained in the premises *taken together;* the conclusion would offend against the rules of the syllogism if it told us anything not contained in the premises. The real act of inference consists in the synthesis (σύνθεσις, putting together) of the premises. When we have got the premises together we have got the conclusion, save for the formal process of expressing it.

It is the fact that, in the syllogism, when you have assumed the premises you have assumed the conclusion, for the whole conclusion is contained in the premises. The student will find that this is true of all inference of every kind. If, then, it were maintained *on this ground* that the syllogism is a *petitio principii* or "begging of the question" (assuming in the premises what we proceed to prove by means of those premises), then this charge is true of all inference whatever, and we could only avoid it by making *all* inference impossible. But when once we understand that the real act of inference consists *in the act of combining* the premises, we see that there can be no question of any *petitio principii* in the matter. For the same reason we can understand that *new* knowledge may be obtained by inference, where "new" knowledge means, not knowledge unconnected with the premises, but knowledge that we did not have in our possession before. We have the new knowledge as soon as we have *put the premises together*, and not before; and we are able to obtain this new knowledge just because it *is* "contained in" the premises *when they are put together.*

The view that the syllogism is a *petitio principii* was adopted by J. S. Mill (*Logic*, Bk. II. ch. iii.) With him, this view is based on a theory, concerning the *major premise*, which is of great importance. Put briefly, what Mill urges is this. Take the syllogism: "All men are fallible; Socrates is a man, therefore Socrates is fallible." How do we know that *all* men are fallible? We are not entitled to make this *general* assertion unless we already know that Socrates is fallible. Hence the conclusion, being already assumed in the general proposition, cannot be proved by it; and *if* the major premise is regarded—as in the Aristotelian view it always is—as the real ground of the proof of the conclusion, then the syllogism is a *petitio principii*. When we have got the *general principle*, we cannot *infer any particulars from it* but those which the principle itself

assumes as known: "for a general truth is but an aggregate of particular truths,—a comprehensive expression by means of which an indefinite number of individual facts are affirmed or denied at once" (*loc. cit.*, § 3). The conclusion about Socrates is inferred from the observed cases in which *other* men have been found fallible. Hence *the inference may take place without the general proposition.* "Not only may we reason from particulars to particulars, without passing through generals, but we perpetually do so reason. All our earliest inferences are of this nature. From the first dawn of intelligence we draw inferences, but years elapse before we learn the use of general language. The child who, having burnt his fingers, avoids thrusting them again into the fire, has reasoned or inferred, though he never thought of the general maxim, fire burns. He knows from memory that he has been burned, and on this evidence believes, when he sees a candle, that if he puts his finger into the flame of it, he will be burned again. He believes this in any case which happens to arise, but without looking in each instance beyond the present case. He is not generalising; he is inferring a particular from particulars. . . . It is not only the village matron who, when called to a consultation on the case of a neighbour's child, pronounces on the evil and its remedy on the recollection and authority of what she accounts the similar case of her Lucy. We all, when we have no general maxims to steer by, guide ourselves in the same way."

The essentials of Mill's view are:—

(1) All inference is from particulars to particulars.
(2) General propositions are merely registers of

such inferences already made, and short formulæ for making more.

(3) The major premise of a syllogism is a formula of this kind; the conclusion is not an inference drawn *from* the formula.

(4) The real logical antecedent or premise consists of the particular facts from which the general proposition was collected.

It is true that in a great deal of our reasoning we do not form general propositions; and it conforms to the instances given by Mill. But we have to ask, What justifies us in passing from one "particular" to another? It is the *resemblance* of the two cases—certain qualities which the two cases have in common. It is the *re*-cognition, in the second case, of attributes found in the first. These *common* characteristics form the only bridge by which we can pass from the one "particular" to the other. What, then, does this perception of similarity *imply?* The cognition and recognition of qualities common to different objects implies the formation in the mind of a general idea of those qualities,—a "universal" (see above, ch. II. § 6). When the child's experience of fire gives him an idea of it which he can extend to a new case, it is a universal idea. And the recognition of this universal is the germ of the recognition of a *general law.* The child may not separate the universal from its embodiment in the particular case, or put it into language even to himself; but he reasons through it. And when the reasoning is explicitly put into words, it must take some such form as this: "The qualities of brightness, movement, &c., found in that object, are also found in this; that object burns, therefore this, which has

the same general nature or is of the same type, burns also." This is *implicit* in the child's thought; and it is in principle a syllogistic argument, bringing a new case under a general principle.

This throws a new light on the nature of the general proposition. It is not "an aggregate of particular truths"; it does not refer merely to a *collection* of things. When I say "hemlock is poisonous," this does not mean merely that in certain cases I have seen it to be fatal; it means that, on the basis of observation, I affirm that there is something in hemlock which makes it fatal. I may gather a universal proposition *from a single instance*, provided that my investigation of it is sufficiently thorough; and the result could not be called an "aggregate of particulars." The characteristic of every truly general proposition is that it does not refer to any definite number or group of individuals, but to a perfectly indefinite number, namely, to all who possess certain attributes. It asserts a connection of attributes.[1] The conclusion of the syllogism is therefore not contained in the major premise. The major premise, when expressed so as to bring out its real meaning, naturally takes the hypothetical form (see above, § 3), since the whole emphasis is laid on the intension of its terms; and the syllogism may be thus expressed:—

"If anything possesses the attribute M, it possesses the attribute P;
S possesses the attribute M;
Therefore S possesses also the attribute P."

We cannot be sure of the conclusion until we have (in

[1] Cf. ch. XI. § 6.

the minor premise) compared the new case S with the general statement made in the major premise, and found their identity in the attribute M. It is entirely on this identity that the validity of the reasoning depends; it is the function of the minor to establish it. The conclusion, therefore, can only be drawn from the two premises in combination.

In those cases where the major premise does express an aggregate of particulars,—where it is no more than a collective statement about a group of facts,—and where the conclusion expresses one of these facts, we anticipate the conclusion in stating the major premise. But even in such cases there is a genuine inference—a discovery of something not known from either premise singly—*whenever we learn the two premises at different times or by different means.* If I learn that the vessel XY was lost at sea with all on board, and learn subsequently, or by some other means, that my friend AB was a passenger on that vessel, then there is no doubt that the conclusion is "something new," although the major states a mere collective fact, which (for those who know, but not for me) already contains the conclusion.

* The last-mentioned example leads us to notice a principle which has been clearly stated by Martineau, and which applies not only to cases of "class-reasoning," as in the last paragraph, but to inference in general. "In the nature of things, or in the sight of a perfect intellect, *all* reasoning must involve a *petitio principii*, the conclusion being already discerned on the first announcement of the premises. Ratiocination itself becomes nugatory in presence of a mind seizing by intuition what others reach by sequence. As soon as we descend to a more tardy and limited intelligence, there will be *some* beliefs that are only mediately reached: the same truths which to one being are contained within their ἀρχή are seen by another lying at some distance from

it. The *petitio principii* is thus entirely relative to the state and range of the individual understanding; and cannot be established as a fault against an argument by merely showing that the inference *might* be thought already in the assumption; but only that it *must* be. . . . The reasoner who, to bring conviction to others, invents the syllogism in question, . . . selects his general principle precisely *because* he foresees what it contains; but in using it he assumes in his hearers a different state of mind—in which the law has been apprehended and the example has been missed. Wherever a teacher and a learner are engaged together, the arguments comprehended in the didactic process involve a *petitio principii* to the former but not to the latter. Upon this difference, the consciousness in one man, the unconsciousness in another, of what, according to the laws of thought, a given proposition may imply, depends the whole business of reasoning as an instrument of persuasion." (*Theory of Reasoning*, Essays, vol. iii. pp. 423, 424.)

* § 7. We must now ask whether all deductive reasoning is capable of being expressed in strict syllogistic form. The usual view is, that not all such reasoning can be so expressed. The student will easily anticipate the fact, that this controversy turns on a possible wider or narrower conception of what "strict syllogistic" inference essentially is. The Aristotelian view is, that *all* inference whatever is capable of being expressed in syllogistic form, and, further, that the real nature of the inference only begins to be apparent when it is so expressed.

It is commonly supposed, however, that this is not the case. (*a*) The most familiar *quantitative relations* produce arguments which are said to be non-syllogistic: *e.g.*, "$A = C$, $B = C$, $\therefore$ $A = B$"; or, "$B = C$, $A = B$, $\therefore$ $A = C$." (*b*) Relations of *time* and *space* also frequently give rise to arguments which are stated to be non-syllogistic: "Bacon lived before Locke, Locke lived

before Hume, therefore Bacon lived before Hume"; "A is north of B, B is north of C, therefore A is north of C." (*c*) The same is true with such arguments as "A is the brother of B, B is the sister of C, therefore A is the brother of C," and (*d*) with such as "A is greater than B, B is greater than C, therefore A is greater than C."

In the *formal syllogism* the copula of the propositions originally expressed only the relation of subject and attribute; and though (as we have seen) it easily expresses the relation of genus and species (class inclusion and exclusion), it does not naturally express all possible relations by the word "is." Hence De Morgan proposed to extend the meaning of the copula, to take it merely as a general symbol signifying some kind of relation be tween subject and predicate; so that the typical syllogism would take the following form: "A is related to B in a certain understood way, B is related to C in the same way, therefore A is related to C in that way." This proposed extension of the meaning of the copula has been called the *Logic of Relatives*. Martineau, writing in 1852, had already suggested a classification of such relations:[1] "The ideas of space and time, of cause and effect, of resemblance and difference, seem to involve distinct laws of thought, to create for themselves special elements and functions of language, and to require special canons of logic. In all these spheres there is room for such a necessary nexus of conceptions as demonstration requires; yet the rules of class-reasoning [the syllogism] have no natural application. Such maxims as that a body cannot be in two places at once, —that *causa causæ causa causati*,—that two things, of

[1] In his essay on *The Theory of Reasoning*, from which we have already quoted; Essays, vol. iii. pp. 421, 422.

which the first is like and the second unlike a third, are unlike each other,—are not less really the basis of frequent reasoning than the *dictum* that what is true of the genus is true of the species." Mr F. H. Bradley, in his *Principles of Logic* (pp. 243, 244), has worked out a classification of the most important types of relation which ordinary judgments express, among which the syllogism takes its place, as dealing *only with propositions that express the relation of subject and attribute.* He considers that there are at least three types of reasoning, in which valid arguments may be and are constructed, and which are distinct from and parallel to the syllogistic type governed by the *dictum de omni et de nullo*, in its mediæval form. For the other types he enunciates separate principles:—

(1) "Synthesis of Identity: where one term has one and the same point in common with two or more terms, there these others have the same point in common." For instances, see examples (*a*) and (*c*) above.

(2) "Synthesis of Degree: when one term stands, by virtue of one and the same point in it, in a relation of degree with two or more other terms, then these others are also related in degree" (example (*d*) above).

(3) "Synthesis of Time and Space: when one and the same term stands to two or more other terms in any relation of Time or Space, there we must have a relation of Time or Space between these others" (example (*b*) above).

Some of the attempts which have been made to express these arguments in syllogistic form have undoubtedly failed. Thus if, in the case of the first argument mentioned in ex. (*a*), we make the general axiom a major premise,—

(*e*) "Things equal to the same thing are equal to one another;
A and B are equal to the same thing;
Therefore A and B are equal to one another,"—

we have not got a true syllogism, because the whole argument is contained in the major premise; and it does not represent the given argument, because C does not appear in it. If, again, we represent the second argument in ex. (*a*) thus,—

(*f*) "Whatever is equal to B is equal to C;
A is equal to B;
Therefore A is equal to C,"—

the new major premise begs the whole question by assuming that "B" and "what is equal to B" are for the purposes of the argument equivalent.

But Mr Bradley's view is not the only alternative; against it the following remarks may be made. One may put the matter as a question of verbal definition, and again as a question of the meaning of the *dictum de omni*.

(1) If by a *syllogism* we mean a piece of "class-reasoning," *formulated* in such a way as always to conform to the type,—

"Each of the individuals which make up the class M, is P;
S is one of these;
Therefore S is P,"—

then there are inferences, scientifically valid, which are not "syllogisms." But (2) if we interpret the *dictum* as Aristotle himself does,—for, when stating it, Aristotle says nothing about *classes* (see above, ch. VI. § 6); and if we regard the "wording" or "formulating" of an argument as not the essence of Logic, but as a process preliminary to the *logical* estimation of it,—as

the spreading out and dissecting of our specimen in order to examine it carefully and see the hidden mechanism,—then all these alleged special kinds of inference, parallel to the "syllogism" in the narrower sense of "class-reasoning," are syllogisms in the Aristotelian sense, which we have adopted. The mistake in ex. (*e*) above, is that the axiom in its most general form, without any intermediate application, is straightway made into the major premise of the syllogism. We should take as our major the general axiom *explicitly applied to* the particular case:

"Things equal to C are equal to one another;
A and B are equal to C;
Therefore A and B are equal to one another."

Similarly in ex. (*f*) we avoid the difficulty mentioned there, by taking as our major, "What is equal to B is equal to that to which B is equal (viz., C)." The actual major premise is (one form of) the general axiom applied to the particular case. The axiom in its most general form may be called the *ultimate major premise.* We shall see in the sequel that Inductive Inference is capable of being similarly expressed in syllogistic form with the principle of Uniformity of Causation as the *ultimate* major, and the application of this principle to the particular case, as the *actual* major.

* NOTE A.

ON SYLLOGISMS INVOLVING NUMERICAL PROPOSITIONS.

These were elaborately investigated by De Morgan in his *Formal Logic.* He pointed out that the following represents a very common type of argument: "If the majority of a public meeting vote for the first resolution, and a majority

also vote for the second, it follows necessarily that some who voted for the first voted also for the second." And from such instances De Morgan argued that two *particular premises* may give a valid conclusion if the actual quantities of the two terms are stated, and if, when added together, they exceed the quantity of the middle term. This is a misleading way of describing such arguments; for the premises are not "particular" in the logical sense. They depend on comparison of numerical relations, and they are at bottom cases of *counting*. They are no more and no less syllogistic than any other kind of calculation is. They share the general nature of arithmetical inference.

This large question cannot be discussed here. A trustworthy introduction to it will be found in Sigwart's *Logic*, Eng. trans., vol. ii. pp. 24 ff. (§§ 65, 66).

* NOTE B.

Aristotle's Defence of the Syllogism.

The objection to the syllogistic form of inference, on which Mill bases his charge that it is a *petitio principii*, was anticipated and answered by Aristotle himself.

In his *Posterior Analytics* he points out that nothing which we *infer*—or, as he expresses it, nothing which we discover by thought as distinct from sense-perception—can be entirely new; it must be at least in part an application of previous knowledge. In the case of deductive or syllogistic reasoning, we require to know—not a mere "collective fact," but—a universal law, and also to know a particular fact; and the inference arises only when we have the former in the mind and the latter is added to it (*An. Post.*, i. 1). Consider any scientific syllogism, *e.g.* :—

Every triangle has its three interior angles together equal to two right angles.

This is a triangle.

Therefore this has its three interior angles together equal to two right angles.

It would seem that some of the Sophists had brought

against the syllogism the same reproach which Mill afterwards brought,—that we have no right to assert the major premise unless we already know the conclusion. Aristotle's reply is as follows:—

"Before the instance is produced or the syllogism completed,[1] in one sense perhaps we must be said to know the conclusion; but in another sense not. For how could any one know in the full sense of the word that *this* triangle, of whose existence he is completely ignorant, has its angles equal to two right angles? Yet it is plain that in a sense he does know it, *inasmuch as he knows the universal;* but in the full sense he does not know it." Aristotle then explains how the objector puts the difficulty. He puts it by asking, "Do you or do you not *know that all* triangles have their angles equal to two right angles?" If the reply is, "I do know it," the objector produces a triangle whose existence was unknown to the respondent, and asserts that as its existence was unknown to him, the equality of its angles to two right angles must have been also unknown; hence he did *not* really know the general proposition which he had asserted. Now there were some who considered the right reply to be, "All the triangles *that we know* have their angles equal to two right angles," not simply "all triangles." This, says Aristotle, is not the correct reply. "They *do* know what they have demonstration of, and the general proposition which they accepted was *a demonstrated principle; it* concerned *not only the triangles which they were aware of as such, but every triangle without qualification.* There is no reason, however, in my opinion, why a man should not know in a sense what he is learning while in another sense he is ignorant of it. The real absurdity would not be this; but that he should know what he is learning in the same sense as when he has learnt it." (*An. Post.*, i. 1.)

In the words which are italicised in this passage, Aristotle consciously and definitely accepts the view that the true universal judgment is a *generic* judgment (ch. XI. § 6). It asserts a *connection of attributes* which depends only on

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[1] By "syllogism" is meant here the two premises.

the attributes themselves; they are such that one must follow from the other—*e.g.*, the equality of the interior angles to two right angles from the Euclidean definition of the triangle. When the major premise of a syllogism is a generic universal, it *includes* any particular instance "in a sense," as Aristotle says; in the sense that the law is potentially applicable to any instance. "In another sense" it does not include the particular case—*i.e.*, not until the latter is explicitly stated, in the minor premise, as an instance of the general law.

Exercise XV.

Questions on Chapter VII.

(i) *Elementary.*

1. What is a hypothetical syllogism? Explain and illustrate the terms Constructive, Destructive, *modus ponens*, *modus tollens*, as used in connection with such syllogisms.

2. What are the rules of hypothetical syllogism? To what rules of categorical syllogism do they correspond? [O.]

3. What aspects of thinking are emphasised by the categorical and hypothetical forms of reasoning respectively?

4. Show that the hypothetical syllogism is not an Immediate Inference.

5. Show the identity of the two forms of the hypothetical proposition.

6. How ought we to deal with the question, "*Are* the alternatives in a disjunctive proposition *exclusive?*"

7. Give two concrete examples of disjunctive syllogisms with negative conclusions, stating on what the validity of the argument depends.

8. Explain and illustrate the terms *modus ponendo tollens*, *modus tollendo ponens*, as used in connection with disjunctive syllogisms.

9. State concisely the structure and rules of the dilemma, and distinguish its four forms. Give an example of a simple destructive dilemma.

10. What is meant by "rebutting" a dilemma? Explain how such "rebuttal" comes to be possible.

11. Explain concisely the grounds on which it has been held that the syllogism is a *petitio principii*.

12. What is meant by saying that "all inference is from particulars to particulars"?

13. Explain and criticise the statement that the general proposition is an aggregate of particular truths.

14. Express the following arguments in strict logical form, so far as necessary, and examine their validity:—

(1) If a man is a tyrant, he deserves to die. Cæsar was not a tyrant, and therefore did not deserve to die.

(2) If virtue is involuntary, vice is also involuntary: but vice is voluntary, and therefore so is virtue.

(3) Logic is either a science or an art; it is a science; therefore it is not an art.

(4) If he succeeded, he must have been either very clever or very rich; but he was neither clever nor rich, and hence he cannot have succeeded. (See § 5 *ad finem.*)

(5) This event happened either at Rome, Naples, or Florence; it did not happen at Rome or Naples, and consequently it must have happened at Florence.

(6) If a man cannot make progress towards perfection, he must be either a brute or a divinity; but no man is either, therefore every man is capable of such progress.

(7) Whenever a syllogism is valid, it has not more than three terms; therefore this syllogism, which has not more than three terms, is valid.

(8) A man holds such opinions, if he is a Buddhist; therefore if *you* hold such opinions, you are a Buddhist.

(9) If education is popular, compulsion is unnecessary; if unpopular, compulsion will not be tolerated. [Fowler.]

(ii) *More Advanced.*

15. Can distinctions of logical *quantity* be made among hypothetical propositions? How far do the different types

of valid hypothetical syllogism correspond to valid moods of categorical syllogism? Give your reasons in both cases.

16. Discuss, with examples, the "reduction" of hypothetical syllogisms to the categorical form.

17 What are the rules which apply to Inference by Disjunctive propositions? Exemplify them.

* Show whether these rules are or are not reducible to the *dictum de omni et nullo*. [L.]

18. Define "Dilemma." Construct a dilemma to prove that "examinations are useless," and rebut it. [O.]

19. "Restrictions on amusement should be avoided, for they are useless if not attended to, and injurious if they give rise to discontent." Exhibit this in logical form, and show that from similar premises an argument may be offered in favour of retaining restrictions. Explain how this apparent dualism arises. [L.]

20. "All M's that I have observed, together with all M's observed in present and past times by other persons, are P." Is this a true expression of the logical significance of the universal proposition? What is the bearing of this question on the Theory of the Syllogism? [L.]

21. Express the following arguments in strict logical form, and examine their validity :—

(1) If all men were capable of perfection, some would have attained it; but none having done so, none are capable of it.

(2) If any objection that can be urged would justify a change of established laws, no laws could reasonably be maintained; but some laws can reasonably be maintained; therefore no objection that can be urged will justify a change of established laws.

(3) If a man is educated, he does not want to work with his hands; consequently, if education is universal, industry will cease.

(4) Giving advice is useless. For you either advise a man what he means to do, in which case the advice is superfluous; or you advise him what he does not mean to do, and the advice is ineffectual.

(5) We must either gratify our vicious propensities or resist them; the former course will involve us in

sin and misery; the latter requires self-denial; therefore we must either fall into sin and misery or practise self-denial.

(6) The laws of nature must be ascertained either by Induction or by Deduction. The latter is insufficient for the purpose, therefore they can only be ascertained by Induction.

(7) If their theories were sound, philosophers would agree among themselves.

(8) If "to improve is to change, and to be perfect is to have changed often," what are we to think of those who oppose change?

(9) He maintains that an action can only be called virtuous if it contributes to the welfare of man; he is therefore bound to maintain that every useful object, an article of food for instance, is virtuous.

(10) The end of human life is either perfection or happiness; death is the end of human life, therefore death is either perfection or happiness.

(11) No man should be punished if he is innocent; this man should not be punished, therefore he is innocent.

(12) If the orbit of a comet is diminished, either the comet passes through a resisting medium, or the law of gravitation is partially suspended. But the second alternative is inadmissible. Hence if the orbit of a comet is diminished, there is present a resisting medium.

[Jevons, Fowler, Creighton ; St A., L.]

* 22. Can the argument *a fortiori* be reduced under the common syllogism? [L.]

* 23. Is the syllogism the type of all reasoning? If not, what is the type? [L.]

* 24. State and examine Bradley's view of the limits of strictly *syllogistic* inference.

State, in its most general and comprehensive form, the essential principle on which syllogistic inference rests.

* 25. It is maintained, on the one hand, that no inference is valid in which the conclusion is not contained in the pre-

mises, and, on the other hand, that no movement of thought deserves to be called inference in which there is not progress from the known to the unknown. Examine the grounds for the two statements, and discuss the possibility of holding them jointly. [L.]

* 26. Examine the following arguments :—

(*a*) "It would seem that a man cannot inquire either about that which he knows or about that which he does not know; for if he knows, he has no reason to inquire; and if he does not know, he cannot inquire, for he does not know the very subject about which he is to inquire" (*from* Plato : *Meno*, 80 E).

(*b*) "Logic can only deal with the *form* of thought; for if it were to take the matter also into account, it would have to deal with all objects without distinction, or make a selection. The former alternative is manifestly impossible: the latter would be quite arbitrary and therefore absurd' (*from* Hamilton).

(*c*) "A beginning of Time is inconceivable, for a beginning must be *in* Time; but Infinite Time is inconceivable; so our thought is brought to a standstill between two contradictory inconceivables" (*from* Hamilton). [D. G. Ritchie.]

CHAPTER VIII.

THE GENERAL NATURE OF INDUCTION.

§ 1 In passing to Inductive Logic, we must return to a point which we reached in the previous chapter (§ 3). The major premise of a hypothetical syllogism, being itself a hypothetical proposition, "affirms only that the relation of Reason and Consequence holds between two judgments or concepts; it does not expressly refer to instances where the relation actually occurs; and about any particular instance it tells us nothing at all." It is a universal proposition of the type to which we referred when examining Mill's theory of the Syllogism (previous chapter, § 6, p. 230). This is **the natural form in which to express a Law of Nature**; and the hypothetical syllogism is the natural form in which to express the application of such a Law to a particular case:—

> Law of Nature: If anything is M it is P;
> Particular fact: S is M;
> Application and Conclusion: S is P.

For simple examples we may give the following: (1) "If a material body is heated, its bulk or volume changes; this pendulum is a material body heated; therefore its volume changes"; (2) "If the pendulum is lengthened, the path in which it swings is lengthened;

the heated pendulum is lengthened; therefore the path in which it swings is lengthened."

A Law of Nature is a general statement of a constant connection between Cause and Effect. When formulated as a hypothetical proposition, the antecedent expresses the cause and the consequent the effect; by saying "If anything is M it is P," we mean that there is a constant connection between M the cause and P the effect. In the given examples, the heating of a material body *causes* its volume to change; the lengthening of the pendulum *causes* its oscillations to widen. Hence Laws of Nature are Laws of Causation. The ordinary business of Science is to *discover such Laws*. The aim of Inductive Logic is to give an account of the methods by which such general principles or Laws of Nature may be established. In other words, **Inductive Logic aims at understanding and classifying the Methods of the Sciences.**

The term *Induction* usually signifies the general process of Science, understood as seeking to establish (discover and prove) Laws. Hence Jevons said that Induction "consists in detecting a general truth among its particular occurrences," "detecting the general laws or uniformities, the relations of cause and effect, or in short all the general truths that may be asserted concerning the numberless and very diverse events that take place in the natural world around us."

Induction and Deduction are not two opposite kinds of reasoning. But the starting-point is different in the two processes; in Deduction, we start with general principles, fitted to serve as major premises; in Induction, we start with facts of observation, not yet raised to the rank of principles.

It is not only in scientific matters that we employ inductive methods. In the commonest affairs we are continually seeking to explain or account for what happens, and in

doing so we employ, in a germinal, elementary form, the genuine method of science.

§ 2. The term Induction has come down to us through the Latin from an Aristotelian term (ἐπαγωγή), which Aristotle uses in a limited meaning. He says that "Induction" reasons from part to whole; we realise, as it were, the truth about the whole by going through the truths about the parts. Thus, to take one of his examples, we see that the skilful steersman is best, and the skilful driver, and so on, and thus we realise that the man who is skilful is best in every occupation. In other words, we illustrate a statement about a whole class by reference to particular cases of it.

The following is Aristotle's most complete account of the process to which he limits the name of "Induction" (*An. Prior.*, ii. 23). It consists in "proving the major of the middle by means of the minor." To understand this, we must first state a syllogism in *Barbara*:—

$$\begin{array}{l} \text{All B is A,} \\ \underline{\text{All C is B ;}} \\ \therefore \text{All C is A.} \end{array}$$

Here, as usual, the major term, A, is proved of the minor, C, by means of the middle, B. But in the inductive syllogism we prove A of B by means of C:—

$$\begin{array}{l} \text{All C is A,} \\ \underline{\text{All C is B ;}} \\ \therefore \text{All B is A.} \end{array}$$

This is a syllogism in fig. iii. and is formally invalid; but it is a cogent argument if we know not only that all C is B but that B and C are convertible, so that all B is C also; for if we then substitute "all B is C" for the old minor premise, we have a valid syllogism in *Barbara*. The possibility of the inductive syllogism depends on our finding, by exhaustive observation, that B and C are convertible.

Thus, to take a concrete example, let A=ductile, B=metal, C=particular ductile kinds of metal, gold, copper, lead, &c. Then the inductive syllogism is :—

Gold, copper, lead, &c., are ductile ;
Gold, copper, lead, &c., are metals ;
∴ All metals are ductile.

This argument is cogent if we know not only that these metals are ductile but that they are *all* the existing metals. The minor premise must give a complete enumeration of all the instances. Otherwise we have only a syllogism in *Darapti* ("gold, copper, lead, &c.," being regarded as a collective singular term), with formally incorrect conclusion.

* Thus the kind of inference which Aristotle calls ἐπαγωγή, induction, is really deductive. In fact, Aristotle does not regard this "induction" as a kind of proof distinct from Deduction. All strict proof is Deduction (ἀπόδειξις), and may be formally expressed as a syllogism in fig. i. (συλλογισμὸς διὰ τοῦ μέσου). What Aristotle calls Induction is—to parody Mill—"not a way in which we *must* reason, but a way in which we *may* reason" to make things clearer (δηλοῦν : or πιθανώτερον, σαφέστερον ποιεῖν) to ourselves and others.[1] It is a mode of arranging a deductive argument so as to enable us to realise, psychologically, the truth of the general principle (ἀρχή) which is the real major premise,—a mode of illustrating the principle by bringing forward instances. Of course, we cannot get "all" the instances, except where the number is limited ; but this fact does not vitiate an illustrative "induction" such as Aristotle had in view.[2]

With the mediæval logicians Induction became simply a process of counting particular things ; and when we have thus found by enumeration that each one has the quality P, the Induction consists in concluding that they all are P. Thus we may prove by complete

[1] See *An. Prior.*, ii. 23, 69 *b* 35 ; *Top.*, i. 12, 105 *a* 16 ; and cf. *An. Post.*, i. 31.
[2] Cf. *An. Post.*, i. 4, 73 *b* 33.

enumeration that "all the months of the year have less than thirty-two days," for the number of months is limited, and so we can ascertain the fact in each particular case before making the general statement. This is **perfect induction.** But it usually happens that we cannot go over all the particular cases, for some of them may occur at future times or in distant parts of the earth or other regions of the universe. When complete enumeration of them all is impossible, the Induction is called **imperfect**: "This crow is black, and that one, and that one, up to all that I have seen or heard of; therefore all crows (without exception) are black." The scholastic "imperfect induction" consists essentially in enumerating all the *known or observed* cases of some object S, and, if it is found that each of them is P, inferring that every S, known *and unknown*, is P. The process rests on observation and counting, and nothing more.

These kinds of induction were vigorously attacked by Bacon and Mill. Mill says, for instance, that Perfect Induction is of no scientific value whatever; the conclusion is only a reassertion in briefer form of the premises. To this Jevons has well replied: "If Perfect Induction were no more than a process of abbreviation, it is yet of great importance, and requires to be continually used in science and common life. Without it we could never make a comprehensive statement, but should be obliged to enumerate every particular. After examining the books in a library and finding them to be all English books, we should be unable to sum up our results in the one proposition, 'all the books in this library are English books'; but should be required to go over the list of books every time we desired to make any one acquainted with the

contents of the library. The fact is, that the power of expressing a great number of particular facts in a very brief space is essential to the progress of science. Just as the whole art of arithmetic consists in nothing but a series of processes for abbreviating addition and subtraction, and enabling us to deal with a great number of units in a very short time, so Perfect Induction is absolutely necessary to enable us to deal with a great number of particular facts in a very brief space."

The case of Imperfect Induction is very different. It is a kind of inference which, as Bacon says, *precarie concludit, et periculo exponitur ab instantia contradictoria.* A simple negative instance will refute it. As regards the example given, few people would care to assert that a grey crow has never been *seen.*[1] It cannot be too strongly impressed on the mind of the student that no mere counting of instances, however many they may be, can make a conclusion more certain. We may know that S and P are conjoined twice or two thousand or two million times; but this does not warrant us in saying that they are *always conjoined* unless we have something more than the mere number to go upon. A mere *enumeratio simplex*, a mere assemblage of positive instances, is simply worthless. Take an old example: "The three interior angles of a triangle are together equal to two right angles." This is known to be true universally, for it is proved from the definition of a triangle and the Euclidean axiom of parallels, which we assume to be universally true, throughout all Space, until the contrary is proved. Suppose that the *proof* of Euclid I. 32 were not known, and that we had to rely only on measurement of the angles of particular triangles to discover what their sum is in each case.

[1] As a matter of fact, grey crows are numerous.

Granting that the measurement could be made with sufficient accuracy to establish the proposition in particular cases, there would be no warrant for taking it to be true of any triangle whose angles we had not measured. There is nothing in the mere measurement of a triangle to show that the sum of its angles *must* be of this particular magnitude. Another example of the difference between the enumeration of positive instances, and real proof, is found in the laws of planetary motion. Newton proved deductively, from the law of gravitation, that the paths of the planets round the sun must be elliptical. Before the discovery of the true law of gravitation, Kepler had attacked the problem of planetary motion; and by laborious calculation on the basis of an immense number of observations, had proved the ellipticity of the orbit of Mars. But this did not prove it of any other planet; the motion of each one in turn would have to be observed with sufficient accuracy to see whether it constituted an ellipse or not. When this had been done with all the *known* planets, it would still be impossible to say that *all* the planets move in ellipses. As a matter of fact, in Kepler's time Neptune, Uranus, and all the asteroids were unknown; and even now there may be another planet beyond Neptune, or one between Mercury and the sun, which we do not know of. Hence if we have nothing but observation and measurement to rely on, we cannot say that *all* the planets move in ellipses. But now we know that if Newton's law of gravitation is true, they *must* do so, whether we have observed them all or not.

But not all "simple enumerations" are turned into demonstrations in this way. Before Neptune and Uranus had been discovered, it was found that all the

satellites in the planetary system went in one uniform direction round their planets. Not only has no reason been found for this, but it has been found that the satellites of Uranus and Neptune move round them in the opposite direction.

There is one condition on which a simple enumeration of positive instances may furnish—not indeed a *demonstration*, but—a strong presumption or probability: when we have reason to suppose that, were there any instances to the contrary, they would have become known to us. A well-grounded conviction that there are no negative instances, even in the absence of complete assurance, may afford a very high degree of probability. This appears to have been the view of Aristotle (*Topics*, viii. 8); and as Aristotle suggests, if any one objects to a generalisation held on such grounds, it rests with the objector to find a negative instance.

What modern Inductive Logic inquires into is, how we establish a *reliable* general statement,—one which goes beyond the range of our particular experience, and yet is more than "probable." How are we justified in concluding from one or more cases known to us, a law for all cases of the same kind? How, in short, can we establish a Law of Nature?

* § 3. To the question stated at the end of the last section, Aristotle paid comparatively little attention; what he says about it is contained in the doctrines of the Enthymeme and the Example (παράδειγμα).

The Aristotelian Enthymeme is of great logical significance; it covers the elementary forms of what later writers have called Induction. And in his treatment of it, Aristotle marks some of the stages by which we pass from guess-work towards scientific knowledge. In one place he speaks of it as "a rhetorical form of

the syllogism," useful for persuasion and for concealing fallacies (*Rhetoric*, i. 2); but it is much more than this.

(i) Before describing the enthymeme, we must indicate Aristotle's conception of a true "scientific syllogism" (συλλογισμὸς ἐπιστημονικός). Consider the premise "if anything is M it is P." Regarded as a logical proposition, in the formal sense, it states that the antecedent is the *reason* of the consequent: looked at in its reference to the real world, it states that M is the *cause* of P; it implies that we have discovered a law of causation in Nature, and M is the cause in question. Now when the syllogism is changed from the hypothetical to the categorical form, M becomes the *middle term*:—

Hypothetical.	*Categorical.*
If anything is M it is P,	All M is P,
S is M;	S is M;
∴ S is P.	∴ S is P.

Hence Aristotle says τὸ μὲν γὰρ αἴτιον τὸ μέσον (*An. Post.*, ii. 2): "the middle term expresses the cause." We may therefore say with Ueberweg (*Logic*, § 101): the worth of the syllogism as a form of knowledge depends on the assumption that general laws of causation hold in nature, and may be known; and that syllogism has the greatest scientific value in which the mediating concept (the middle term), by which we know the truth of the conclusion, expresses the real cause of the fact stated in the conclusion. This is essentially the Aristotelian doctrine.

It will be seen that the typical form of the "scientific syllogism" closely resembles the inferences discussed in the last section of the previous chapter. The Major Premise or Law of Nature is really the axiom of Universal Causation

(see below, § 7) applied to the particular case. The general axiom is, "the same cause will always produce the same effect." This, in its explicit application, becomes: "The cause M (having produced P) will always produce the same," which is the real meaning summarily expressed in the hypothetical proposition, "If anything is M it is P." This is the *actual* major premise; the axiom of causation is the *ultimate* major premise (ch. VII. § 7, *ad finem*).

(ii) An enthymeme is "an argument from probabilities or signs" (*An. Prior.*, ii. 27: ἐνθύμημα μὲν οὖν ἐστὶ συλλογισμὸς ἐξ εἰκότων ἢ σημείων). The word ἐνθύμημα is derived, not from ἐν and θύμος, but from ἐνθυμεῖσθαι, to reflect upon, or hold as probable. By εἰκός, the "probable," Aristotle means the rough generalisations of ordinary practical experience (ἐμπειρία), such as are embodied in proverbs, &c. Enthymemes ἐξ εἰκότων, from "probabilities" of this kind, are all in fig. i.; but, having only a probable major premise, they have only a probable conclusion. The best illustrations of the "sign" (σημεῖον) are medical; the word might be rendered "symptom,"—the σημεῖον being the symptom from which the physician makes his diagnosis. To state it more generally, the "sign" is a fact which is found to accompany some other fact. The two facts may go together in time, as when the carnivorous habits of certain animals are a sign of great ferocity and strength; or one may follow the other, as lightning and thunder may be signs of one another. The union of the two facts may have all degrees of probability, from absolute necessity down to the most groundless opinion, as when the flight of birds is taken to be a sign of coming events. The conclusion of course cannot be more certain than the sign.

The forms of the enthymeme correspond to the three figures of the syllogism. We begin with the third figure.

In conversation and writing, one of the premises is frequently omitted, when it is obvious.

(*a*) In the third figure, the enthymeme gives an instance of an accepted or suggested rule: "Wise men are good, for Pittakos is good" (Aristotle, *loc. cit.*). Stated in full, this becomes:—

Example 1.

Pittakos is good;
Pittakos is wise;
Therefore wise men are good (*i.e.*, the individual instance of Pittakos is the *sign* from which we infer a real connection between the two qualities which he possesses).

What we are usually inclined to do in such a case is to make the conclusion universal, thus committing the formal fault of illicit minor in fig. iii. (*Darapti*). Nevertheless the universal conclusion, though *formally* unsound, may be justified by the one example, if we have examined it thoroughly enough to discern a real *connection* between the wisdom and the goodness. Otherwise, their combination in this instance may be merely accidental, and we are justified only in concluding that wisdom and goodness are not incompatible; they are united in this case, and may be so in other cases as well.[1] The following is an instance where we should go quite wrong if we leapt, without further examination of the case, to a universal conclusion:

Ex. 2.

Potassium floats on water;
Potassium is a metal;
Therefore metals float on water.

[1] This is what the formally correct conclusion, "some wise men are good," really means (see ch. III. § 2, p. 57).

The enthymeme in fig. iii. may be compared to the beginning of a scientific investigation. It points out the circumstances under which a conjunction of facts—which is popularly believed, or has been suggested to be true—really takes place. The following, as Prof. Bosanquet says, is little more than "an observation and a guess":—

Ex. 3.

Yesterday it rained in the evening;
All yesterday the smoke tended to sink;
Therefore smoke-sinking may be, or is sometimes, a sign of rain.

The following is rather more than a guess. Prof. Bosanquet calls it "enumerative suggestion":—

Ex. 4.

Three species of butterfly, genus x, closely resemble three species of y;
The species of x would be protected by resembling y (because y is distasteful to birds);
Therefore the resemblance may be a "protective resemblance"—*i.e.*, a resemblance brought about by the survival of those thus protected.

What we call Induction by simple enumeration in the absence of a contradictory instance, is really an example of an enthymeme in fig. iii. with universal conclusion:—

Ex. 5.

x, y, z, are ductile;
x, y, z, are metals;
Therefore all metals are ductile.

An argument of this sort is unreliable as long as the instances are merely counted; unless we have good reason to believe that if there were any negative in-

stances, we should have met with them (see the preceding section).

(*b*) In the second figure, the enthymeme comes nearer to giving us real knowledge than in fig. iii.[1] It does not merely adduce one or more instances; it *compares* two cases.

Ex. 1.

Fever-stricken patients are excessively thirsty;
This patient is excessively thirsty;
Therefore he is fever-stricken.

Formally, all enthymemes in fig. ii. are invalid, for they attempt an affirmative conclusion; but practically they are of extreme importance. The "sign" or "symptom" is not conclusive, for it might have another cause; but the conclusion has a certain probability. And when we have a number of independent symptoms all suggesting the same conclusion, we regard the conclusion as practically certain. In legal investigations, a "coil" of *circumstantial evidence* consists of nothing else than a series of enthymemes in fig. ii. For example: a person is found in an uninhabited house, dead from the effects of a wound; and on that same evening, a man, A.B., is seen running away from the neighbourhood of the house.

Ex. 2.

Murderers flee from the scene of the crime;
A.B. flees from the scene of the crime;
Therefore A.B. may be the murderer.

[1] This is true, although *formally* the enthymeme in fig. ii. is more invalid than in fig. iii., where the only formal fault is an A conclusion instead of I. This is why Aristotle considers that the enthymeme in fig. ii. is the *least* certain, and may be quite fallacious (ἀεὶ λυσιμός). Still, it is a step nearer to scientific knowledge than the mere enumeration in fig. iii.

This, by itself, is of course very inconclusive. But if, when A.B.'s house is searched, it is found that his clothes are blood-stained, then we may make another enthymeme in fig. ii., with conclusion pointing in the same direction. Similarly with other items of evidence —*e.g.*, A.B.'s boots fit the fresh foot-marks going from the house where the murder was committed; and so on. Many times a group of such enthymemes has led, rightly or wrongly, to the execution of a prisoner.

The following examples afford tentative justifications of what is suggested by the last two examples in fig. iii.:—

Ex. 3.

Smoke that goes downwards is heavier than air;
Particles of moisture are heavier than air;
Therefore particles of moisture are in the descending smoke.

This conclusion is probable; for the cause would naturally act in the way suggested. For the other example, we may find a rather stronger justification.

Ex. 4.

Protective resemblances naturally increase through series of species from slighter to closer resemblance;
The resemblances in question increase in genus x from slighter to closer resemblance to y;
Therefore the resemblances in question show important signs of being protective.

The student should notice, finally, that our ordinary perceptive judgments are enthymemes in fig. ii., when their implication is expressed in words:—

Ex. 5.

An oak-tree has such and such appearances;
This object has the same appearances;
Therefore this object is an oak-tree.

Again:—

My brother has such and such an appearance;
That person has the same appearance;
Therefore that person is my brother.

Most of our mistakes in identification arise from the *formal* invalidity of the inference into which the perceptive judgment may be expanded.

(*c*) In the first figure, as we said, the enthymeme becomes a formally valid syllogism whose truth depends on the truth of the major premise. The enthymeme in fig. i. differs from the scientific syllogism (συλλογισμὸς ἐπιστημονικός) in fig. i., only through having as middle term the symptom or effect, not the cause or ground. The following examples will make the difference clear:—

Ex. 1. Enthymeme in fig. i.

All such combinations of symptoms mean consumption;
Here we have such a combination;
Therefore this is a case of consumption.

The physician's diagnosis would run thus; and the middle term—the combination of characteristic symptoms—does not express the cause, but the effect, of the disease. But in a treatise on the subject, he would begin by describing the specific microbe or bacillus and the effects of its introduction into the human organism: "When bacillus x is introduced, such and such things follow; here it is introduced; observe the consequences." And when this argu-

ment is expressed formally as a syllogism, it would run thus :—

Ex. 2. Scientific syllogism in fig. i.

If bacillus x is introduced, such and such things follow;

This is a case of the introduction of the bacillus;

Therefore the results in question must follow.

And observation shows that the results do follow. When expressed categorically, the syllogism has, as its middle term, "the introduction of the bacillus"—*i.e.*, the *cause* of the disease.

We may also sum up the result of the discussion as to the connection of smoke and rain, in the form of an enthymeme in fig. i. :—

Ex. 3.[1]

All particles that sink in the air in damp weather more than in dry are loaded with moisture when they sink;

Smoke that descends before rain is an example of particles that sink in the air in damp weather more than in dry;

Therefore smoke that descends before rain is loaded with moisture when it descends—*i.e.*, is really connected with the cause of rain.

(iii) The argument called by Aristotle "example" (παράδειγμα) is practically equivalent to what we now call analogy. It is what Mill called reasoning from particular to particular, from one instance to another. "Athletes are not chosen by lot, therefore neither should statesmen be," is one of Aristotle's examples (*Rhetoric*, ii.

[1] Examples *a* (3 and 4), *b* (3 and 4), and *c* (3) are from Prof. Bosanquet's *Essentials of Logic*, where, however, they are used in another connection.

20). Aristotle thus describes it: "The Paradeigma reasons from particular to particular (ὡς μέρος πρὸς μέρος). It brings both cases *under the same universal,*—one being known [to come under it]." Aristotle saw what Mill did not: if we argue from one particular to another which resembles it in certain attributes, it is only because we have formed in our minds a concept, a universal, which represents those attributes of the first object, and we find it to be applicable to the second. All that Mill proved was that we do not, or need not, consciously express the universal in the form of a general proposition.

In order to bring out clearly that this kind of reasoning depends on a universal, Aristotle arranges it as an Imperfect Induction followed by a syllogism. Aristotle's example of an analogical argument is as follows: "The war between the Thebans and Phocians was a war between neighbours, and an evil; hence war between the Athenians and Thebans will be evil, for it is a war between neighbours." We have, first, an incomplete induction:—

War between Thebans and Phocians was disastrous;
This war was one between neighbours;
Therefore war between neighbours is disastrous.

This brings out the universal which connects the two cases, and which is then applied deductively to the second case:—

War between neighbours is disastrous;
War between Athenians and Thebans is war between neighbours;
Therefore war between Athenians and Thebans is disastrous.

The principle of this analysis is quite sound; we form a universal from the first case and apply it to the second.

The argument from Example may also be arranged—more concisely and not less correctly—as an Aristotelian enthymeme in fig. ii. :—

This disastrous war (referring to the instance of Thebes and Phocis) is a war between neighbours;
War between Athens and Thebes is a war between neighbours;
Therefore war between Athens and Thebes will probably be disastrous.

This would be formally incorrect as a syllogism in fig. ii., for it has an undistributed middle; but as an enthymeme it gives a real probability.

§ 4. Before dealing with the central problem of Induction, we must consider the argument from Analogy as it is explained in modern Logic.

In ordinary language the word Analogy is often used very loosely, to signify any kind of resemblance between two things. In Logic, Analogy is an inference from one instance to another which resembles it in certain respects: "Two things resemble each other in one or more respects; a certain proposition is true of the one, therefore it is true of the other" (Mill, *Logic*, III. xx. § 2). The inference may have all degrees of value,—from being worse than worthless (when the resemblance lies in merely accidental qualities), to being a ground for a practically certain conclusion. Its worth depends on the importance of the points of resemblance on which it is based.

On what does the "importance" of the points of resemblance depend? Not on the mere number of resemblances, as Mill said, "the extent of ascertained resemblance compared first with the amount of ascertained difference, and next with the extent of the unexplored region of unascertained differences." The

"unexplored region" here referred to cannot be used as a standard of comparison, simply because it *is* "unexplored." And the *unknown* range of points of difference between the two cases makes it impossible to take the mere ratio of known resemblances to known differences as a valid ground for an inference, as Mill maintains (*Logic*, III. xx. § 3). Two cases may resemble one another in a very large number of unimportant respects, affording not the least ground for inferring a resemblance in any other quality. For instance, two boys may resemble one another in height, features, strength, and other physical gifts, may be of the same age, born in the same town, educated in the same way, come from families of similar social position and cultivation; yet could we infer that because one of them has native mental abilities of a high order, the other will have the same? If the *number* of points of resemblance were the essential thing, the argument ought to possess some force; but it is clearly worthless. The reason is that none of the points of resemblance are *fundamental*. Hence, as Prof. Bosanquet says, in Analogy we must *weigh* the points of resemblance, not simply *count* them.[1] For a like reason we must weigh the points of difference, and see whether the two cases differ in any fundamental quality. **The resemblances must be essential, the differences unessential.** General experience, and systematic knowledge of the subject to which the given analogy belongs, are the only means of distinguishing the essential and the unessential.

The following example has been frequently used as an

[1] If we insist on regarding it only as a case of counting, Analogy becomes (what Fowler makes it) an inferior form of Induction, corresponding in Intension to Imperfect Enumeration in Extension. (See Fowler, *Inductive Logic*, pp. 227-234.)

illustration of Analogy. Prof. Minto quotes it from Reid (*Intellectual Powers*, Essay I. ch. iii.): "We may observe a very great similitude between this earth which we inhabit and the other planets. They all revolve round the sun, as the earth does, though at different distances and in different periods. They borrow all their light from the sun, as the earth does. Several of them are known to revolve round their axis like the earth, and by that means have like succession of day and night. Some of them have moons, that serve to give them light in the absence of the sun, as our moon does to us. They are all, in their motions, subject to the same law of gravitation as the earth is. From all this similitude it is not unreasonable to think that these planets may, like our earth, be the habitation of various orders of living creatures." The inference, as Reid states it, is, however, defective in two ways. (1) Though all the points which he mentions are important, he does not mention the absolutely necessary conditions for the existence of life; (2) he neglects the possibility that the other planets may *differ* from the earth in such ways that those essential conditions are not fulfilled. What are the essential conditions of the possibility of life?[1] "By life we mean the existence of organisms which depend upon the possession of a nitrogenous compound, protoplasm, for the chemical changes by which the phenomena of living are exhibited; and upon the presence in the atmosphere, or dissolved in water, of the element oxygen, with which their nitrogenous constituents combine." This requires also a temperature free from extremes of heat and cold much greater than those found on the earth. Now some of the planets may resemble the earth in all the ways enumerated by Reid, and yet may not provide for these strictly essential conditions.

* We have seen that Analogy may be expressed as an inconclusive but probable argument in fig. ii.,—an enthymeme from a "sign." Hence, as we may have a convergence of "signs," so we may have a convergence of analogical arguments, leading to practical certainty; thus:—

[1] That is, of "life" in the only sense of the word which we can conceive.

(*a*) In districts of the earth now exposed to glacial action we find scored or "striated" rocks;
In such and such a valley in Great Britain we find striated rocks;
Therefore this valley probably has been exposed to glacial action.

(*b*) In districts now exposed to glacial action we find perched boulders;
In the same valley we find perched boulders;
Therefore this valley has been exposed to glacial action.

(*c*) In districts now exposed to glacial action we find lateral and terminal "moraines";
In the same valley we find lateral and terminal moraines;
Therefore this valley has been exposed to glacial action.

Such a convergence of analogies, each inconclusive if taken by itself, leaves no room for doubt. Of one such case, Darwin said: "A house burnt down by fire did not tell its story more plainly than did this valley."[1]

§ 5. We now come to the essential problem of modern Induction, as we have formulated it at the end of § 1 and again at the end of § 2. Our experience is fragmentary and incomplete; it gives us events one by one, whose real connections have to be discovered. What Science does is to seek for *causal connections* between fact and fact; and we want to know what conditions must be satisfied before we can legitimately infer such a connection between two facts, so that we can say

[1] The student should notice that the English word *analogy* comes from the Greek ἀναλογία, but has changed the meaning of its Greek original. The word ἀναλογία is used for what we call *proportion* in mathematics,—an equivalence of ratios, ἰσότης λόγων. In this sense of the word it would be said that the relation of four to two is analogous to that of six to three. Hence Euclid speaks of "Analogy, or Proportion."

that one is the Cause of the other,—that one must have preceded in order that the other may happen.

Such an assertion is a universal law, in the form "S is P" or "S must be P," or, to bring out the real meaning, "if S is M it is P" (where M and P are causally connected). And *if the knowledge of such a law is properly reached*,—that is, if we are sure that M and nothing but M is the cause of P,—then the connection between M and P is independent of time and place. We can reason backwards to unobserved cases in the past, and dip into the future and be sure that P will always be produced by M.

There are two different questions concerning our discovery of a Law of Nature. How came the inquirer to think of the principle, as a suggestion or a possibility? How, when once suggested, is it to be *proved?* We will attend to the latter only for the present, as it is logically the most fundamental. It will be most advantageous to consider first the case where such Laws are most easily obtained,—Induction in Mathematics.

(*a*) In illustration of Geometrical Induction we may quote a forcible passage from Jevons:—

"When in the fifth proposition of the first book of Euclid we prove that the angles at the base of an isosceles triangle are equal to each other, it is done by taking one particular triangle as an example. A figure is given which the reader is requested to regard as having two equal sides, and it is conclusively proved that if the sides be really equal then the angles opposite to those sides must be equal also. But Euclid says nothing about other isosceles triangles; he treats one single triangle as a sufficient specimen of all isosceles triangles, and we are asked to believe that what is true of that is true of any other, whether its sides be so small as to be only visible in a microscope, or so large as to reach to the

farthest fixed star. There may evidently be an infinite number of isosceles triangles as regards the length of the equal sides, and each of these may be infinitely varied by increasing or diminishing the contained angle, so that the number of possible isosceles triangles is infinitely infinite; and yet we are asked to believe of this incomprehensible number of objects what we have proved only of one single specimen. We do know with as much certainty as knowledge can possess, that if lines be conceived as drawn from the earth to two stars equally distant, they will make equal angles with the line joining those stars; and yet we can never have tried the experiment."

In this passage Jevons has well shown the "universality" of the results of Geometrical reasoning. But he does not clearly bring out what is the most essential point, the reason why this universality is attainable. By examination of a single case we have reached an absolutely universal law. How is this possible? It is possible for two reasons. We know by definition what are the *essential qualities* of the isosceles triangle; and we argue from these essential qualities and from no others. Hence we are certain that the result will be true of *every* isosceles triangle; for every isosceles triangle, simply because it is isosceles, must agree with our specimen in all the qualities necessary for the proof. The length of any of the sides, or the size of any of the angles,—points in which any triangle may differ from any other,—are not included in the definition of the triangle, and they are not the points on which the proof depended.

The universality of the result depends on our being absolutely certain of what are the essentials of the kind of triangle in question; and we can be certain of these

because in geometry definitions have not to be *discovered.* The geometrician can frame his own definitions, and change them, if necessary.

(*b*) Let us next consider an algebraical formula which is true universally—*i.e.*, true whatever quantities the letters may represent. It may easily be proved that

$$(a+b)(a-b) = a^2-b^2.$$

Having proved this in the single case, we know that the result is of absolutely universal validity, whatever the quantities may be. How do we know this? Because the proof depended only on the definitions and rules of a few fundamental algebraical operations, on the "essential qualities," so to speak, of these operations, and not on any quantity that the terms a and b might represent. And the definition and rules of operation have not to be discovered; the algebraist, like the geometrician, frames his own definitions.

(*c*) There is a process technically termed "Mathematical Induction," which reaches a universal conclusion from two or three instances. It illustrates the same principle as the previous inductions; but it is specially applicable to terms which may be arranged in a regular series whose order of progression is known. The following, which is a fairly simple example, is given by De Morgan:—

"Observe the proof that the *square* of any number is equal to as many consecutive odd numbers, beginning with unity, as there are units in that number: thus $6\times 6=1+3+5+7+9+11$. Take any number, n, and write down $n\times n$ dots in rank and file, so that a dot represents a unit. To enlarge this figure into $(n+1)\times(n+1)$ dots, we must place n more dots at each of two adjacent sides, and one more at the corner. So that the square of n is changed into the square of $(n+1)$ by

adding $2n+1$, which is the $(n+1)$th odd number. (Thus 100×100 is turned into 101×101 by adding the 101st odd number, or 201). If then the alleged theorem be true of $n \times n$, it is therefore true of $(n+1) \times (n+1)$. But it is true of the first number, for $1 \times 1 = 1$; therefore it is true of the second—*i.e.*, $2 \times 2 = 1 + 3$; and therefore of the third—*i.e.*, $3 \times 3 = 1 + 3 + 5$; and so on."

Here we have a series of terms (1, 2, 3, &c.) in which we know the relation between every pair of *consecutive* terms. We wish to establish a fact about every term in it. We *suppose* that the fact holds of *any one* term, which we therefore denote by n; and prove that it holds of the *next* term, which is $n+1$. We then find by observation that it holds of the first term, 1; *therefore* it must hold of the second, 2; and so on. The universality of the result depends on the fact that the *essential relation* (which is simply a numerical one) between any pair of consecutive terms is known as n, $n+1$; and the proof depends on this alone.

On the other hand, where this proof from the essential conditions cannot be obtained, we may verify a theorem in case after case, without being sure that it holds universally. This is a case of "incomplete induction by simple enumeration of positive instances." Complete enumeration is impossible, for from the very nature of quantity the number of cases is infinite. Thus, the great mathematician Fermat believed that $2^{2x}+1$ was always a prime number, whatever value x might have. He could not, however, prove that it *must* be so. Case after case was tested until $x = 16$, and the result amounted to 4294967297. This large number was found not to be a prime; it is divisible by 641. A rule based only on observation, in the absence of demonstration, cannot be asserted to be always true.

Now, with these instances before us, what can we say as to the *conditions of proof* for a general law, from an individual case? The proof depends on two conditions. (1) We must be sure that we have really grasped something essential or fundamental in the particular case, and are not arguing from changeable or accidental qualities; (2) we must be sure that any new case exactly resembles the old in those characteristics on which the proof depended. In mathematics both conditions are absolutely secured, for the mathematician makes his own definitions of what is essential, and argues from them. But in Nature the essential conditions have to be discovered and proved. This is the great difference between mathematical and physical induction, and all the difficulties of physical induction result from it. There are always the two possibilities of error. The original case may not have been examined with sufficient thoroughness; or, in applying the general rule which we derive from it, we may be mistaken in thinking that the new case really resembles the old. If the result of induction is "uncertain," it is only for these reasons. Jevons and other writers constantly speak of the results of induction as only "probable," as containing an element of "uncertainty." This is true, if we are careful to put the uncertainty in the right place. If there is any uncertainty it arises *not because we go beyond the present or immediate experience of our senses in stating a law*, but because experience is subject to the double misinterpretation of which we have spoken.

We may define Induction, then, as the **legitimate inference of general laws from individual cases.** By this means—as Jevons observes in the suggestive sentences quoted above (§ 1)—we detect the *presence* or *operation* of the general law *in* the particular cases.

Fowler gives two definitions of Induction. (*a*) "The legitimate inference of the general from the particular."[1] This agrees with what we have said above; for the word "particular" is not here used in the rigid narrow sense in which it is objectionable (the sense in which it means something that points to nothing beyond itself, unconnected with other things: such a thing could never be *known*, simply because that would destroy its isolation). (*b*) "The legitimate inference of the unknown from the known" (of the future from the past).[2] This definition is based on Mill, and is very misleading. If the new or "unknown" or "future" cases are strictly *unknown*, we could not apply to them the results of our investigation of the "present" or "known" cases. We can only do this so far as we *know* the constitution of the new cases in this respect at least—they must contain the same conditions as the one which was first examined. We can hardly speak of passing from the "known" to the "unknown," when we know that there must be a complete identity between them in certain respects. It would be less incorrect to speak of passing "from the *comparatively known* to the *comparatively unknown*"—which is probably Fowler's real meaning—but this is much better expressed by saying that what we reach is a *general proposition*.

Bain—also following Mill—talks about "the inductive hazard," "the leap to the future";[3] but he is putting the difficulty in the wrong place. He also speaks as if the mere lapse of time could have an effect on the action of a cause. Time might produce other causes which would counteract the first; but if we have ascertained the presence and action of the same cause in a subsequent instance, the passage of time alone makes no difference to the certainty of the effect.

§ 6. How shall we define a Cause? From the standpoint of Inductive Logic, which aims at giving a general account of scientific method, this question means, What is the best definition of Cause from the scientific point of view? For simplicity's sake, and as leading on to

[1] *Inductive Logic*, p. 10. [2] Ibid., p. 9.
[3] *Inductive Logic*, Bk III ch. i. § 1.

a more detailed discussion, we shall first quote from Jevons. "By an *antecedent* we mean any thing, condition, or circumstance which exists before or, it may be, at the same time with an event or phenomenon. By a *consequent* we mean any thing, circumstance, event, or phenomenon, which is different from any of the antecedents and follows after their conjunction or putting together. It does not follow that an antecedent is a *cause*, for the effect might have happened without it. Thus the sun's light may be an antecedent to the burning of a house, but not the cause, because the house would burn equally well in the night; but *a necessary or indispensable antecedent is identical with a cause*, being that without which the event would not take place. . . . There are usually many different things, conditions, or circumstances necessary to the production of an effect, and all of them must be considered causes or necessary parts of the cause. Thus the cause of the loud explosion in a gun is not simply the pulling of the trigger, which is only the last apparent cause or *occasion* of the explosion; the qualities of the powder, the proper form of the barrel, the proper arrangement of the percussion-cap and powder, the existence of the surrounding atmosphere, are among the circumstances necessary to the loud report of a gun: any of them being absent it would not have occurred." What Jevons here calls the "occasion" of the effect, we shall call the *immediate cause;* the *other* circumstances present and necessary for the effect we shall call *causal conditions* or, briefly, *conditions.*

The account which Jevons gives of the meaning of Cause in science is based on Mill. Let us consider Mill's account (*Logic*, Bk. III. ch. v.)

(*a*) He defines the Cause first as the invariable ante-

cedent: "Invariability of succession is found by observation to obtain between every fact in Nature and some other fact which has preceded it; . . . the invariable antecedent is termed the cause, the invariable consequent the effect. And the universality of the law of Causation consists in this, that every consequent is connected in this manner with some particular antecedent or set of antecedents" (III. v. § 2). (*b*) He then points out that the "invariable antecedent" is not usually *one* particular circumstance, but a group of conditions, as when a person eats of a particular dish and dies in consequence—"that is, would not have died if he had not eaten of it"; not only the food, but the taking of it in combination with a particular constitution, state of health, climate, &c.,—these constitute the group of conditions which is the "invariable antecedent" (III. v. § 3). Among these conditions we choose one, which we call, in a special way, the Cause; and as far as popular thinking (apart from science) is concerned, this choice is somewhat arbitrary. Afterwards he states more exactly what this reference to a whole group of conditions implies: "The Cause is the sum-total[1] of the conditions, positive and negative, taken together; the whole of the contingencies of every description, which being realised, the consequent invariably follows. The negative conditions . . . may be all summed up under one head, namely, the absence of preventing or

[1] Some writers have engaged in verbal criticism of the expression "sum-total" and complained that it implies a mere mechanical addition of the component conditions, and that this idea is altogether inadequate to the complexity of the combinations and interactions of natural events. But no such suggestion is intended in Mill's words, which convey no more indication of the *kind* of combination of the conditions in question than would be indicated by the use of such a word as "totality."

counteracting causes" (III. v. § 5). (*c*) Finally, he shows that *invariable* sequence is not an adequate definition, unless the sequence is also regarded as *unconditional.* "This," says Mill, "is what writers mean when they say that the notion of Cause involves the idea of necessity; that which is necessary, that which *must* be, means that which will be, whatever supposition we may make with regard to other things" (III. v. § 6)

We will comment on each of the steps in Mill's development of the idea of Cause. By laying stress, in his first statement (*a*), on the *antecedence*, or priority in time, of the Cause, Mill raises the question, What is the relation of *time sequence* to causation? It is only when we have two distinct events that we can have a relation of *before* and *after.* Can cause and effect be regarded as two distinct events? Some cases of causation may seem to lend support to such a conception—*e.g.*, we have (1) the cause, the introduction of microbes into a living body; (2) the effect, the appearance of a certain disease some hours afterwards. But this is going too far: even popular thought never regards the effect as *separated* from the cause; it regards them only as *distinct* in time. The apparent separation in the above case arises from the fact that we have not considered the *immediate* effect, but have waited until it has reached an advanced stage of development and have called *this* the effect. Cause and effect are divided simply by a mathematical line—a line destitute of breadth—which is thrown by our thought across the current of events; on one side we have the cause, on the other the effect. There is no pause in reality; the whole process is continuous; the immediate cause comes into full action only at the very moment when the effect begins to be produced. The point to be borne in mind

is the **continuity of cause and effect.** But the relation of antecedent and consequent, of two distinct events, one following the other, is not the essential aspect of the causal relation. It is, of course, true that the entrance of microbes into a human body is "followed" by a certain disease; but this is no essential aspect of the case. The essential matter is that as soon as the microbes effect a lodgment in the human body they begin to produce injurious substances. In Chemistry, again, the union of Oxygen and Hydrogen in the proportion by weight of eight to one is not an event separate from the formation of water, although the whole process is continuous. Whenever this is understood, there is no objection to speaking of cause and effect in terms implying *duration in time*, for duration is an essential aspect of continuity.

* We have not committed ourselves to the statements made by some critics of Mill, that the relation of cause and effect has *no essential reference* to time (that its temporal aspect is merely accidental), and that cause and effect are *identical.* A certain plausibility is lent to such statements by reference to such an example as that of the contact of a drop of ink with the paper causing a blot; of which it may be said that the contact of the ink and paper simply *is* the blot. This is merely to play with the word "blot." What we have insisted on is the continuity of all natural events, so that cause and effect are not two separate things.

* The statement that cause and effect are "identical"—whatever may be thought of it as a metaphysical principle—becomes an extravagant paradox if taken seriously and applied to any particular case of *causation determined by scientific experiment.* The object of Inductive Logic is to render the method of such cases intelligible, and it is by reference to them that we must form our definition of Cause.

The truth of Mill's second statement (*b*) is obvious:

the causal antecedent of an event is usually a complex group. This is also pointed out by Jevons, in the passage quoted above. Further, it is quite true that in popular thought, or in everyday life, we take as the Cause an antecedent selected arbitrarily or for some special practical purpose.

We do not go beyond the preceding circumstances out of which the immediate cause arose—*e.g.*, in the case of one shot through the heart, we take as cause the action of the person who fired the bullet. Such antecedent circumstances, which are striking and important from some practical point of view, are the "causes" with which we concern ourselves. Sometimes, what is practically the most important is scientifically the least important: it may be of great importance to know what circumstances will produce an event without knowing *how* they produce it. For instance, it may be of importance to clear the premises of rats; traps, strychnine, phosphorus, and terriers are various "causes" between which we must choose: but we do not as a rule hold *post-mortems* on dead rats.

But when in Science we select from the concurrent conditions one, which we call the Immediate Cause, the selection is in no sense arbitrary. It is made according to strict methods which we shall endeavour to analyse in the sequel (ch. IX. §§ 3 to 7).

* Undoubtedly, in Science, as in common life, the cause is sought and investigated "for a purpose"; but the purpose of scientific, unlike that of popular thought, is not special and limited. The scientific purpose is that of discovering how the effect may *always* and under *any* circumstances be produced; and this aim can be fulfilled only by discovery of the *law* of its occurrence.

Mill's third statement raises a very difficult question, which, however, is of more importance for general Philosophy than for Inductive Logic: What,

precisely, and in the last resort, can we mean by saying that one event is *necessarily connected* with another? We shall form a sufficient idea of the meaning of "necessary connection"—that is, an idea sufficient for the purposes of Inductive Logic—if we compare Mill's third statement (*c*) with his first (*a*). His first statement means only that the cause is that circumstance (or group of circumstances: this is always understood) in the presence of which the event always takes place; M is the cause of P when M is always followed by P. This does not exclude the possibility that there are other and different circumstances which may also be uniformly capable of bringing the same event to pass. There may be a circumstance N which is always followed by P, another circumstance R which is always followed by P, and so on; so that P may be due at one time to M, at another to N, at another to R, and so on. On this assumption, we can argue *from cause to effect*, saying, "if M, then P," or "if N, then P," &c.; but we cannot argue *from effect to cause*, and say, "if P, then M." For popular thinking, this is sufficient. But Mill's third statement (*c*) means that the circumstance on which the event always follows is not to be taken as the cause (in the scientific sense) *until we know* that it is necessary or indispensable for the production of the effect: in other words, the cause is *the circumstance in whose presence the event always takes place, and in whose absence it never takes place.* In this case we may argue *not only* from cause to effect (if M, then P) *but also* from effect to cause (if P, then M).

Mill's first statement is consistent with the doctrine which he calls "Plurality of Causes" (*Logic*, III. x. § 1); his third statement is not consistent with it. The doctrine of "Plur-

ality of Causes" does *not* mean that the cause of an event may be (as we have pointed out above, p. 275) a complex group of many conditions; it means, what we have explained in the preceding paragraph, that the same effect may be due sometimes to one cause, sometimes to another. How far is this true? It is only true as long as the "cause" is understood in the popular way. *The plurality disappears before any exact scientific investigation.* We may illustrate this by some of Mill's instances. "There are many causes of motion,"—visible impact; heat; electrical and magnetic action; gravitation. Yet the doctrine of the Conservation of Energy, which rules modern Physics, means practically that all motion in matter is produced in the same way, namely, by other motions in matter. "There are many causes of death." But life is a complex process consisting of a multitude of co-operating processes, of which some are directly essential. If any one of these essential processes is interfered with, life ceases; and the interference can only be of one kind. Hence there are many causes of death only because there are many kinds of death; "death" is a fact as complex as "life." Again: "A disease may have many different causes." But the youngest and most successful of recent scientific studies—sometimes called *Bacteriology*—has proved beyond doubt that each kind of disease—among those most inimical to life—is produced by the entrance into the human body of one particular kind of the extremely minute living organisms known as "microbes." Thus, when the apparent "many causes" of the disease are analysed, there is found to be something fundamental, common to them all—namely, the presence of these minute forms of life. Each disease has its characteristic "microbe."

The doctrine of plurality is only a practical working caution. In the absence of scientific knowledge of the immediate cause, we have to bear in mind that different combinations of circumstances may bring about the same event Practically we have to begin the investigation by examining those different combinations of circumstances in which the event is produced—considering them, at first, as so many different "causes." They are not the immediate cause;

but it is *operative in them.* As a practical caution, "plurality of causes" is equivalent to the rule which forbids arguing from the negation of the antecedent or the affirmation of the consequent.

Fowler adopts Mill's first statement as a sufficient definition of causation for scientific purposes; but he is forced to acknowledge its insufficiency when he comes to speak of what is admittedly the most effective of the Inductive Methods,—the "Joint Method"; for in so far as we can control our subject-matter sufficiently to apply this method, it establishes the Cause as that with which the Effect takes place, and without which it does not, "so that the two are (unless counteracting circumstances intervene) always present together and always absent together" (see Fowler's *Inductive Logic*, pp. 4, 127, 163).

We are now in a position to make our statement of the meaning of a "Law of Nature" more definite. We said (§ 1) that it meant "a constant connection between a cause M and an effect P." We are now able to say the connection must be of such a kind that whenever the cause M is present its effect P is present, and whenever M is absent P is absent. When we have established a connection of this kind between two events, we have established a **Law of Nature.** Strictly speaking, we cannot stop at any *limited* totality or combination of circumstances and say "these, and nothing else, constitute the cause"; for all events are connected together, —when a stone is dropped, there is a sense in which it has an effect through all time and all space. In reality, any event is the effect of all causes operative throughout the universe, and the real "sum-total of conditions," which is the cause, is nothing less than the universe itself. But all these further and more remote conditions are usually taken for granted,—in fact, necessarily so. What we want to know is the **immediate cause.** The scientific investigator seeks to isolate the

event in various ways, and examine it under conditions which if possible he arranges for himself, so as to discover *among those conditions* some definite circumstance **with which the event will occur, and without which it will not occur.** This is what we mean by the "immediate cause."

The immediate cause is but one out of a group of conditions necessary for the effect; but sometimes it is more convenient to regard the immediate cause itself as a group of facts acting together rather than as a single fact.

For example, in the formation of water by the passage of an electric spark through a vessel containing two parts (by volume) of Hydrogen and one of Oxygen, the immediate cause is the one fact of the action of the electric energy, whatever it may be. On the other hand, in the modification of a species of the animal kingdom in the course of ages, it is more convenient to consider the possibility of several different immediate causes—*e.g.*, Natural Selection; the direct action of the environment; the inheritance of characteristics acquired by the creatures' own activities, &c. Again, in the case of a person's death through being shot in the heart, the immediate cause is the piercing of the heart by the bullet, which we may regard as a single fact. This stops the heart's action; and the heart's action is one of the processes necessary in order that the complex process of physical life should continue.

We have thus, among the various ambiguities of the word Cause, distinguished the following different meanings: (1) the cause is a group of antecedents which always produces the given effect; this admits "plurality of causes," and includes (2) the popular view of the cause as some "invariable antecedent" which is striking or important; (3) the cause is the "totality of conditions" necessary for the production of the effect, as for instance, the whole circumstances, instruments, and arrangements of an *experiment;* (4) among these conditions, the cause

is that one (simple or complex) on the introduction of which the effect occurs, and without which it does not occur,—the "immediate cause"; (5) the cause is the "*totality* of conditions" in the *full* sense of the word; which means, in the last resort, the universe itself. Of these meanings, (3) and (4) are of predominant importance in Inductive Logic and in Science. Many traditional disputes concerning Causation are settled by distinguishing these various meanings: *e.g.*, the validity of the statement *cessat causa cessat et effectus*.

§ 7. There is a fundamental assumption involved in induction. There is a principle which must be granted if scientific investigation is to be possible,—a necessary presupposition of scientific method. We must grant beforehand that **every event has a cause.** This principle or *postulate* is called the **Law of Universal Causation.** Fowler states it thus: "No change can take place without being preceded or accompanied by other circumstances, which if we were fully acquainted with them would fully account for the change" (*Inductive Logic*, pp. 4, 5). This principle may be shown to be implied in all thinking.[1] Even children, and the lower races of men, though they do not think *of* it, think *according to* it. If the savage were content to leave any event unexplained, he would not imagine that all events are controlled by spirits, malevolent or benevolent. It is in fact **impossible to think of an event without referring it to a cause,** known or unknown. Even if we had a state of affairs where the past gave scarcely any assurance as to the future, our way of conceiving it would not be contrary to the principle of the Universality of Causation. We should think that some capricious

[1] In other words, it is an Axiom as defined above (ch. II. 14, p. 45).

power had added itself to the conditions, and was turning them now this way and now that.

By the side of the Law of Universal Causation Fowler places, as another fundamental presupposition of induction, the law that **the same cause must have the same effect**; when the same conditions are fulfilled the same result will follow (*Inductive Logic*, pp. 5, 6). This is sometimes referred to as the principle of the Uniformity of Nature; it is better described as the "Unity of Nature," or, less abstractly, as the "Uniformity of Causation." The student will see on reflection that this principle is *included in* the principle of Universal Causation; for by Cause is at least meant a condition on which the effect *always* follows (§ 6). If it sometimes followed and sometimes did not, there would be no object in trying to discover it; you would simply not have a cause at all.

Jevons speaks of the "Uniformity of Nature" as liable to exceptions; asking, for example, whether we can be "certain that the sun will rise again to-morrow morning, as it has risen for many thousand years, and probably for some hundred million years." To answer this question we must make an important distinction between two meanings of the Uniformity of Nature: (1) the Uniformity of Causation, (2) the maintenance of the present order of things in the universe. Experience shows us that there are general "laws"—*i.e.*, kinds of orderly succession in the outward course of events: such as appear in the succession of day and night, summer and winter, seed-time and harvest, life and death. The regular succession of events in a thousand different ways accustoms us, from force of habit, to expect things to happen in a regular order; and we find that the expectation is fulfilled. This

constitutes an overwhelming presumption in favour of the maintenance of the present arrangements in Nature; but it does not show that deviations from this order are impossible. An expectation, bred by experience and custom, that events will occur in a certain way is not the same as a knowledge that they must so occur; and this knowledge is not in our possession. We have no grounds for affirming that the sun *must* rise to-morrow morning; there is only an overwhelming presumption in favour of the expectation that it will. But the principle of Uniform Causation tells us nothing as to the permanence of the present "choir of heaven and furniture of earth." It only says that the same cause will have the same effect; and to this there are no exceptions. The same cause may conceivably never act again; but this does not affect the truth of the principle that *if* it did it would have the same effect.

Mill expresses the principle of Uniformity by saying that "the unknown will be similar to the known, and the future resemble the past" (*Logic*, III. iii. § 2). This is not the scientific principle of Uniformity; it is the practical presumption of which we have spoken, and there is no intellectual necessity about it. "The future," says Green, "might be exceedingly unlike the past (in the ordinary sense of the words), without any violation of the principle of inductive reasoning, rightly understood. If the 'likeness' means that the experiences of sensitive beings in the future will be like what they have been in the past, there is reason to think otherwise. Present experience of this sort is very different from what it was in the time of the ichthyosaurus."[1] And even at present experience has an

[1] Green, *Lectures on the Logic of J. S. Mill* (Philosophical Works, vol. ii. p. 282).

aspect of chaos as well as one of regularity. There are indeed in the infinite variety of Nature many ordinary events; but others appear uncommon, perplexing, or even contradictory to the general run of things. This is fully admitted by Mill: "The course of Nature is not only uniform; it is also infinitely various. Some phenomena are always seen to recur in the very same conditions in which we met with them at first; others seem altogether capricious; while some, which we had been accustomed to regard as bound down exclusively to a particular set of combinations, we unexpectedly find detached from some of the elements with which we had hitherto found them conjoined, and united to others of quite a contrary description" (*Logic*, III. iii. 2).

Mill, Bain, and Fowler regard the belief in Uniformity as based on induction from uninterrupted experience. This is only true of the belief that the present order of Nature will continue in the future. This belief is a late development. In early ages human beings believed that the course of Nature was always being capriciously interrupted. But the belief in the Universality and Uniformity of Causation is not a late development; it can be traced, as we have seen, even in the speculations of savages. And there is no evidence that it can be manufactured by experience. It seems essentially impossible that experience, with the irregularity that actually exists in it, can *of itself* have produced a belief that every event has a cause, and that the same cause will always produce the same event. And if it were so,—if the laws of Causation are wholly based on experience as given to our senses,—then this means that the whole of inductive reasoning is based on what Mill and all others admit to be the weakest kind of induction, "simple enumeration of merely positive instances."

§ 8. Mill and his followers[1] have laid great emphasis on a distinction between *Uniformities of Coexistence* and *Uniformities of Causation.* Hitherto we have spoken as if the latter type of Uniformity were alone considered in Inductive Logic; but this is not the case. By cases of Uniform Coexistence, as distinguished from cases of Causation, we mean cases where (*a*) two distinguishable attributes (we speak of *two* only for simplicity's sake) are always found conjoined throughout some particular kind of being, and where (*b*) we cannot find that either of these attributes is the cause of the other, *i.e.*, that either of them is indispensable for the existence of the other. This definition does not imply that the attributes in question are not causally connected: though neither of them is the cause of the other, they may be causally connected *through something else*,—more precisely, they may be *joint-effects* of previous causes.

It is generally admitted that very many cases of Uniform Coexistence can be so explained. Thus, Bain observes (*Inductive Logic*, p. 11): "The numerous coexistences of Order in Place, or the distribution and arrangements of material objects throughout the Universe, are all the results of causation, starting from some prior arrangements. The distribution of sea and land, the stratification of the earth's crust, the existence of an atmosphere, the distribution of the materials of the globe generally,—are the result of natural agencies or forces, operating upon prior arrangements."

Mill and Bain, however, consider that there really are cases of uniform coexistence where the connection can never be causally explained. Instances are alleged to be found both in the inanimate world and among the varieties of plant and animal life. (*a*) The several

[1] Mill, *Logic*, Bk. III. ch. xxii.; Bain, *Inductive Logic*, Bk. III. ch. iii.; more recent treatment in Venn, *Empirical Logic*, ch. iii.

kinds of inorganic matter known in chemistry as "elements," and the many classes of minerals as set forth in Mineralogy, are all distinguished from one another by different groups of ascertained qualities, and the qualities in each group uniformly accompany one another: thus, the *diamond* has always a characteristic colour, brightness, and form; is always combustible, and when burnt produces "carbonic acid" gas; cannot be cut by any other substance; and so forth. We can also distinguish certain coexisting qualities which pervade all matter of every kind: such are *inertia* (resistance to motion and force when moved) and *gravitation* (mutual attraction), which together characterise all matter. (*b*) The sciences which investigate the structure and characteristic properties of the different kinds of living beings are sciences dealing only with coexistences. Thus, anatomical laws are laws affirming coexistences between the different parts of living bodies. Anatomy is "a vast congeries of such propositions" (Bain). The same is of course true with reference to the vegetable kingdom. Along with these coexistent facts of structure, there are found facts of *colour* and other characteristics not "anatomical" in the strict sense. It is on such "uniformities of coexistence" that *classification* is based. All classification implies a previous observation of the fact that certain attributes always accompany one another (for examples see p. 143 above).

We have mentioned merely the chief types of coexistence on which Mill and his followers laid stress. To attempt to make a complete list would be useless and misleading; for, though it may be true that in every such case we have a group of coexistent qualities no one of which is the cause of any of the others, yet

science as it progresses is continually explaining them as joint-effects: more especially since the general acceptance of the doctrine of the evolution of species. Mill greatly exaggerates the range of the unexplained differences, and the difficulty of causally accounting for them.

* This exaggeration is due to Mill's doctrine of "real kinds" (criticised above, ch. V. pp. 154-6). He conceived these "kinds" to exist both in the organic and in the inorganic worlds. It is remarkable that one of the most evident tendencies in the science of the twentieth century is the extension to the inanimate world of the idea of evolution and survival. In the universal process of continuous change which we call Nature, a case of invariable coexistence is a case of the survival and perpetuation of the given combination of qualities in a comparatively permanent form, owing to its stability. This hypothesis is applicable both to the molar and molecular forms of matter,—to the mechanical structure and motions both of visible bodies and of their ultimate particles. Remembering that a "quality" or "attribute" is always a *mode of action*, we find this conception of comparatively permanent "configurations or modes of motion of matter" to be in agreement with the conception of "Essence" outlined above (p. 156).

* What we have said of the comparatively persistent or permanent forms of inorganic matter is *a fortiori* applicable to the case of those great classifications which correspond to the chief differences contained in the world of life. If these coexistences are resolved into cases of causation, it is by conceiving every such group as a group of joint-effects in evolution. The ancient doctrine of fixity of species can no longer be maintained (p. 155); but though the species are not conceived as absolutely and inviolably stable, they are conceived as having a relative or comparative stability which has been able to resist destruction.

J. S. Mill is a writer whose greatest mistakes are at the worst only misstatements of real truths which he

sees and wishes to enforce. The truth underlying his doctrine of the inexplicability of coexistences is this: the very considerations which compel us to recognise that uniformities of coexistence are capable of causal explanation, "compel us also to recognise that there must be one class of coexistences which cannot depend on causation; the coexistences between the ultimate properties of things—those properties which are the causes of all phenomena, but are not themselves caused by any phenomenon, and a cause for which could only be sought by ascending to the origin of all things" (*Logic*, Bk. III. ch. xxii. § 2). These words remain true although very many of the "coexistences" which Mill regarded as "ultimate" have been or will be shown to be only derivative.

EXERCISE XVI.

Questions on Chapter VIII.

(i) *Elementary.*

1. What is the import of the kind of proposition represented by the Major Premise of a hypothetical syllogism?
2. Explain and illustrate the terms—Antecedent, Consequent, Cause, Occasion, Condition, Law of Nature.
3. "The Aristotelian 'Inductive Syllogism' is really *deductive*." Explain this statement.
4. In what different senses has the word Analogy been used? What is meant by Reasoning from Analogy? Give an example of good and one of bad analogical reasoning. [E.]
5. Distinguish between the so-called "Perfect" and the so-called "Imperfect" Induction. What is the use of the former? What is meant by "Induction by simple enumeration"? Discuss its value.
6. (*a*) From what circumstance arises the certainty and generality of reasoning in Geometry? (*b*) Find other instances of certain and general reasoning regarding the

properties of numbers. (*c*) Why is a single instance sometimes sufficient to warrant a universal conclusion, while in other cases the greatest possible number of instances, without any exception, is not sufficient? (see pp. 265-9 and cp. p. 230). [Jevons.]

7. What do you understand by the term "Induction"? In what sense exactly, and why, are the results of Induction only "probable"? State clearly the aim of "Inductive Logic."

8. (*a*) Define precisely what Science understands by the Cause of an event, and illustrate your answer by a typical example of Causation as determined by experiment. (*b*) What is the relation of the scientific conception of Cause to the conception of Cause as the "sum-total of conditions" on which the event depends? (*c*) Can you account for the divergence between the scientific and popular views as to the Cause? [L.]

9. Indicate the chief points brought out by Mill in his treatment of the meaning of Causation. Is his treatment "inconsistent"?

10. In what sense may it be affirmed, and in what other sense may it be denied, that "a phenomenon can only have one cause"? [L.]

11. Examine the relation of Causation (*a*) to Experience, (*b*) to the Uniformity of Nature. [G.] *Or*—

In what relation does the antecedence and sequence of phenomena stand to the principle of Causation, and to the Uniformity of Nature? [L.]

(ii) *More advanced.*

12. "Induction is the discovery of Major Premises." Comment on this statement, and illustrate by reference to the essential structure of the Categorical and of the Hypothetical syllogism.

13. Explain, illustrate, and estimate the value of Analogical Reasoning. [G.] *Or*—

"Logically considered, Analogy is always a weak argument." Examine this carefully. *Or*—

Define Analogy, and consider carefully the nature of

analogical reasoning. Is the conclusion that the lower animals suffer pain an argument from Analogy? [L.]

14. Illustrate Induction as used in Arithmetic and Geometry. Is it governed by the same principles as physical Induction? [L.]

15. Account for the preponderance of affirmations of *proprium* in Mathematics as compared with physical science. How far can non-mathematical instances of such predication be found? [L.]

16. "Induction is legitimate inference from the known to the unknown." "The inductive hazard—the leap to the future." Examine the view of Induction implied in these statements. [St A.]

17. What variety of meaning has been assigned to the word Cause? [E.]

18. Carefully explain the term "Uniformities of Co-existence." How far can such uniformities be distinguished from Uniformities of Causation?

19. "Every person's consciousness assures him that he does not expect uniformity in the course of events." "Uniformity pervades all Nature." Discuss these statements. [L.]

* 20. Explain concisely the Aristotelian doctrine (*a*) of the Enthymeme, (*b*) of the Paradeigma. How are these arguments treated in modern Logic? What is their relation to the syllogism?

* 21. Τὸ μὲν γὰρ αἴτιον τὸ μέσον (Aristotle). Comment on this.

* 22. What did Aristotle understand by the Inductive Syllogism? What was his view of Induction *per enumerationem simplicem?*

* 23. Discuss the logical character of what has been called *circumstantial evidence.* Is it ever conclusive? How would it be expressed in syllogistic form?

* 24. Can a "materially" sound argument always be expressed in a "formally" correct syllogism? If not, why not? Express each of the following in syllogistic form: (*a*) A typical argument from Analogy; (*b*) A statement based on perception, such as "this is the footprint of a horse"; (*c*) A generalisation suggested as possibly true by the observation of a number of instances. [L.]

✻ 25. Distinguish between *sign* (or *symptom*) and *cause* (or *causal condition*), giving examples. [L.]

✻ 26. Enumerate and carefully distinguish the presuppositions involved in Inductive Inference, and estimate the degree of certainty which this kind of argument yields. [E.] *Or*—

Consider the *necessity* attaching to the conclusions of mathematical science and natural science respectively. [L.]

✻ 27. "The cause must be contiguous to the effect"; "The cause must precede the effect"; "*Cessante causa, cessat et effectus.*" Discuss these statements. [St A.]

✻ 28. "The conception of Cause is ultimately identical with that of the Reason." Carefully examine this.

CHAPTER IX.

THE THEORY OF INDUCTION OR SCIENTIFIC METHOD.

§ 1. EXPERIENCE presents to us a chaos of innumerable events, together and in succession. In this chaos, science has first to ascertain the facts; then, to ascertain "what follows what"—*i.e.*, what facts are invariably connected together; and then, to account for these regular connections, to show how or why they are so connected.

The first step in Science is to gain possession of the facts: this is impossible without "observation." **Observation** is a mental as well as a physical activity;[1] for in order to observe, not only must the attention take a particular direction, but we must be more or less conscious of what we are looking for. In other words, observation, like ordinary perception, is **selective.** A man's experience consists, indeed, only of what he agrees to be interested in. Millions of events that pass before a man never enter into his experience at all; they have no interest for him, and hence he does not notice them. It is a well-founded doctrine of modern psychology that without selective interest, experience would be an utter chaos. "Interest alone gives accent and emphasis, light and shade, background and foreground,—intelligible perspective, in a word. Our own interest lays its weighty index-finger on particular items of experience, and may

[1] To overlook this was Bacon's great mistake.

emphasise them so as to give to the least frequent associations far more power to shape our thoughts than the most frequent ever possess." And in science the interest springs from previous knowledge; the simplest fact, when noticed by a well-prepared mind, may become an observation of immense importance. The too-familiar anecdotes of James Watt's observation of the force of steam in lifting the kettle-lid, and Newton's observation of the falling apple, will illustrate our point. The true observer brings to his observation more than he finds in it, and yet knows how to abandon one by one his most cherished preconceptions if the facts will not support them.

We must carefully distinguish between **observation** and **experiment**. In simple observation, the facts observed are due to Nature; in experiment, they are arranged by ourselves. Jevons has excellently described the difference between the two:—

"To **observe** is merely to notice events and changes which are produced in the ordinary course of nature, without being able, or at least attempting, to control or vary those changes. Thus the early astronomers observed the motions of the sun, moon, and planets among the fixed stars, and gradually detected many of the laws or periodical returns of those bodies. Thus it is that the meteorologist observes the ever-changing weather, and notes the height of the barometer, the temperature and moistness of the air, the direction and force of the wind, the height and character of the clouds, without being in the least able to govern any of these facts. The geologist, again, is generally a simple observer when he investigates the nature and position of rocks. The zoologist, the botanist, and the mineralogist usually employ mere observation when they examine animals,

plants, and minerals, as they are met with in their natural condition.

"In **experiment**, on the contrary, we vary at our will the combinations of things and circumstances, and then observe the result. It is thus that the chemist discovers the composition of water by using an electric current to separate its two constituents, oxygen and hydrogen. The mineralogist may employ experiment when he melts two or three substances together to ascertain how a particular mineral may have been produced. Even the botanist and zoologist are not confined to passive observation; for by removing animals or plants to different climates and different soils, and by what is called domestication, they may try how far the natural forms and species are capable of alteration."

We must remember that it is impossible to draw a line of complete separation between the two processes, so as to say "just here the one ends and the other begins." They have this much *in common:* observation (as we have pointed out) is itself not only an active but a selective process, and so is experiment. And if we look for a transition-process leading from observation to experiment, and sharing the characteristics of both, we can find it in the use made of **instruments of observation** such as the telescope and the microscope. Still, *experiment* in the full sense of the word goes much further, and deliberately *arranges the facts*, in order to obtain knowledge regarding them. We can clearly distinguish the sciences according to the extent to which they depend on experiment in this sense. Without experiment Mechanics, Physics, and Chemistry could scarcely exist; and these are fundamental sciences in an advanced state. In Physiology experiment naturally plays a much smaller part, for, if

made at all, it has to be made on the organs of the living body. In the sciences of description and classification,—Botany, Zoology, Mineralogy,—the range of experiment is still more restricted; while in Astronomy, Geology, Meteorology, we may say that experiment, as far as we are concerned, is impossible. We say "as far as we are concerned," because Nature sometimes produces phenomena of so remarkable a character that she may be said to be making an experiment herself—as in an "eclipse of the sun."

§ 2. The foregoing discussion of experiment introduces us to the second step in Science, which is to ascertain the Cause of the fact. This is usually impossible except by experimental investigation. We have to look for the Cause among those events which invariably accompany the one under investigation; but the *causal conditions* and the *immediate cause* (see ch. VIII. § 6) are not given,—they have to be discovered. Mere *sequence*, as Minto puts it, does not prove *consequence;* to suppose so would be to commit the fallacy of *post hoc ergo propter hoc.* The question is, in the ever-changing succession of events which Nature presents, what events are *causally* connected, in distinction from those which are *casually* conjoined? When do observations of *post hoc* warrant a conclusion *propter hoc?* This is decided by **varying** as much as possible the circumstances of the phenomenon[1] under investigation, so as to **eliminate what is unessential** or casual in them.

In an elementary work it is best to base our account of the methods of causal investigation on that of J. S.

[1] The word "phenomenon" (φαινόμενον, that which appears) is used synonymously with "fact" and "event" to signify anything that can be obseived by our senses.

Mill. Mill elaborated five rules for such investigation, stating five distinct processes which he called respectively the Method of Agreement, the Method of Difference, the Joint Method of Agreement and Difference, the Method of Concomitant Variations, and the Method of Residues. For these methods Mill makes high claims, which in other parts of his work he is obliged to retract. We shall see that they are not independent of one another, and not equally fundamental; and when we thus understand their relations to each other, and the work that they will do, we shall see that they are sound in principle, and are actually employed in scientific investigations. In fact, Mill's account of them is based on Herschel's description of the methods of Induction in his *Discourse on the Study of Natural Philosophy*.

Mill explains that in all the methods there are only two principles involved. "The simplest and most obvious modes of singling out from among the circumstances which precede or follow a phenomenon, those with which it is really connected by an invariable law, are two in number. One is by comparing together different instances in which the phenomenon occurs. The other is by comparing together instances in which the phenomenon does occur with instances (in other respects similar) in which it does not. These two methods may be respectively denominated the Method of Agreement and the Method of Difference" (*Logic*, III. viii. § 1). These are the two primary methods; the "Joint Method," as described by Mill, is in the main a double application of the method of "Agreement"; the method of "Concomitant Variations" is a quantitative application of either of the two primary methods; and the method of "Residues," as Mill conceives it, is a variety of the method of "Difference."

§ 3. Mill states the Method of Agreement—better named the method of Single Agreement—as follows:

"If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause or effect of the given phenomenon." We may express the rule more simply by saying that facts which may be eliminated (may be present or absent) without affecting the event are not causally connected with it. It is then probable that the remaining fact, which is present whenever the event occurs, is causally connected with it. Thus, let A be an event whose cause is sought for. We observe the circumstances in which A occurs, in order to find what other facts are invariably present with it. Mill indicates distinct facts by different letters. Suppose, then, that we are able to analyse the various instances of A as follows: first instance, A b c d; second, A c f e; third, A g h c; and so on. Then *c* is the only other fact in which the instances of A agree; hence there is a probability that A and *c* are causally connected.

Mill's statement of this method ignores a preliminary difficulty. Nature not only fails to show us, at a glance, what events are really connected with a given one; she does not give us events marked off into distinct and separate phenomena. To denote the facts learnt through observation by letters *a*, *b*, *c*, &c., is to take for granted that the hardest part of the work of observation is already done. When phenomena have been analysed into their elements in this manner, it is a very simple affair to ascertain the common facts in the different instances. We must not forget that the Method of Single Agreement starts with prepared material, taking for granted the very thing that is most difficult to discern. Further, it is very difficult to be sure that the instances have *only one* material circumstance in common. Practically, the

force of the method depends on the number and variety of the instances; the more numerous and varied they are, the greater is the probability that A and *c* are causally connected.

The "plurality of causes" is also a serious obstacle to this Method. We have seen (ch. VIII. § 6) that the plurality will probably disappear before a more searching analysis; but still, there is a popular sense in which it is true—for instance, that heat, light, and motion may be caused in different ways. But *until* scientific investigation has reduced the various "causes" to a single *immediate cause*, the Method of Single Agreement breaks down. If heat, for instance, is produced by friction, combustion, electricity, all these real causes would be eliminated by this method, for they are points in which the different instances of heat differ.

Hence the real worth of the method is seen when we regard it not as a proof of a case of causation, but as a stage in scientific inquiry. It "points to the probability of some law of causation which, if discovered, would explain more satisfactorily the facts disclosed to our observation," and "paves the way for other and more effective methods."[1] Its real significance appears when we state Mill's canon thus: **When observation shows that two events accompany one another (either simultaneously or in succession), it is probable that they are causally connected; and the probability increases with the number and variety of the instances.** The student should notice the difference between this method and the method of simple enumeration (*i.e.*, counting instances). As Mr Laurie says, in the Method of Agreement stress is laid on the variety as well as on the

[1] H. Laurie, *Methods of Inductive Inquiry*, *Mind*, vol. ii. (1893), pp. 319-338.

number of the instances; to enhance the probability, we must deliberately assemble not only as many but as varied instances as possible.

We may add two concrete examples of the application of this method.

(*a*) An interesting application of the Method was made by Roger Bacon in the fifteenth century. He wished to ascertain the cause of the colours of the rainbow. "His first notion," says Minto, "seems to have been to connect the phenomenon with the substance crystal, probably from his thinking of the crystal firmament then supposed to encircle the universe. He found the rainbow colours produced by the passage of light through hexagonal crystals." But in extending his observations, he found that the passage of light through other transparent materials of certain forms was attended by the same phenomenon. He found it in dewdrops, in the spray of waterfalls, in drops shaken from the oar in rowing. This afforded a good indication that the production of rainbow colours is somehow connected with the passage of light through a transparent globe or prism. These observations were made, and extended, by other investigators; but the true analysis of the causal connection remained for Newton to accomplish by another method (§ 4).

(*b*) An extremely important chemical or biological problem was suggested by applications of the Method of Single Agreement, in 1838. This affords an excellent illustration both of the value of the method and the limits of its power. When sugar is changed into alcohol and carbonic acid in the ordinary alcoholic fermentation, the process is in some way related to the vegetable cells of the yeast plant. "For many years these minute organisms received little or no attention; but in 1838 Schwann, one of the founders of the cell theory, and Cagniard de la Tour demonstrated the vegetable nature of these yeast cells, and showed that they grew and multiplied in saccharine solutions."[1] Hence on the basis of the Method of Single Agreement it was asserted that these minute living things were the immediate cause

[1] M'Kendrick, *Helmholtz* (*Masters of Medicine Series*), p. 26.

of fermentation. But this was to go further than was warranted by this method alone. It only gave a probability that there is a causal connection. Accordingly, a counter-theory, supported by Liebig, held its ground for a considerable time. He maintained that the connection between the fermentive process and the living organisms is altogether indirect; that the yeast cells form a substance which by purely chemical action produces the chemical change called fermentation. Between these two theories the Method of Single Agreement is powerless to decide. We shall see what other methods were called in, by which in the end the original hypothesis was established (§ 7).

This method is applicable where our control over the phenomena under investigation is very limited, so that experiment, unless of an extremely rudimentary kind, is not possible. When our object is to discover the cause *of a given effect*, we are compelled in the first place to have recourse to the Method of Single Agreement. The method to be described in the next section is specially adapted to the discovery of the effects of given or suggested causes.

§ 4. When the Method of Single Agreement has *suggested* a causal connection—and this, as we have seen, is all that it can do—an important means of testing the supposition is provided in the **Method of Single Difference.** This is essentially the **Method of Experiment.**[1] Experiment is the characteristic of this method; and where a cause is not experimentally tested, the Method of Difference is not employed. When we can produce the phenomenon ourselves, we are not content with the mere general probability which the Method of Agreement yields. We take the agent believed to be the cause, and introduce it into definite circumstances

[1] Under "experiment" we here include what may be called 'Nature's Experiments"—*e.g.*, an eclipse of the sun.

arranged by ourselves, where we know therefore that whatever change follows must be due to the agent which we have introduced. Sometimes we add the agent to the known circumstances, at other times we subtract it; logically the results are the same. Mill's statement of the canon is as follows: "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the cause or an indispensable part of the cause of the phenomenon." The canon may be more simply and clearly expressed as follows: **When the addition of an agent is followed by the appearance, or its subtraction by the disappearance, of a certain event, other circumstances remaining the same, that agent is causally connected with the event.** When the suspected agent is present, we have the **positive instance**; when it is absent, the **negative instance.** What cannot be eliminated without doing away with the event, is causally connected with it.

One of the simplest illustrations of this method is seen in the coin and feather experiment, designed to show that the resistance of the air is the cause of a light article, as a feather, falling to the ground more slowly than a heavier one, as a coin. The phenomenon to be investigated is the retardation of the feather. "When the two are dropped simultaneously in the receiver of an air-pump, the air being left in, the feather flutters to the ground after the coin. This is the instance where the phenomenon occurs (the positive instance). Then the air is pumped out of the receiver, and the coin and feather, being dropped at the same instant, reach the ground together. This is the instance where the phenomenon does not occur (the negative instance)." The single circumstance of difference is the presence of the air in the former case,

and with its removal the retardation of the feather's fall is removed.

In further illustration of this method, we may return to the first of the examples given in the previous section. The production of colours by light passing through spherical and prismatic glasses had already been noticed; and Newton proceeded to make it the subject of exact experiment by repeated applications of the Method of Single Difference. A beam of the sun's light admitted through a small hole in an otherwise darkened room, produces on a screen a circular image of the sun (negative instance). But on passing the beam through a prism, the image becomes nearly five times as long as it is broad, and is coloured from end to end by a succession of vivid tints (positive instance). Hence *something in the glass* is the cause of the colours. Newton now proceeded to vary the size of the prism, to vary the quality of the glass, to pass the beam through different parts of the same prism, and to try other minor suppositions; but none of these changes made any difference in the colours. Hence he concluded that the *prismatic shape* of the glass was the real cause. He eliminated this by placing on the original prism a second one of exactly the same angle, but inverted, so that together the two prisms formed a solid with parallel surfaces. The light, passing through both, came out uncoloured and gave a perfect undistorted image of the sun. Hence the prismatic shape of the glass was proved to be the cause of the colours. Newton now adopted the idea that white light is really compound, being composed of differently-coloured primary rays, each undergoing a different degree of refraction (change of direction on passing into the glass of the prism). So he proceeded to test the actual properties possessed by each ray separately. Through a hole in the screen, any one ray could be transmitted while the rest were stopped. The transmitted ray was passed through a second prism, and was found to undergo only a change of direction. When this was done to each of the distinct coloured rays, the latter were found to be refrangible by the second prism in different degrees—the violet most, the red least; precisely, that is, in the same order as by the first prism in forming the elongated spectrum. Thus the composite character of white

light was proved, and the fact that the primary rays composing it have different degrees of refrangibility corresponding to the differences of colour.[1]

The student should notice that in every case the Method is applied in order to test a *suggested* cause; although the suggestion does not always arise from a deliberate application of the Method of Single Agreement.

The successful application of the Method of Single Difference depends on our knowledge of the negative instance, where the phenomenon under investigation is absent. Only when—as in the above examples—we have control of *all the material circumstances* acting in the negative instance, can we be sure (*a*) that the introduction of the suspected cause makes no other change, and (*b*) that the apparent effect of its introduction is not due to some circumstance which was present before in the negative instance. In the examples given, the negative instance was deliberately and carefully *prepared beforehand* in the apparatus of the air-pump, and the arrangements of the darkened room; but it is by no means always possible to do this. When we cannot *prepare* the negative instance, the experiment is of little or no value, unless we can control the field of negative instances in some other way. For example: (*a*) If the attempt is made to measure the force of gravity by delicately suspending a small and light ball, and suddenly bringing a large and heavy ball close to it, the mass of the large ball would attract the small one. But the experiment would not be of the least value unless performed with the utmost precaution; the sudden motion of the large ball would cause currents of air,

[1] Cf. Baden-Powell's *History of Natural Philosophy*, p. 279.

vibrations, &c., which would disturb the small ball far more than the force of gravity. The experiment has been successfully performed by reason of the very ingenious methods adopted to control the negative instance from the action of any circumstance other than the sudden appearance of a large mass of matter.[1] (*b*) Suppose, again, it is required to test the result of using artificial manure for clover. This might seem a very easy matter to determine; for a portion of ground is sown with the manure, another portion is not, and the weight of clover obtained from the one is compared with that obtained from the other. But several questions remain: "How are we to tell what the result would have been had the season been wet instead of dry, or dry instead of wet? How are we to tell whether the manure is equally useful for light soils and heavy, for gravels and marls and clays?" The result is only established for the particular circumstances of season and soil in which the trial was made.

Hence the Method of Single Difference proves *a* cause, but to prove that this cause is the only cause, we require to pay special attention to the possible negative instances.

In connection with the method of Single Difference, we may explain the term "crucial instance" (from Bacon's expression, *instantia crucis*) or "crucial experiment" (*experimentum crucis*). It is an observation or experiment which enables us at once to decide between two or more rival suggestions, suppositions, or hypotheses: this use of the term rests on the metaphor of a guide-post (*crux*) showing us which of two or more ways we ought to take. An example which is commonly given is one taken from the history of theories of Light. From the time of Newton

[1] See Tait's *Properties of Matter*, ch. vii. (p. 127, second edition).

until towards the middle of the nineteenth century, there were two rival theories as to the nature of Light: the "corpuscular" or "emission theory" according to which Light consists of material particles emitted from luminous bodies; and the "undulatory" theory, according to which it consisted of wave-like vibrations of an elastic medium ("ether") pervading all space, and "imponderable" (not subject to the law of gravitation). To put the case as simply as possible: it was proved by mathematical calculation that if the corpuscular theory were true, the velocity of Light would be greater in water than in air, and that if the undulatory theory were true, the velocity of Light would be greater in air than in water. Experiment (by means of a highly ingenious apparatus for ascertaining the velocity of Light) proved that as a matter of fact the velocity of Light is greater in air than in water. This was a "crucial experiment" in favour of the undulatory theory.

§ 5. We have seen that, to apply the Method of Single Difference successfully, we require to obtain control of as many negative instances as possible. In the case of a suspected causal connection between A and *c*, we require to establish the two propositions:—

If A, then *c*;
If not A, then not *c*.

Now in order to prove the second of these two statements, it is usually necessary to conduct **an independent investigation into all the material negative instances.** "Material" negative instances are of course those falling in the same department of investigation. It is obvious that the positive and negative instances must be *in pari materia*—*e.g.*, that, if the subject of inquiry be Chemistry, the negative as well as the positive instances must be sought in the department of Chemistry.[1] We require, then, to exhaust the field of negation by proving

[1] Cf. Fowler, *Inductive Logic*, p. 160.

that if A is absent, c is absent; and this is far more difficult than to exhaust the field of affirmation by proving that if A is present, c is present. The *independent investigation* of the negative instances which is thus required, always presupposes the employment of the Method of Single Difference, and supplements it in the way we have indicated. But it is usually most important and most necessary that the Method of Single Difference *should be* thus supplemented; and when this is done, we have what is practically a new method, which—by adapting one of Mill's expressions—we will call the "Joint Method of Difference and Agreement." The "Difference" refers to the causal connection experimentally determined in the positive instance; the "Agreement" refers to the absence of the effect together with its suspected cause in *all* the negative instances examined. This is the fundamental method of science, and all other methods are only imperfect approximations to it.

The method of Single Agreement may also be supplemented, and in a corresponding way, where experiment is not possible. When the positive instances are taken in accordance with the Method of Single Agreement, and negative instances, agreeing in the absence of the effect together with its suspected cause, are sought for, then we have what we will call the "Double Method of Agreement."

The distinction between the "Double Method of Agreement" and the "Joint Method of Difference and Agreement" rests in the first place on the way in which the positive instances are determined: in the former method by observation, starting usually with an observed event regarded as an effect whose cause is to be sought for; in the latter method by experiment, starting with a suggested cause whose effect is experimentally investi-

gated. The distinction between the two methods rests in the second place on the fact that in the former method the negative instances also have to be found by observation merely; in the latter method, they have very often to be *constructed* in such a way that the cause cannot occur in any of them (see below, § 7).

Mill's third method is called by him the "Joint Method of Agreement and Difference"; but it is really not a combination of Agreement and Difference at all: and Fowler describes it correctly as the "Double Method of Agreement." But neither Mill nor his followers appear to have seen the importance of the "Joint Method of Difference and Agreement" as above defined.

§ 6. The **Double Method of Agreement** is stated by Mill in the following canon: "If two or more instances in which the phenomenon occurs have only one circumstance in common, while two or more instances in which it does not occur have nothing in common save the absence of that circumstance, the circumstance in which alone the two sets of instances differ is the cause or the effect or an indispensable part of the cause of the phenomenon." This is vague, and it is incorrect in more than one point: *two* positive instances would never be enough, still less could two negative instances; and it is not necessary that the negative instances should have "nothing in common." The following simpler canon may be proposed: **Whatever is present in numerous observed instances of the presence of the phenomenon, and absent in observed instances of its absence, is probably connected causally with the phenomenon.** This method presupposes that we have had a wide and varied experience of the conjunction of two events, and that we have failed to find any instance where one has occurred without the other; then

it is probable that they are causally connected, and the probability increases with the number and variety of the negative instances. It presupposes the ordinary Method of Single Agreement before proceeding to "marshal the negative instances"; and like that Method, it is appropriate where exact experiment is not possible. We start with an event regarded as an effect whose cause is to be sought. The Method of Single Agreement leads us to suspect a certain cause for it. We then look for instances all agreeing in the absence *both* of the effect and of its suspected cause.

In illustration of the Double Method of Agreement we will take Darwin's investigation of the theory that "vegetable mould" is produced by earthworms. He devoted a special treatise (*Vegetable Mould and Earthworms*) to the proof that these creatures are thus performing a work of vast magnitude and importance for the maintenance of life on the surface of the earth.

The phenomenon to be investigated is the production of vegetable mould on the surface.

(a) *Positive Instances.*—These were rightly made as numerous and varied as possible—*i.e.*, the surfaces examined consisted of widely different kinds of land, and the objects which sunk were of different kinds. Many observations were made, of which we quote a few. "In the spring of 1835, a field, which had long existed as poor pasture, and was so swampy that it trembled slightly when stamped on, was thickly covered with red sand, so that the whole surface appeared at first bright red. When holes were dug in this field after an interval of about two and a half years, the sand formed a layer at a depth of three-quarters of an inch beneath the surface. Seven years after the sand had been laid on, fresh holes were dug, and now the red sand formed a distinct layer, two inches beneath the surface." The original surface-soil, which consisted of black sandy peat, was found immediately beneath the layer of red sand. Another instance was that of a Kentish chalk formation. "Its surface, from having been exposed for an immense

period to the dissolving action of rain-water, is extremely irregular, being penetrated by many deep well-like cavities. During the dissolution of the chalk, the insoluble matter, including a vast number of unrolled flints of all sizes, has been left on the surface and forms a bed of stiff red clay, from six to fourteen feet in thickness. Over the red clay, wherever the land has long remained as pasture, there is a layer a few inches in thickness of dark-coloured vegetable mould." In another case chalk spread over the surface of a field was buried seven inches in thirty years; in another, a field whose surface had been originally thickly covered with flints of various sizes, was in thirty years covered with compact turf growing out of vegetable mould, beneath which lay the flints. In the latter case also, the worm-castings increased in numbers as the pasture improved. In yet another case, objects such as chalk, cinders, pebbles, &c., of different degrees of heaviness, were tried on the same land; and it was found that they sunk to the same depth in the same time, being covered by vegetable mould. The only material circumstance common to all the different cases of the formation of vegetable mould on the surface is the presence of earthworms, which are estimated, on the basis of careful observation and calculation, to number from thirty to upwards of fifty thousand in an acre, and to yield castings weighing in the mass from seven and a half to over eighteen tons in an acre. There is therefore no doubt of the *adequacy* of the cause which the Method of Single Agreement suggests.

(b) *Negative Instances.*—The suggestion was found to be confirmed as follows. Boulders, of sufficient size to keep the earth beneath them *dry*, do not *sink* although the surface of the ground is raised all round their edges. But in permanently dry earth very few earthworms exist. In one case a stone in length about five feet and in breadth three, had only sunk two inches in thirty-five years; but "on digging a large hole to a depth of eighteen inches where the stone had lain, only two worms and a few burrows were seen, although the soil was damp and seemed favourable for worms. There were some large colonies of ants beneath the stone, and possibly since their establishment the worms

had decreased in number." Among other negative instances recorded, is the case of a dense forest of beech-trees, in Knole Park. "The ground," says Darwin, "was thickly strewed with large naked stones, and worm-castings were almost wholly absent. Obscure lines and irregularities on the surface indicated that the land had been cultivated some centuries ago. It is probable that a thick wood of young beech-trees sprang up so quickly, that time enough was not allowed for worms to cover up the stones with their castings, before the site became unfitted for their existence."

Hence we have good grounds for believing that earthworms are the agency by which vegetable mould is formed, and that it is formed by no other means.

§ 7. The nature of the **Joint Method of Difference and Agreement** may be thus expressed: When one phenomenon has been shown to be *the cause of another under given conditions*, by the Method of Single Difference; and when we fail to find or to construct any instance where the one phenomenon occurs without the other: then it is probable that the first is the "*unconditionally* invariable antecedent" of the second—*i.e.*, that the latter can be produced in no other way than by the former; and the probability increases with the number and variety of the negative instances all agreeing in the absence both of the effect and its suspected cause. The Method presupposes that of Single Difference, and goes beyond it in examining the negative instances *independently*. Very often persevering work is necessary in constructing various negative instances in such ways that the effect (or the suspected cause) cannot occur in any of them: if then these constructed instances agree also in the absence of the suggested cause (or the effect), the conclusion of the Method of Single Difference is very greatly strengthened.

The *extent* of the field over which we must range in

assembling negative instances is a question which the trained investigator, possessing wide and accurate knowledge of the subject, alone can decide. It depends on the kind of problem in question, and the advanced state (or the reverse) of the science to which it belongs. In Chemistry, there is reason to believe that we have experimental knowledge of nearly all the elements to be found on earth. Hence, when by the Method of Single Difference an element yields a particular reaction (*i.e.*, "If A, then *c*"), the investigator is justified in assuming that our knowledge of the negative instances (the properties of the other elements) warrants the statement that no other element will produce that particular reaction (*i.e.*, "if not A, then not *c*"). But the limited number of the elements places Chemistry in an exceptional position. In other branches of science, the great difficulty lies in the examination of the negative instances.

In illustration of the Joint Method of Difference and Agreement, we shall analyse the investigation occasioned by the suggestions made through the Method of Single Agreement, mentioned in § 3, Example (*b*).

A suggestion had been made (and controverted) that the process of fermentation was directly connected with the presence of living yeast-cells. Accordingly a series of searching experimental investigations into the negative instances (of no fermentation) was undertaken: these afford a beautiful example of the successful treatment of the negative instance. "Gay-Lussac showed that clean grapes or boiled grape juice, passed into the Torricellian vacuum of a barometer-tube, remained free from fermentation for any length of time, but that if a single bubble of air were admitted, fermentation soon appeared." Here the negative instance was prepared beforehand, and an application of the Method of Single Difference suggested that *something in* atmospheric air was the cause of fermentation. Then the construction of independent negative instances was

begun, in order to answer the question, when does fermentation *not* take place in the presence of atmospheric air? Schwann repeated Gay-Lussac's experiment, and showed that if the air were admitted to the vacuum through a red-hot tube then fermentation did not occur. Thus the "something in atmospheric air" that caused fermentation was destroyed by heat. The effects of temperature were then further studied. A temperature of from 20° C. to 24° C. was most favourable to it; while the process was stopped at freezing point (0° C.) and again at 60° C.; and boiling destroyed it. Afterwards Helmholtz showed that the oxygen produced by electrolysis in a sealed-up tube containing a boiled fermentible fluid, did not cause fermentation. This was simply oxygen that was absolutely unmixed with any organic or other foreign matter, and differed in this respect from atmospheric oxygen, since air always has extremely minute living organisms in it. Hoffmann showed that air filtered through cotton-wool was incapable of causing fermentation. All these negative instances agreed *both* in the absence of living germs or any foreign matter in the air *and* in the absence of fermentation; and they went to confirm the theory that the yeast-cells were the *immediate* cause of the process, especially the fact that the cause of the process was destroyed by heat. But the most ingenious negative instance was constructed by Helmholtz. "He placed a sealed bladder full of grape-juice in a vat of fermenting juice, and found that the fluid in the bladder did not ferment. Thus the cause of the fermentation could not pass through the wall of the bladder. If the fermentation were excited, as Liebig held, by a separate substance formed by the yeast-cells, and presumably soluble, one would have expected it to pass through the wall of the bladder; but if the process were caused by the small yeast-cells, then one can see why fermentation was not excited, as the yeast-cells could not pass through the membrane."

The theory of causation by yeast-cells was not *proved* by these applications of the Joint Method; but this Method proved a number of facts about the cause, which lent support to that theory, and laid the foundation for the splendid researches of Pasteur.

§ 8. In the most "exact" sciences, where the causes and effects which we examine are susceptible of degrees of intensity, or at any rate of being "more or less," we may not only observe and compare events but *measure* them. Jevons's statement is fully justified: "Every question in science is first a matter of fact only, then a matter of quantity, and by degrees becomes more and more precisely quantitative"; in the middle of the nineteenth century most of the phenomena of electricity and electro-magnetism were known merely as general facts; now they can be for the most part exactly measured and calculated.

As soon as phenomena can be measured, there arises the possibility of a more exact form of either of the two primary methods. This is the **Method of Concomitant Variations**, the canon of which is thus stated by Mill: **Whatever phenomenon varies in any manner whenever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation.** When the variations are ascertained by experiment, this may be regarded as a modification of the Method of Single Difference. To make the above canon apply to cases where experiment is not possible, we must add "other circumstances varying quite independently or with no correspondence." It then becomes a modification of the Method of Single Agreement.

A simple but excellent example of this Method is given by Mill (*Logic*, Bk. III. viii. 7),—the experimental proof of the First Law of Motion. This law states that all bodies in motion continue to move in a straight line with uniform velocity until acted on by some new force. "This assertion," says Mill, "is in open opposition to first appearances; all terrestrial objects, when in motion, gradually abate their velocity and at last stop. . . . Every moving body, however,

encounters various obstacles, as friction, the resistance of the atmosphere, &c., which we know by daily experience to be causes capable of destroying motion. It was suggested that the whole of the retardation might be owing to these causes. How was this inquired into? If the obstacles could have been entirely removed, the case would have been amenable to the Method of Difference. They could not be removed, they could only be diminished, and the case, therefore, admitted only of the Method of Concomitant Variations. This accordingly being employed, it was found that every diminution of the obstacles diminished the retardation of the motion; and inasmuch as in this case the total quantities both of the antecedent and consequent were known, it was practicable to estimate, with an approach to accuracy, both the amount of the retardation and the amount of the retarding causes or resistances, and to judge how near they both were to being exhausted; and it appeared that the effect dwindled as rapidly as the cause, and at each step was as far on the road towards annihilation as the cause was. The simple oscillation of a weight suspended from a fixed point, and moved a little out of the perpendicular, which in ordinary circumstances lasts but a few minutes, was prolonged in Borda's experiments to more than thirty hours, by diminishing as much as possible the friction at the point of suspension, and by making the body oscillate in a space exhausted as nearly as possible of its air. There could therefore be no hesitation in assigning the whole of the retardation of motion to the influence of the obstacles; and since, after subducting this retardation from the total phenomenon, the remainder was a uniform velocity, the result was the proposition known as the First Law of Motion."

The Method may be applied where *exact measurement* is not possible; it is available whenever the *intensities* of two phenomena can be compared, as they vary from more to less or the reverse. A specially important case for its application is when a phenomenon goes through *periodic changes*—*e.g.*, alternately increases and decreases, of which the tides are the most obvious example. If

other phenomena can be found which go through changes in the same periods of time, there is probably a causal connection between them and the first phenomenon. This is the case with the *apparent* motions of the sun and moon round the earth.[1]

§ 9. Mill lays down a fifth canon for a method which, like that of Concomitant Variations, is specially appropriate to quantitative investigations. This is the **Method of Residues.** Its canon is thus stated by Mill: "**Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents.**" Thus, if we are able to show that the complex event *efgh* is caused by ABCD, and is caused in no other way; and that *e* is caused by A, and in no other way, *f* by B, and *g* by C; then we know that *h* is caused by D. Typical instances of the employment of this method are found in chemistry, as Jevons says: "In chemical analysis this method is constantly employed to determine the proportional weight of substances which combine together. Thus the composition of water is ascertained by taking a known weight of oxide of copper, passing hydrogen over it in a heated tube, and condensing the water produced in a tube containing sulphuric acid. If we subtract the original weight of the condensing tube from its final weight, we learn how much water is produced; the quantity of oxygen in it is found by subtracting the final weight of the oxide of copper from its original weight. If we then subtract the weight of the oxygen from that of the water, we learn the weight of the hydrogen which we have combined with the

[1] For further examples of Concomitant Variations, see Fowler, *Inductive Logic*, pp. 189-204.

oxygen. When the experiment is very carefully performed, as described in Dr Roscoe's *Lessons in Elementary Chemistry*, we find that 88·89 parts by weight of oxygen unite with 11·11 parts of hydrogen to form 100 parts of water."

We must observe that the Method assumes that we have performed several conclusive inductions of causation: we must know that ABCD is the "unconditionally invariable antecedent" of *efgh*, and similarly with the various component causes, A, B, and C. If we do not know that ABCD is the "unconditionally invariable antecedent" of *efgh*, we cannot, after the subtraction, infer that D is the cause of *h*, or even that they are causally connected in any way,—for *h* may be connected with other antecedents which co-operate with ABCD.

Many examples which Mill and his followers give as coming under this canon are really instances of a distinct rule, which has been expressed thus: **When any part of a complex phenomenon is still unexplained by the causes which have been assigned, a further cause for this remainder must be sought.** There is no indication in the inquiry, as far as it has gone, of what this cause may be, and hence the "Method" becomes a *finger-post to the unexplained.* It calls attention to "residual phenomena" which have to be accounted for. Such phenomena have frequently led to discoveries of the first importance, such as that of *argon* by Lord Rayleigh and Professor Ramsay in 1894. Their investigations started from the detection of an unexplained residual phenomenon: nitrogen obtained from various chemical sources was of uniform density, but atmospheric nitrogen was about ½ per cent heavier. They proved that the increased weight was due to the fact that the nitrogen in the atmosphere is

mixed with an inert gas hitherto undetected. Sir J. Herschel says: "Almost all the greatest discoveries in astronomy have resulted from the consideration of residual phenomena of a quantitative or numerical kind. . . . It was thus that the grand discovery of the Precession of the Equinoxes resulted as a residual phenomenon, from the imperfect explanation of the return of the seasons by the return of the sun to the same apparent place among the fixed stars." Herschel's remarks received afterwards a most remarkable illustration in the discovery of the planet Neptune by Adams and Leverrier in 1846. The sun and the known planets have a calculable effect in disturbing the path of Uranus in its elliptic orbit; but there were residual perturbations which could not be thus accounted for. From these the orbit and position of Neptune were calculated before the planet had been observed.

Mill refers to the Method of Residues as available when special difficulties arise in observation, because several causes act at once, and their effects are all blended together, producing a joint effect of the same kind as the separate effects (*Logic*, Bk. III. x. §§ 3 and 4). Mill's view of what he calls "intermixture of effects" has been simply explained by Jevons: "If in one experiment friction, combustion, compression, and electric action are all going on at once, each of these causes will produce quantities of heat which will be added together, and it will be difficult or impossible to say how much is due to each cause separately. We may call this a case of the **homogeneous intermixture of effects**, the name indicating that the joint effect is of the same kind as the separate effects. There are several causes, each producing a part of the effect, and we want to know how much is due to each." It is true that the

Method of Residues (in the form implied in Mill's own canon quoted above) is available in such cases, under the conditions stated (p. 315), when the magnitude of the several known causes and their respective effects can be quantitatively estimated.

This "intermixture" or combination of homogeneous effects may assume very complicated forms. The action of any cause may be (*a*) augmented, (*b*) diminished, (*c*) diverted, (*d*) wholly counteracted, by that of another cause;[1] and all these various kinds of interaction may occur at the same time among the effects. Fowler observes truly that in every case *each cause produces its appropriate effect*, even though it may have disappeared wholly or partly in the total result. "An object may remain at rest, when subject to two equal forces acting in opposite directions, but we cannot say of either of these forces that it is inoperative: each, it is true, prevents any visible effect resulting from the other: but then this is the very effect which it produces." When the full consequences of a Law are thus affected (modified or neutralised) by other Laws, it is called a *tendency*. There are no real *exceptions* to Laws of Nature.

§ 10. We may, therefore, sum up the characteristic features of what we have called the second step in the process of science:—

(1) It is suggested or assumed, from previous observations or by some other means, that A is the immediate cause of *c*.

(2) Positive instances, of *c* occurring in connection with A, are then sought for, experimentally if possible, in order to establish the proposition "if A, then *c*."

[1] Thus, for simple examples we may suppose a body (*a*) pulled by two forces in exactly the same direction; (*b*) pulled by two forces of different magnitudes in exactly opposite directions; (*c*) pulled in one direction by one force, and by another force pulled in a direction at right angles to the former; (*d*) pulled by two *equal* forces in exactly opposite directions, when no motion takes place.

(3) Negative instances, including apparent exceptions, are then investigated in order to establish the proposition "If not A, then not *c*."

How far precisely do the Methods hitherto examined carry us? The answer is, they cannot do more than establish a causal law that *c* results from A under all circumstances; and this only by the application of the most powerful method, that of Joint Difference and Agreement. What more than this do we want? We want, if possible, to bring this law into harmony with other scientific laws, and more especially to deduce it or anticipate it on the ground of previous knowledge. This is the meaning of **Explanation** in science. The chief object and the great difficulty of the Methods hitherto examined is to *isolate* a cause—that is, prove that A produces *c* by getting A to act as far as possible in isolation (§§ 4 and 7). The chief object and the great difficulty of scientific explanation is to *break down this isolation* by connecting the action of A with the action of other causes. This we will call the third step in the process of science.

When a law is ascertained, and we do not know how to connect it with other laws, it is said to be an **empirical law** (ἐμπειρία, experience or trial).

Mill says that "scientific inquirers give the name of empirical laws to uniformities which observation or experiment has shown to exist, but on which they hesitate to rely in cases varying much from those which have been actually observed, for want of seeing any reason *why* such a law should exist" (*Logic*, Bk. III. xvi. 1). When we speak of the "reason why" a fact or a law exists, we mean "its connections with other laws or facts." To this we must add that the degree of reliability of such a law varies according to the method by which it was established. (1) "Horned animals are ruminants": this is an instance of Agreement

which is scarcely more than a simple enumeration, and affords no presumption of causal connection. Hence there is a certain doubt in extending it to any new case of a "horned animal." (2) "Where dew is formed, the dewed surface is colder than the surrounding air": this connection has been ascertained in many instances, varying from one another in other respects. The resulting empirical law may therefore be extended to new cases "differing from those previously observed," with greater confidence than in case (1). The same remark applies to many instances of the Method of Single Agreement, such as those given in § 3. (3) The Method of Single Difference gives us reliable knowledge of the action of a cause under the prepared conditions of the experiment; but as we have seen, it does not give us knowledge of the action of the same cause under *new* conditions. It does not warrant the empirical law "if A, then *c*, whatever the circumstances may be." The conclusion which it actually does warrant, that "the cause A in the circumstances *bdf* has the effect *c*," *if the experiment is careful enough*, may be made the ground of a universal law by the principle of the Uniformity of Causation: "the cause A under the same circumstances will always have the same effect." For instance, we may know from observation and experiment that "quinine affects beneficially the nervous system and the health of the body generally, while strychnine has a terrible effect of the opposite nature." But we can give no other reason for the truth of such generalisations. (4) The Double Method of Agreement, and the Joint Method, as we have shown, serve to make the resulting generalisations more trustworthy—*i.e.*, we are able to affirm with greater confidence that A, and A only, is *always* the cause of *c*, but still, so far it remains an isolated law.

So far as we are able to connect a law with other laws, so far we are able to "explain" it. We must not expect too much of science, or read too much into the word Explanation: the truth is, to "explain" a fact, in Science, comes in the last resort only to this,—that we show it to be part of a wider fact.[1]

[1] Cp. ch. VIII. § 8 *ad finem.*

§ 11. It was remarked in passing that the methods which we have already explained cannot get to work without the aid of a preliminary guess, supposition, or suggestion of a possible cause for the phenomenon under investigation.

First, then, we must have an assumption as to the locality, and possibly the nature, of the cause; and the Methods previously examined exist in order to test such suggestions. Every research by which we seek to discover truth must be guided by some conjecture: whether it be a theoretical suggestion of a cause, or the practical suggestion of something to be accomplished.

If we are in doubt as to the cause of any phenomenon, we make a guess, supposition, or conjecture,— we imagine what seems a sufficient cause, and proceed to test it by the methods previously described. Such a conjecture is called in scientific language an **hypothesis** (ὑπόθεσις, *suppositio*, "placing under"). Hypotheses, then, are continually employed throughout all the Inductive Methods.

* Mill's great mistake (in which Bain and Fowler follow him) lay here. He recognised, indeed, the validity of the Method of Explanation by Hypothesis, which he calls the Deductive Method; and he grants that to this method "the human mind is indebted for its most conspicuous triumphs in the investigations of Nature." But he treats it as a different kind of proof from "induction," or the "experimental methods"; it is available where these methods fail. In complex cases, where numerous causes are interacting to produce an effect, the laws of the separate causes must be ascertained by "induction" (*i.e.*, by one or more of the five methods mentioned by Mill, — Agreement, Difference, Double Agreement, Concomitant Variations, Residues); then follows the suggestion or hypothesis, that a particular combination of these causes is at work in the particular case; the effect of the supposed combination is

ascertained by deductive reasoning, and the results of the deductions are compared with the facts. He refuses to call this process "induction," and restricts this term to the process of generalising from experience by the five methods. He must have regarded these methods as applicable directly, without any previous assumption, to the masses of fact which ordinary experience presents to us: by this means the facts are to be made to disclose uniform laws. This is just what the methods will not do. They require prepared material; and this means that they require much preliminary scientific arrangement of facts. They require also preliminary theories or conjectures to be tested.

We now proceed to deal with the question, How are these "hypotheses of immediate causation" *suggested?* There are two principal means by which facts may be made to suggest the supposition of a possible immediate cause—

(*a*) By the **Method of Agreement** itself;
(*b*) By **Analogy.**

The account already given of the Method of Agreement has shown how it may suggest an hypothesis of immediate causation. We find the event P in a number of instances A, B, C, D, &c.; examining further, we find that the fact S is the only other material circumstance in which they agree,—hence a connection of S and P is suggested. This differs from simple enumeration; for we do not merely count the instances,—we take them so that they differ as much as possible from each other, except in the presence of S and P. Expressed syllogistically, the argument becomes—

A, B, C, D, &c., have the property P;
A, B, C, D, &c., have the property S;
∴ S and P are or may be causally connected.

The student will notice that, since the expression "A, B, C, D, &c." may be regarded as a collective singular

term, this argument is really a syllogism in *Darapti;* for the logical form "Some S are P" implies "S may be P" (see p. 57),—in other words, there may be a connection between S and P.

* The argument is also an Aristotelian Enthymeme in fig. iii., whose probability depends on the number and variety of the instances which collectively form the subject in both premises. In fact, Aristotle's "Enthymeme in the third figure" expresses the principle of Mill's "Method of Agreement" more correctly than Mill himself did.

If no more powerful method than that of Single Agreement can be employed, we may fall back on counting. We may try to count *all* the instances of S in order to see if P is present in each. If so, then by complete enumeration every S is P. The instances are 100 per cent; the total is limited, and we have reached it. This is usually impossible. All that remains (if we cannot go beyond counting) is to estimate the *probability* of S always being P. This leads to the calculation of chances and the quantitative Theory of Probability. In this work we do not propose to touch on these subjects. The natural sequel, however, to a suggestion by Single Agreement is to test and investigate it by one or more of the further methods described in §§ 4 to 9.

§ 12. Hypotheses of immediate causation are also frequently suggested by **Analogy.**

If by any means whatever the *possibility* of a connection between S and P is suggested to us, then we may raise the question: "Is there anything in S which is already known to be capable of producing P, or *vice versa?*" This is to connect SP (by Analogy) with previous knowledge.

Analogy may be regarded first as a special kind of argument, as Aristotle regards it. We have already

discussed it from this point of view (ch. VIII. § 4). The student will remember that Analogy is "any resemblance between things which enables us to believe of one what we know of the other," and that the value of the inference depends on whether the resemblance is in the material (or essential) points. We now proceed to discuss the value of Analogy as a means of suggesting hypotheses of causation. At what point does an argument from Analogy pass into such a suggestion?

In analogical inference, a new case is shown to be probably an instance of a cause whose working is known to be illustrated in a case with which we are familiar. It is, as Aristotle said, an argument from particular to particular, from one example or instance to another, depending on the resemblance between the two cases in some material circumstance. Hence it has been said that Analogy—as long as it remains Analogy only —"sticks in the particular instances"; it does not work out a law of connection between the two cases, or compare other cases, according to the canons of inductive observation. But it suggests that both cases may be instances of a general law under which they fall. It prompts us to extend our knowledge of the first case and found on it a law of connection which includes the second.

* Thus, suppose we have a suggested connection, S is P. It *may* be suggested, in the way we have described, by the Method of Agreement (otherwise, by an Enthymeme in fig. iii.); or in some other way. If we can find some fact M to be an important circumstance in both S and P, we may justify the original suggestion by an analogical inference, thus :—

P is M,
S is M;
∴ S and P are probably causally connected through M.

This is of course an Aristotelian Enthymeme in fig. ii. The "suggested explanation" is, to investigate the connection of M and P further, in order to determine whether M is the cause of P. If this relation can be established, then we may "explain" P, bringing both S and P under the universal M. Stated syllogistically, this becomes a valid syllogism in fig. i (a syllogism "of cause"):—

M is P,
S is M;
∴ S is P.

For concrete examples, the student may refer to ch. VIII. § 3. In § 3 (*a*), Ex. 3 is a suggestion based on Agreement; in § 3 (*b*), Ex. 3 is an analogical justification of the same suggestion; in § 3 (*c*), Ex. 3 is an explanation of the suggested connection by a law of real causation. Similarly, § 3 (*a*), Ex. 4, is a suggestion based on Agreement; and § 3 (*b*), Ex. 4, is an analogical justification of it.

It thus appears that an analogical inference is a stage or step in the complete inductive process. If the analogical inference to the new particular case is justifiable, there is ground for going beyond analogy and inferring a general law under which both cases come; although, so far, the law is only a suggestion, an hypothesis. And if there is no ground for an induction of a general law from the two cases, there is no ground for a good analogical inference.

Analogy may be described as the application of previous knowledge to a new set of facts; this broadens our conception of their scope, suggesting restatements and revisions. Thus, "our knowledge of the various functions of plants—digestion, reproduction, &c.—has been obtained by ascribing to the various organs of the plant purposes analogous to those which are fulfilled by the various parts of animal bodies. And in turn the study of plant physiology has thrown light upon animal physiology, and enlarged and modified many of its

theories." This "reforming of the old by the new" is a general characteristic of the *growth* of knowledge. A conspicuous instance of it is seen in the early researches of Pasteur and his friends into bacteriology, as described in the *Life of Louis Pasteur* by his son-in-law. The old belief was that many contagious diseases were due to a *virus* or poison introduced into the blood. Further research was undertaken on the assumption that the cause of the diseases was something in the blood, but not necessarily a *virus*. This was a suggestion by analogy with the former belief, and it was experimentally proved by inoculating healthy animals with a drop of infected blood. Afterwards the presence of minute animalculæ, visible only by the microscope, was detected in the blood of diseased animals; but at first it was supposed that these minute organisms could not produce such great effects. But subsequently Pasteur proved that a phenomenon of such magnitude as fermentation was caused by the growth of an invisible vegetable organism; hence analogy suggested that the animalculæ, whose presence was detected in the infected blood, might after all be the true cause of the diseases in question. This hypothesis, being experimentally verified, was proved to be true by applications of the Joint Method. The old theory, that these diseases were caused by a *virus* introduced into the blood, could only give a forced explanation of many known facts; and it had to give way to a new theory, harmonising all the facts. But the new theory was originally suggested by analogy with the old; and the speculations with regard to the action of the *virus* which were based upon facts did not lose their value; they simply had to be *revised* by the aid of the new light shed on the question.

§ 13. In the methods of Induction analysed in §§ 3 to 9 of the present chapter, the function of hypothesis is restricted to the suggestion of possible causes. In the Method of Explanation, by which we seek to connect things together with one another and with previous knowledge, as indicated in § 10, the function of hypothesis is more fundamental.

The **Method of Explanation** has four stages :—

1. Preliminary ascertainment of fact or causal law, either by simple observation, or by more complex processes, as in the Methods of Agreement, Difference, &c.
2. Formation of an hypothesis to *explain* the fact or law which these Methods disclose.
3. Deduction according to the first figure: the hypothesis being treated as a general principle from which conclusions are drawn.
4. Verification, or comparison of these consequences with the facts of Nature.

This might be called the **Newtonian Method**, since all its stages are exemplified in the process by which Newton established his theory of Gravitation. Before illustrating it further, we must examine the second stage again. We have already said that to explain anything is to connect it with other things, and that this means **connect it with what we already know.** The following more abstract definition is frequently given: **Explanation is essentially a bringing of the particular, or less general, under the universal, or more general.** This may be done in different ways. The following examples illustrate both definitions, and show that they are consistent :—

(*a*) We may " explain " **facts by a law**, as when many different and (at first sight) disconnected events are shown to be instances of one and the same Law of

Causation. One of the most famous examples of such explanation is Kepler's discovery that the planet Mars moves in an elliptic orbit. The observations of Tycho Brahe had determined a great number of successive positions of that planet to a high degree of accuracy; and the resulting orbit appeared to be extremely irregular. But the earth itself, from which the observations were made, is in motion round the sun; hence it was necessary to distinguish that part of the irregularity of the orbit of Mars which was due to the earth's motion, and then to ascertain what curve corresponded to the true positions of the planet. He assumed the earth's motion to be circular, which is approximately true; but the orbit of Mars was evidently not circular. "The picture which Kepler presents to us of the working of his own mind while pursuing this research is full of the most intense interest. It would be impossible, without entering into mathematical details, to explain the process by which the ultimate suggestion was brought under his consideration; and it would be equally so to convey an idea of the immense mass of calculation through which he toiled in putting each of his successive theories to the test of agreement with the observations. Finally, after working his way in alternate exultation at anticipated triumphs, and bitter disappointment when, one after another, they vanished in air,—driving him, as he says, 'almost to insanity,'—he at length had the intense gratification of finding that *an elliptic orbit described about the sun in one of the foci* agreed accurately with the observed motions of the planet Mars."[1] The irregularity of its movement vanished; all its observed positions became intelligible, were "explained," when seen to be successive points on this simple and symmet-

[1] Baden-Powell, *History of Natural Philosophy*, p. 150.

rical curve. The hypothesis, thus proved for Mars, was extended by analogy to the other known planets, and proved true of them also, by observations as accurate as were then available,—Kepler perceiving that his original assumption as to the motion of the earth was only an approximate one. Thus was established "Kepler's first Law." There could be no better instance of how disconnected facts are "explained" by being brought under general laws. In Kepler's case the law had to be discovered; but in the same sense we "explain" an event when we can show it to be a new instance of a *known* law.

(*b*) We may "explain" **law by law.** Of such explanation there are two kinds. A given law may be shown to result from the combined operation of other laws. Thus, the motion of a projectile, if we neglect the resistance of the air, is a parabola. This is "explained" by proving it to be the result of two known laws governing the motion of the projectile; these are, the first Law of Motion, that a body in motion continues to move in a straight line with uniform velocity until acted on by some external body; and the attraction between the moving body and the earth, according to the Law of Gravitation, that any two bodies attract each other with a "force" varying (1) *inversely* as the square of the distance between them, and (2) *directly* as the product of their masses. But the most fundamental explanation of "law by law" is attained when a given law can be shown to be a particular case of a more general law. Newton's explanation of Kepler's Laws by the Law of Gravitation affords an impressive instance of this, and is also a perfect example of what we have called the complete Inductive Method. The process by which the first (and essential) part of Newton's great

generalisation was established may be analysed as follows, according to the four stages mentioned earlier in this section.

Newton's own genius, taking up facts of observation and suggestions thrown out by previous investigators, led him to formulate this law as an *hypothesis:* Any two bodies attract one another with a force varying inversely as the square of the distance between them. If this hypothesis is true, the weight of an object (the pull exerted upon it by the mass of the earth[1]) should decrease as its distance from the earth increases. Within those short distances from the earth's surface to which our observation extends, the intensity of gravity does not appreciably diminish as we recede from the earth. But in the case of an object removed as far from the earth as the *moon* is, it must have appreciably diminished. Now the intensity of gravity at the earth's surface is measured by the space through which a body falls in one second; and its intensity at the distance of the moon, by the space through which the moon would fall towards the earth in one second, if she were not prevented by another cause. Also, the object at the surface of the earth is distant from the centre of the earth (which is the centre from which gravity acts) by the length of the earth's radius; and the distance of the moon from the earth's centre is known. Hence it became a calculation in Proportion, to ascertain the distance through which the moon ought to fall towards the earth in one second, if the Newtonian hypothesis were true. How can we compare this result with the actual fact of the moon's falling, since no such experiment can

[1] The pull which the said object exerts upon the earth is of course a real fact, but, in comparison with the earth's attraction, may be reckoned as practically nothing.

be tried upon her? "Newton saw that such an experiment is in fact constantly exhibited to us. The moon performs a revolution in an orbit whose dimensions had been ascertained by astronomers; consequently, the *velocity* with which she moves was known. But this velocity impressed upon such a body must, if nothing else interfered, carry it off in a straight line through space. The actual motion of the moon is in an orbit round the earth; and in any given portion or 'arc' of that orbit, the distance through which, at the end of one second, the moon has deflected from the straight line which is a tangent to the orbit at the commencement of that second, is known. This is the space through which the moon is actually 'falling' (*i.e.*, is actually pulled) towards the earth in one second. Newton, then, in his calculation, had only to take the distance of the moon from the centre of the earth, and the distance of the surface from the centre (*i.e.*, the radius of the earth); and, squaring these numbers, the inverse proportion would be that of the spaces fallen through in one second by the moon, and by a body at the surface of the earth. If this calculated result agreed with the result actually observed, his conjecture would be verified; and the very same force of gravity which causes bodies to fall near the earth would be that which causes the moon to 'fall,' or, in other words, to be deflected from a rectilinear course, and to describe her orbit round the earth." In this calculation Newton took for the radius of the earth the length which at that time (about 1666) was considered accurate, and the result did not verify his conjecture; there was a difference of two feet per second between the actual and the calculated deflection of the moon. This small discrepancy was large enough, in Newton's opinion, to show

that his cherished hypothesis could not account for the facts; and he dismissed the subject from his thoughts for some time. But in 1682 the radius of the earth had been more accurately calculated. Newton substituted the new value in his former proportion, and "having proceeded a little way in the calculation, was utterly unable to carry it on, from the overpowering excitement of its anticipated termination; and he requested a friend to finish it for him." The result was that the moon's deflection, as calculated from his hypothesis, agreed with the deflection calculated from observation. This great result sufficed as a clue to the whole mechanism of the planetary system, and afterwards of the universe. Newton proceeded to show, by his unrivalled powers of mathematical calculation, that Kepler's Laws are a necessary consequence of the Law of Gravitation. If we have bodies freely revolving round a common centre of force, which attracts them with a "pull" varying inversely as the squares of their distances from it, then the following laws must hold good: (α) Their orbits must be ellipses with the "centre of force" in a focus; (β) the radius drawn from each moving body to the centre must describe equal areas in equal times; (γ) the periodic times of their revolution vary as the cubes of their mean distances from the centre. These were the same three laws which Kepler had shown, from Tycho's observations, to be true of the motions of the planets round the sun, and which other observations showed to be true of the motions of satellites round their planets, as was most evident in the case of Jupiter and Saturn. Newton went further, and proved that if his law were absolutely true, Kepler's could only be *approximately* so; for the attraction holds not only between the sun and the planets, but between the planets themselves. Hence it

was impossible that their orbits should be perfectly elliptical; and the more accurate observation which afterwards became possible, showed that just such "perturbations" take place as would be expected if Newton's law were true. And by rigorous deductions it has been shown that his law is competent to account for the complex motions actually observable in the solar system. They are "accounted for," or "explained," in being proved to be consequences of the law. It is this demonstration, that the consequences of a law do actually agree with facts, that forms for Science the verification of that law.

§ 14. We shall now give a brief general statement of the characteristics of a good hypothesis, and the conditions under which we may regard an hypothesis as proved.

The characteristics of a good hypothesis depend on the principles stated at the beginning of the previous section, and most essentially on the second of these,—that the hypothesis must be a possible "explanation" of some fact or law which is under consideration.

Before we go further, however, we must be clear as to one general truth. We must understand that **the invention of hypotheses is the work of the scientific genius.** In the previous section we were discussing the ways in which hypotheses of causation might be "suggested"; but before any hypothesis can be suggested there must be a mind prepared to receive the suggestion. Hypotheses are the creations of the investigator's mind. There is such a thing as genius in science as well as in poetry and art; and the scientific genius stands out clearly from the common run of scientific workers. To such a mind, trained by previous observation and thought, a few facts will suggest, almost as if by inspiration, hypotheses of far-

reaching importance. This is what Tyndall expressed in the passage so often quoted from an essay on "The Scientific Use of the Imagination," in his *Fragments of Science*. "With accurate experiment and observation to work upon, imagination becomes the architect of physical theory. Newton's passage from a falling apple to a falling moon was an act of the prepared imagination; out of the facts of chemistry the constructive imagination of Dalton formed the atomic theory; Davy was richly endowed with the imaginative faculty, while with Faraday its exercise was incessant, preceding, accompanying, and guiding all his experiments. . . . Without the exercise of this power, our knowledge of Nature would be a mere tabulation of coexistences and sequences."

The conditions of a good hypothesis may be stated as follows:—

(*a*) The hypothesis must be **based on facts** (using "fact" in the wide sense of something ascertained to be real in Nature). It is suggested only because it is a possible explanation of the facts. It is not created by the scientific imagination "out of nothing"; it is not independent of facts, as are the impulses of the artistic imagination. It is intimately dependent, as Tyndall says, on the suggestions of accurate experiment and observation, and also on whatever knowledge the investigator already possesses. His previous acquaintance with the subject suggests the limits within which probable hypotheses must lie, and opens his eyes to obscure analogies and insignificant residual phenomena to which the ordinary mind would pay no attention. And as in its origin it depends upon facts, so for its verification we must examine the relevant facts with the most rigorous exactness, and if there is any discrepancy,

the hypothesis must be rejected or modified. It is no paradox to say that "the first thing is to form an hypothesis; the second, to be dissatisfied with it." The instances of Kepler and Newton show that the greatest investigators are those who are most ready to abandon cherished theories, the fruit of laborious research, if they cannot be shown to harmonise with fact. What Francis Darwin says of his father is true of the scientific genius in every branch of inquiry: "It was as though he were charged with theorising power, ready to flow into any channel on the slightest disturbance; so that no fact, however small, could avoid releasing a stream of a theory, and thus the fact became magnified into importance." In this way many untenable theories naturally occurred to him; but his richness of imagination was equalled by the power of judging, and if necessary condemning, his theories by comparing them with facts.

(*b*) It must be capable of being **brought into accord with received knowledge**, by mutual modification, if necessary. This rule depends on the essential nature of an hypothesis, as we have already stated it,—*i.e.*, it is a connection of new knowledge with old (with mutual modification if necessary). We often find mistaken statements on this subject. It is sometimes said that the hypothesis must be "conceivable." This is, of course, true if "conceivable" means "not self-contradictory" (see ch. II. § 12); but it is hardly necessary to state as a special rule that the hypothesis must not contradict itself. If, on the other hand, "conceivable" means "easy to imagine" in the sense of picturing the meaning to one's mind, then it is not true to say that a legitimate hypothesis must be "conceivable." It is not easy to imagine the antipodes, where "to go up" means to go in a direction diametrically opposite to that which

we so describe; it is not easy to imagine that we are moving through space with great velocity in two different directions at once. It is not easy to imagine that if an organ were played by machinery in a hall, and there were no living creatures in or near it, it would make no *sound.* It is harder still to imagine that we live and move in a perfectly *solid* and *elastic* medium, possessing no weight, and capable of nine hundred millions of millions of vibrations in a second of time. For similar reasons it is a mistake to make the rule say that a legitimate hypothesis must not *conflict* with any of the "received" or "accepted" laws of Nature. The rule means that, though an hypothesis may be new or strange —*i.e.*, may conflict with the apparent implications of previous knowledge—it may still be legitimate. And it is legitimate if, when we consider both what the hypothesis implies and what is implied in our previously accepted knowledge, the discrepancy can be shown to be only apparent. This may require a modification of the received knowledge, by which it is set in a new light; and it may require a modification of the hypothesis also. Thus, the supposition of the "antipodes" was once believed to be in conflict with ordinary experience, for it seemed to mean that on the other side of the earth were people living with their heads "downwards." The difficulty was removed when it was understood that "down" means only the direction in which the mass of the earth attracts bodies by gravitation; and that direction is always in a straight line towards the earth's centre.

(*c*) It must **furnish a basis for deductive inference** of consequences. This rule implies not only that the hypothesis must be clearly and distinctly conceived in itself; it must also be conceived after the analogy of

something in our experience. To assume something utterly unlike all that we are previously acquainted with, is to assume what can be neither proved nor disproved, for we could not draw any conclusions from it. Even the hypothesis of an absolutely solid and elastic *something*, to explain the phenomena of light, is not of this kind. Jevons says truly that if this "luminiferous *ether*" were wholly different from everything else known to us, we should in vain try to reason about it. "We must apply to it at least the laws of motion—that is, we must so far liken it to matter. And as, when applying those laws to the elastic medium air, we are able to infer many of the phenomena of sound, so by arguing in a similar manner concerning ether we are able to infer many of the phenomena of light. All that we do is to take an elastic substance, increase its elasticity immensely, and denude it of gravity and some other properties of matter; but we must retain sufficient likeness to matter to allow of deductive calculations."

Newton did not use the word *hypothesis* as we now use it. He used it to signify just such unprovable assumptions as are excluded by this third rule. Hence he said *hypotheses non fingo*, "I do not imagine hypotheses." The word is still occasionally used by scientific writers in this sense.

(*d*) **The inferred consequences must agree with the facts** of Nature. This condition is one that we have already illustrated. The consequences of the hypothesis must be deduced as rigorously as possible, and then compared with the results of accurate observation. The greater the extent of agreement, the more justified we are in accepting the hypothesis as true. The hypothesis must of course agree entirely with the facts which it was invented to explain; but it requires to be compared

with a wider range of facts, and to have a place found for it in the general body of knowledge bearing on the subject. And when, by this means, we have found that it is the only supposition which can be made in the circumstances, and that it is competent to explain the facts in question, we may regard it as fully established; and then it may be spoken of as a "fact."

The student should notice the ambiguities of the words "fact" and "theory." "Fact" is frequently used, as we have used it in previous chapters, to signify what is observable by our senses; and in contrast, "theory" is frequently used for an hypothesis which is suggested but not yet established. Many writers restrict the meaning of "theory" to "hypotheses which are fully established"; but, none the less, when an hypothesis is thus established beyond the possibility of doubt, we tend to speak of it as a "fact." The two meanings of "fact" are not so unrelated as might appear.

Finally, we must understand that **hypotheses are not limited to science.** Even primitive savages, in conceiving all living and moving Nature to be possessed by innumerable ghosts or spirits, were forming an hypothesis to explain the facts—not, of course, with full consciousness of what they were doing. And whenever we try to account for anything given to us by testimony or by perception, we are forming an hypothesis; and the method is always the same,—we "account for" the event, or "explain" it, by connecting it with the knowledge which we believe ourselves already to possess. The hypotheses of Common-Sense are made for practical purposes; no more is required of them than that they should answer these purposes, as they do. The hypotheses of scientific thought are made with the purpose, before all else, of helping us

to understand; hence they must be *thought out* as completely and accurately as possible. Here we have stated the essential difference between Science and Common-Sense. It is not so much a difference of subject-matter. Professor W. K. Clifford said roundly that "scientific thought does not mean thought about scientific subjects with long names; there are no scientific subjects. The subject of science is the human universe; that is to say, everything that is or has been or may be related to man." Common-Sense is content to know and understand this universe just far enough to satisfy practical needs; hence Common-Sense is knowledge in a disorganised and sometimes chaotic state. Science, on the contrary, seeks for the real causes of events, and seeks to connect these causes together by means of explanatory laws. Common-Sense is usually contented with the outside of things. Science seeks for clear and distinct conceptions which shall give us, not the appearance only, but something deeper, which is more true and real.

We must add that though in the strictest sense a "good" hypothesis must satisfy our fourth condition; yet in a less strict sense an hypothesis may be considered good and useful in the absence of such complete verification. The truth is, that the fourth condition states an *ideal*, which may be more and more realised, but which no hypothesis as yet fully realises. The law of gravitation probably realises the condition more fully than any other principle as yet worked out by Science. But a supposition may fall far short of such complete verification, and yet be scientifically useful. If it is suggested by a real knowledge of the facts dealt with, it will open out new and important lines of inquiry, in addition to providing a conception of the facts under

which they may be conveniently collected, in thought, and harmonised. An hypothesis which is thus fruitful, but insufficiently verified, is called a **working hypothesis.** In illustration of what has been said, the reader should study Professor G. H. Darwin's Addresses before the British Association for the Advancement of Science (1905),[1] which afford an instructive example of how a representative scientific thinker deals with a comprehensive working hypothesis and collects under it a number of subsidiary hypotheses.

* References for Reading.

The following references relate chiefly to the topics dealt with in chapters VIII. and IX. For the Inductive Methods: Mill, *Logic*, Bk. II. ch. i. to iii., v., viii., x., xi.; Venn, *Empirical Logic*, ch. xiv., xv., xvii. (cf. H. Laurie, *Methods of Inductive Inquiry*, "*Mind*," N.S., vol. ii. p. 319, March 1893); Sigwart, *Logic*, vol. ii. §§ 93-97 (pp. 288 ff.); Bosanquet, *Logic*, vol. ii. ch. iv. (pp. 108 ff.). For subsidiary processes: "Methods of Observation," Venn, *op. cit.*, ch. xviii.; Sigwart, *op. cit.*, vol. ii. §§ 86-92 (pp. 234 ff.); "Enumeration" (logical theory of), Bosanquet, vol. i. ch. iii., iv. (pp. 128 ff.); "Arithmetical Calculation" (logical theory of), Sigwart, vol. ii. § 66 (pp. 30 ff.) for introductory treatment, and more fully in Bosanquet, vol. ii. ch. ii. (pp. 43 ff.); "Analogy," Mill, Bk. III. ch. xx.; Bosanquet, vol. ii. ch. iii. (pp. 83 ff.); Hypothesis, Bosanquet, vol. ii. ch. v. (pp. 155 ff.), Jevons, *Principles of Science*, ch. xxiii.; Venn, ch. xvi.; "Explanation," Venn, ch. xxi., xxiv.; Sigwart, vol. ii. §§ 98-100, 105 (pp. 417 ff., 548 ff.); Theory of the *ultimate* meaning of Cause, Bosanquet, vol. i. ch. vi. (pp. 252 ff.), vol. ii. ch. vii. (pp. 205 ff.). For logical aspects of Probability and Statistical Enumeration: Mill, Bk. III. ch. xvii., xviii.; Hobhouse, *Theory of Knowledge*, pt. ii. ch. x., xi. (pp. 289 ff.); Sigwart, vol. ii. §§ 85, 101-2 (pp. 216 ff., 480 ff.); Bosanquet, vol. i. ch. viii. (pp. 352 ff.).

[1] See *Nature*, vol. 72, Nos. 1868, 1870.

We shall also give some references to guide the student in following up a subject which is of great importance from the point of view of Inductive Logic: Methods of Classifying the Sciences. The varying classifications put forward depend at bottom on different views of the nature and aim of Science. English writers have usually paid most attention to the arrangements given by Comte and by Spencer: by Comte, in *Cours de Philosophie Positive*, vol. i. (Harriet Martineau's English edition, vol. i. pp. 15 ff.), criticised by Spencer, in articles on "The Genesis of Science" and "The Classification of the Sciences," *Essays*, vol. ii., and by James Martineau, *Types of Ethical Theory*, Bk. II. § 2 (B), second ed., vol. i. pp. 431-4; expounded by Lévy-Bruhl, *Philosophy of Comte*, Bk. I. ch. iii. (Eng. Trans., pp. 49 ff.). Spencer's own classification is set forth in the Essays already named. Bain, *Deductive Logic*, pp. 25 ff., 232 ff. (App. A)[1] has given a valuable and suggestive classification, and has exhaustively criticised Spencer's arrangement. The student should make Bain's treatment a starting-point in his study of the subject. Recent works dealing most elaborately with the mutual relations of the Sciences are Masaryk, *Versuch einer Concreten Logik*, and (for physical science) Ostwald, *Naturphilosophie*.

Of great historical interest is the classification given by Francis Bacon, *De Augmentis Scientiarum* (reproduced in the *Advancement of Learning*); which was further elaborated by the authors of the famous *Encyclopédie* published at Paris in the middle of the eighteenth century (Encyclopédie, *Discours Préliminaire*, by J. D'Alembert). The history of Classifications of the Sciences is described in outline by Flint, *Philosophy as Scientia Scientiarum.*

EXERCISE XVII.

Questions on Chapter IX.

(i) *Elementary.*

1. How does Experiment (*a*) resemble, and (*b*) differ from, Observation?

[1] Cp. the arrangement in *Inductive Logic* (Bk. V. pp. 199 to 367).

2. Give, briefly and in outline, a classification of the sciences on the basis of their employment of "passive" observation, experiment, or both.

3. Show the logical character of the Method of Single Agreement. Under what conditions or pre-suppositions is it applicable? How does it differ from simple enumeration?

4. Give some instances of simple experiments fulfilling completely the conditions of the Method of Single Difference.

5. Compare and contrast (*a*) the Double Method of Agreement and the Method of Single Agreement, (*b*) the Joint Method of Difference and Agreement and the Method of Single Difference, showing the advantages of the first method in each case.

6. What place does *hypothesis* hold in the above-mentioned Methods of causal investigation? By what means may such hypotheses be suggested?

7. Explain concisely the Method of Concomitant Variations, and give two instances of its use. What is its relation (*a*) to the Method of Single Difference, (*b*) to that of Single Agreement?

8. Explain the Method of Residues. When can it prove that one event is the cause of another, and when can it only suggest an inquiry into causation?

9. What is an "Empirical Law"? Give examples.

10. What is meant by "Explanation" in Science? Can we distinguish different kinds of "Explanation"?

11. What are the three steps of the "Method of Explanation"?

12. Trace the steps in the progress of the theory of Gravitation, showing that it was established by this method. [Jevons.]

13. What is the place of hypothesis in the "Method of Explanation"?

14. State the conditions of a good hypothesis. What is a "working hypothesis"?

15. Explain the expressions "intermixture of effects," "counteracting cause," "tendency."

16. Supposing us to be *unacquainted* with the causes of

the following phenomena, by what methods should we investigate each?—

(*a*) The connection between the barometer and the weather.

(*b*) A person poisoned at a meal.

(*c*) The connection between the hands of a clock.

(*d*) The effect of the Gulf Stream upon the climate of Great Britain. [Jevons.]

(ii) *More Advanced.*

17. Distinguish Experiment from Observation, and show by examples how the several methods of experimental research have been of use in Scientific discovery. [L.]

18. "No theorising apart from observation, and no observing save in the light of theory." Comment on this.

19. How far is the validity of the several Inductive Methods affected by the possibility of a "plurality of causes"? [St A.]

20. The Inductive Methods have been called "weapons of elimination." Discuss the appropriateness of this description. [L.]

21. Is hypothesis an essential factor in Inductive investigation? [L.]

22. Consider the relations that have been held to exist between Analogy and Induction. Do you think there is ever proof from Analogy? If not, what place does Analogy hold in the process of inference? [L.]

23. Distinguish hypothesis from theory. Explain the use of hypothesis in scientific procedure. Give examples showing how far the imagination, and how far the reason, has entered into the construction of a workable hypothesis. [L.]

24. Select any of the great conclusions of modern science, and show how hypothesis has given rise to discovery, tracing the stages by which approximate certainty has been reached. [L.]

25. *Vere scire est per causas scire.* Examine this.

26. "What is called the scientific *explanation of a fact* is nothing more than showing it to be a case of a more general fact, which though *more general*, is still a *fact* merely, and is as much in need of 'explanation' as the original fact was." Examine this.

27. (*a*) "The aim of Science is to *explain*." (*b*) "Science never *explains*; she only reduces complex events to simple ones of the same kind, as when she deals with certain phenomena of magnetism by supposing every ultimate unit of the substance to act as if it were a magnet."

Carefully consider these statements. [L.]

28. Point out the successive questions which would have to be decided in the investigation of the following phenomena:—

(*a*) Friction alters the temperature of the bodies rubbed together.

(*b*) The sun is supposed to move through space.

(*c*) A ray of light passing into or out of a denser medium is deflected. [Jevons.]

29. In the following cases, what are the conclusions, and by what methods?—

(*a*) A bell struck *in vacuo* gives no sound; while light traverses an airless medium.

(*b*) As a body passes from a lower to a higher temperature, it invariably undergoes a change of volume, generally in the direction of expansion, but sometimes (as in the melting of ice) in that of contraction.

(*c*) When the cork of a bottle of soda-water slightly warmer than the surrounding air is expelled by the elastic action of the "carbonic acid" gas, the bottle becomes cooler than the surrounding air.[1]

30. Both mosquitoes and cases of malarial fever have, in certain parts of Italy, in West Africa, and elsewhere, become much rarer since these districts have been well drained. Is malarial fever the effect of the presence of mosquitoes? Suggest additional tests of this conclusion.

31. Describe the logical characters of the following arguments, and discuss their validity:—

(1) "That *The Tempest* belongs to the latest period of Shakespeare's literary activity is shown, *inter alia*, by the absence of rhyme, the large number of 'run on' (unstopped) lines, the high proportion of weak and light endings, and the comparative rarity of puns in the low scenes."

[1] Cf. Fowler, *Inductive Logic*, vol. ii. p. 157.

(2) "I own myself entirely satisfied . . . that there is no such thing as colour really inhering in external bodies, but that it is altogether in the light. And what confirms me in this opinion is that in proportion to the light the colours are still more or less vivid, and if there be no light, then there are no colours perceived."

[Stock; from Berkeley.]

(3) "The increase of agrarian crime, say the judges, was coincident with the activity of the Land League, and the decrease of agrarian crime with the inactivity of the Land League." [Stock.]

(4) "The place of a planet at a given time is calculated by the law of gravitation; if it is half a second wrong, the fault is in the instrument, the observer, the clock, or the law; now the more observations are made, the more of this fault is brought home to the instrument, the observer, and the clock."

[W. K. Clifford.]

(5) "Sir D. Brewster proved that the colours seen upon mother-of-pearl are not caused by the nature of the substance, but by the form of the surface. He took impressions of mother-of-pearl in wax, and found that, though the substance was entirely different, the colours were exactly the same."

[Jevons.]

(6) "A person is in sound health mentally and physically. The breaking of a minute blood-vessel in the brain causes a clot of blood there, which is followed immediately by unconsciousness and soon afterwards by death. Hence the existence of mind depends on the healthy functioning of the brain."

(7) "There are no great nations of antiquity but have fallen by the hand of time; and England must join them to complete the analogy of the ages. Like them, she has grown from a birth-time of weakness and tutelage to a day of manhood and supremacy; but she has to face her setting. Everything that grows must also decay." [E.]

(8) "No coal can be found in that district; for if the rock nearest the surface is Laurentian, the Carboniferous strata must be absent; for the Laurentian formation is older than the Silurian, and the Silurian older than the Old Red Sandstone, and the Old Red Sandstone older than the Carboniferous strata." [St A.]

(9) Goldscheider proved that muscular sensations play no considerable part in our consciousness of the movements of our limbs, by having his arm suspended on a frame and moved by an attendant. Under these circumstances, where no work devolved on the muscles, he found that he could distinguish as small an angular movement of the arm as when he moved and supported it himself. [Creighton.]

(10) He also proved that the chief source of movement-consciousness is pressure-sensations from the inner surface of the joints, by having his arm held so that the joint surfaces are pressed more closely together, and finding that a smaller movement was now perceptible. [Creighton.]

* 32. "The Third is distinctively the *Inductive Figure.*' Discuss this view of the nature of Induction.

* 33. Criticise Mill's Canons of Induction, and show in what way they may be amended. *Or—*

Can Mill's Methods of Induction be reduced to one Method? Are they logically valid? [St A.]

* 34. What are the inductive difficulties in arguing from a negative? Give appropriate examples. [E.]

* 35. Analyse the process of scientific observation, and in the light of your answer consider whether or how far it is possible to have a *logic of observation.*

* 36. Consider in detail the nature and relation to one another of the processes called Description and Explanation.

* 37. "Whatever is inconceivable must be false." Discuss the ambiguities in this statement. In what sense is it true?

CHAPTER X.

FALLACIES.

§ 1. THE word Fallacy is sometimes used to signify any false statement, erroneous belief, or mental confusion of any kind. This leaves the meaning of the word too vague. In the logical sense, a fallacy is a violation of some rule or regulative principle of logical thought. There are such principles governing the formation of conception, of judgment, and of inference, deductive and inductive; of these we have been treating in the preceding pages in an elementary way. From this point of view the chief types of fallacy might be classified according to the logical principle violated. We say "chief types" only, for we could not take account beforehand of every possible kind of mistake which might be made.

The traditional logical doctrine has generally narrowed the meaning of the word fallacy to mistakes in *reasoning*, limiting the latter term to that type of reasoning which can be expressed in syllogistic form (ch. VI.) The traditional *classification* of fallacies is based on that of Aristotle as given in his treatise on sophistical arguments (περὶ σοφιστικῶν ἐλέγχων), usually referred to as the "Sophistical Refutations." As the title suggests, the aim of his discussion is entirely practical,—to enumerate the various tricks which might be employed in contro-

versy, and were employed by many of the "Sophists" (ch. I. § 3).

A false argument, says Aristotle, may err either in the thoughts expressed or in the signs (words) which express them. Hence he indicates two main classes of fallacy: (*a*) those which are directly due to language (fallacies *in dictione*, παρὰ τὴν λέξιν), and (*b*) those which arise from the thought rather than the language (fallacies *extra dictionem*, ἔξω τῆς λέξεως). Of the first class he enumerates six forms: some of them are trifling, being indeed dependent on the peculiarities of Greek syntax.

I. *Fallacies due to Language.*

(1) Ambiguity of word (ὁμωνυμία, "equivocation"). This consists in the ambiguous use of one of the three terms of a syllogism, so that in reality there are four terms. Its most important case is the *fallacy of ambiguous middle*, already referred to (ch. VI. § 3).

An old example is given by De Morgan :—

"Finis rei est illius perfectio,
Mors est finis vitæ,
Ergo, Mors est perfectio vitæ ;"

where the ambiguity may be laid on *perfectio* or on *finis*. Some instructive examples are given by Jevons: "Often the ambiguity is of a subtle and difficult character, so that different opinions may be held concerning it. Thus we might argue :

"'He who harms another should be punished. He who communicates an infectious disease to another person harms him. Therefore he who communicates an infectious disease to another person should be punished.'

"This may or may not be held to be a correct argument according to the kinds of actions we should consider to come under the term *harm*, according as we regard negligence or

malice requisite to constitute harm. Many difficult legal questions are of this nature, as for instance:

> Nuisances are punishable by law;
> To keep a noisy dog is a nuisance;
> To keep a noisy dog is punishable by law.

"The question here would turn upon the degree of nuisance which the law would interfere to prevent. Or again:

> Interference with another man's business is illegal;
> Underselling interferes with another man's business;
> Therefore underselling is illegal.

"Here the question turns upon the *kind of interference*, and it is obvious that underselling is not the kind of interference referred to in the major premise."

The serious confusion of ambiguous terms can only be met by careful definition (ch. V. Part ii.)

(2) Ambiguity of structure (ἀμφιβολία, "amphiboly"). This arises when the ambiguous grammatical structure of a sentence produces misconception:

> "The Duke yet lives that Henry shall depose."
> —"K. Henry VI.," Part II., Act I. sc. iv.

Ambiguities of this kind are more possible in the classical languages than in English, owing to the possible variations of order in a sentence and to "oblique" constructions, as in the Latin version of the oracle given to Pyrrhus: "Aio te, Æacida, Romanos vincere posse." One of Aristotle's examples is *τό βούλεσθαι λαβεῖν με τοὺς πολεμίους*.

(3) *Composition* (σύνθεσις). Aristotle explains this fallacy to consist in taking together words which ought to be taken separately. He *seems* to have been considering only verbal mistakes of this kind—*e.g.*, "Is it possible for a man who is walking not to walk?" "Yes." "Then it is possible for a man to walk without

walking." Again: "Can you carry this? and this? and this? &c." "Yes." "Then you can carry this and this and this, &c. [together]."[1] But the latter example suggests that what he really had in view was the important logical fallacy of arguing from the distributive to the collective use of a term.

Jevons has explained this very clearly: "In the premises of a syllogism we may affirm something of a class of things *distributively*, that is, of each and any separately, and then we may in the conclusion infer the same of the whole put together. Thus we may say that 'all the angles of a triangle are less than two right angles,' meaning that *any* of the angles is less than two right angles; but we must not infer that all the angles put together are less than two right angles. We must not argue that because every member of a jury is very likely to judge erroneously, the jury as a whole is also very likely to judge erroneously; nor that because each of the witnesses in a law case is liable to give false or mistaken evidence, no confidence can be reposed in the concurrent testimony of a number of witnesses. It is by a fallacy of Composition that protective duties are still sometimes upheld. Because any one or any few trades which enjoy protective duties are benefited thereby, it is supposed that all trades at once might be benefited similarly; but this is impossible, because the protection of one trade by raising prices injures others."

Accordingly, in modern text-books the fallacy of Composition is defined as arguing from a common term (*i.e.*, one used distributively) to one used collectively.

(4) The fallacy of Division (*διαίρεσις*) is treated, both by Aristotle and modern writers, as the converse of the fallacy of Composition.

[1] The example in the text of *Sophistici Elenchi* (ch. iv.) is one of verbal confusion only—*i.e.*, of a phrase which may be read in either of two ways:—

τὸ | ἓν μόνον δυνάμενον φέρειν | πολλὰ δύνασθαι φέρειν and
τὸ | ἓν μόνον | δυνάμενον φέρειν πολλὰ | δύνασθαι φέρειν.

Aristotle's examples are of separating *words* which should be taken together, and so changing the meaning of a sentence; as though one made the statement "four and three are six and one" mean that "four is six" and "three is one." In modern text-books, the fallacy of Division means to argue from the collective to the distributive use of a term, as in the very common mistake of making a statement about a group *as a whole*, and then taking for granted that it is true of *each individual* member of the group. The statement that a certain political party is a "blatant faction" does not imply that the opinions of every one of its members are blatant and factious; to say that "the Germans are an intellectual people" does not warrant the conclusion that this or the other German is intellectual; and so on.

(5) The fallacy of Accent (προσοδία) is explained by Aristotle simply as the mistaken accentuation of a word in writing Greek.

In modern text-books it is taken to be the quibble of altering the meaning of a sentence (when speaking) by emphasising some particular word above the rest. More important is the observation of De Morgan, that if in quoting an author we italicise a word which he has not italicised, or leave out words, in the quotation or its context, we are guilty of this fallacy.

(6) The fallacy of Figure of Speech (τὸ σχῆμα τῆς λέξεως) is the confusion of supposing that words similar in grammatical form (case, declension, conjugation, termination, &c.)—or similar in being derived from the same root—are similar in meaning. It is really a trivial kind of false analogy, *e.g.*, to suppose that *poeta* is feminine because *mensa* is so; or to confuse the meanings of forms resembling one another, as do *art*, *artful*, *artificer*.

The two most important fallacies in the foregoing list are those of *Composition* and *Division*.

II. *Fallacies due to the Thought rather than the Language.*

Aristotle mentions seven types of this kind of fallacy.

(1) The fallacy of Accident (τὸ συμβεβηκός) consists in confusing an unessential with an essential difference or resemblance.

Thus: "Is Plato different from Socrates?" "Yes." "Is Socrates a man?" "Yes." "Then Plato is different from man." It does not follow that because the one differs from the other in one or more respects, they therefore differ in every respect. In the same way, it does not follow that because the one resembles the other in one or more respects, that the two are similar in all respects. Of this mistake the following is a crude example: "To call you an animal is to speak truth; to call you an ass is to call you an animal; therefore to call you an ass is to speak truth." Any typical fallacy of Accident, when stated in syllogistic form, will be found to be an example of Four Terms.

(2) Next in Aristotle's list stands a form of fallacy to which subsequent Latin writers gave the name of *a dicto secundum quid* (πῇ) *ad dictum simpliciter* (ἁπλῶς). It consists in assuming that what holds true in some particular respect, or under some special circumstances, will hold true without any restriction or as a general rule. Aristotle, in speaking of this fallacy, refers chiefly to illustrations of it which appear to deny the Law of Contradiction (ch. II. § 12); thus, he says that we should be committing this fallacy in arguing that an object which is partly black and partly white is both white and not white. It is only white in a certain respect (*secundum quid*, πῇ), not absolutely (*simpliciter*, ἁπλῶς).

The fallacy, which is a very common one, consists essentially "in getting assent to a statement with a qualification, and then proceeding to argue as if it had been conceded without qualification." We commit this fault if we prove

that the syllogism is useless for a certain purpose, and then claim to have proved that it is useless for any purpose. For another example: it is undoubtedly true that to give to beggars promotes mendicancy and causes evil; but—as Jevons says—if we interpret this to mean that assistance is never to be given to those who solicit it, we fall into the fallacy under consideration "by inferring of all who solicit alms what is only true of those who solicit alms as a profession."

There is a converse form of this fallacy which is quite as common, and consists in assuming that what holds true as a general rule will hold true under some special circumstances which may entirely alter the case. "For example," says Professor Minto, "it being admitted that culture is good, a disputant goes on to argue as if the admission applied to some sort of culture in particular—scientific, æsthetic, philosophical, or moral." Fallacies of this kind seek to argue *a dicto simpliciter ad dictum secundum quid*—*e.g.*, every man has a right to inculcate his own opinions; therefore a magistrate is justified in using his power to enforce his own political views. We cannot infer of his special powers as a magistrate what is only true of his general rights as a man.[1]

To the two fallacies already mentioned in this connection, De Morgan rightly proposes to add a third—that of arguing from one special case to another special case, which does not resemble it in material circumstances. The student will see that this is really identical with false analogy.

(3) The next fallacy was called by the Latin writers *Ignoratio Elenchi*, after Aristotle's ἐλέγχου ἄγνοια, "ignorance of [the nature of] refutation." To *refute* an adversary's assertion, we must establish the exact logical contradictory of it (ch. III. § 7). To prove a conclusion which is not the contradictory is *ignoratio elenchi*.

[1] Some writers identify the fallacy *a dicto simpliciter ad dictum secundum quid* with the fallacy of *Accident*, and accordingly call the fallacy *a dicto secundum quid ad dictum simpliciter* the "converse fallacy of Accident."

In modern text-books the scope of the fallacy is extended to cover all cases of "proving the wrong point,"—all cases in which, instead of the required conclusion, a proposition which may be mistaken for it is defended. Mr Welton quotes a concise example from Spencer's *Education:* "Throughout his after career, a boy, in nine cases out of ten, applies his Latin and Greek to no practical purposes." As the same writer observes, Mr Spencer's argument "ignores the fact that the advocates of a classical education do not claim that Latin and Greek are of direct use in practical life. What they do urge is that the study of the classics furnishes an unrivalled mental training; and it is this proposition which a true ἐλέγχος must disprove." Jevons truly says, "The fallacy usually occurs in the course of long harangues, where the multitude of words and figures leaves room for confusion of thought and forgetfulness." "This fallacy is, in fact, the great resource of those who have to support a weak case. It is not unknown in the legal profession, and an attorney for the defendant in a lawsuit is said to have handed to the barrister his brief marked, 'No case; abuse the plaintiff's attorney.' Whoever thus uses what is known as *argumentum ad hominem* —that is, an argument which rests, not upon the merit of the case, but the character or position of those engaged in it—commits this fallacy. If a man is accused of a crime, it is no answer to say that the prosecutor is as bad. If a great change in the law is proposed in Parliament, it is an Irrelevant Conclusion to argue that the proposer is not the right man to bring it forward. Every one who gives advice lays himself open to the retort that he who preaches ought to practise, or that those who live in glass houses ought not to throw stones. Nevertheless there is no necessary connection between the character of the person giving advice and the goodness of the advice."

The *argumentum ad populum* is another form of Irrelevant Conclusion, and consists in addressing arguments to a body of people calculated to excite their feelings and prevent them from forming a dispassionate judgment upon the matter in hand. It is the great weapon of rhetoricians and demagogues.

To these we may add what is called the *argumentum ad ignorantiam*, trading on the ignorance of the person or persons addressed; the *argumentum ad verecundiam*, an appeal to veneration for authority instead of to reason; and the *argumentum ad baculum*, which is not an argument at all, but an appeal to physical force.

(4) The fallacy of the *Consequent* is vaguely explained in some modern text-books as meaning "any kind of loose or inconsequent argument," and described by the phrase *non sequitur*.

Aristotle meant by it simply the invalid "argument from the affirmation of the consequent" in a hypothetical proposition. He mentions cases of it in arguments from presumptive evidence—*e.g.*, "this man has no visible means of support, and must therefore be a professional thief." Even if we grant that "if a man is a professional thief, he will have no visible means of support," the particular conclusion will not follow. Of equal importance, as we have seen (ch. VII § 2), is the fallacy of denying the antecedent. When long pieces of reasoning are being dealt with, the "denial of the antecedent" often takes the form of assuming that because the conclusion is supported by invalid arguments, it is false.

(5) The fallacy of *Petitio Principii* (τὸ ἐν ἀρχῇ αἰτεῖσθαι καὶ λαμβάνειν)—*i.e.*, to assume the conclusion which is to be proved. Aristotle says that this may take place in five ways.

(*a*) To assume the proposition which is to be proved, and in the very form in which it is to be proved. As Aristotle hints, this is hardly possible unless the assumption is concealed by some sort of verbal confusion.

An illustration is to be found in Mill's account of the ground of the axiom which lies at the basis of Induction—the Uniformity of Nature. This principle, says Mill, is the "ultimate major premise of all Induction," and yet is itself

founded on Induction of the weakest kind, *per enumerationem simplicem;* it is therefore only an "empirical law," true within the limits of time, place, and circumstance which have come under our actual observation. But if it is the ultimate major premise of all Induction, it must be a law of the nature of things, true without exception of past, present, and future. On this difficulty Mill says: "The precariousness of the Method of Simple Enumeration is in an inverse ratio to the largeness of the generalisation. The process is delusive and insufficient, exactly in proportion as the subject-matter of the proposition is special and limited in extent. . . . If we suppose, then, the subject-matter of any generalisation to be so widely diffused that there is no time, no place, and no combination of circumstances, but must afford an example *either of its truth or of its falsity*, and if it be never *found* otherwise than *true*, its truth cannot be contingent upon any collocations, unless such as exist at all times and places. . . . It is therefore an empirical law co-extensive with all human experience, at which point the distinction between empirical laws and laws of Nature vanishes."[1] Stated briefly, Mill's argument is this. The Law of Uniform Causation is of so universal a character that *every* time and place must afford an instance either of its *truth* or of its *falsity*. It is *observed to be true* at those times and places which have come within our actual experience; *therefore it is true* of every time and place, independently of our experience. This is a neat example of proving the universality of a principle by assuming it. It is usual to call this form of fallacy, when committed in a single step of inference, a ὕστερον πρότερον (*hysteron proteron*); when the assumption and conclusion are separated by various steps of inference, a *circulus in probando*.

(*b*) The same type of fallacy is committed when we take for granted a general principle which involves the required conclusion, and which is just as much in need of proof as the conclusion itself; or, indeed, when any general truth is falsely taken to be self-evident.

[1] *Logic*, Bk. III. ch. xxi. § 3. The italics are mine.

Mr Welton quotes an example from Spencer's *Education* (ch. i.) "After stating that 'acquirement of every kind has two values—value as *knowledge* and value as *discipline*'—Mr Spencer goes on to discuss the value of different subjects from the point of view of knowledge. He then turns to the disciplinary value of studies, and commences his disquisition with the following flagrant *petitio:* 'Having found what is best for the one end, we have by implication found what is best for the other. We may be quite sure that the acquirement of those classes of facts which are most useful for regulating conduct, involves a mental exercise best fitted for strengthening the faculties. It would be utterly contrary to the beautiful economy of Nature, if one kind of culture were needed for the gaining of information and another kind were needed as a mental gymnastic.'"

(*c*) Aristotle says that if we assume the particulars to prove the universal which involves them, we commit the same kind of fallacy. This is induction *per enumerationem simplicem*—*e.g.*, assuming that "some S is P" warrants "all S is P": it is an inductive fallacy.

(*d*) The fourth mode which Aristotle refers to is only a more prolix form of the first. It is to prove a general proposition by breaking it up into parts and assuming the truth of each part.

(*e*) The fifth mode rests on immediate inference by converse relation (ch. III. § 13): to assume, for instance, that A is south of B in order to prove that B is north of A.

(6) In his list of fallacies, Aristotle next enumerates *τὸ μὴ αἴτιον ὡς αἴτιον*, afterwards rendered *non causa pro causa*. This is not an inductive fallacy, for Aristotle's *αἴτιον*, *causa*, here signifies *reason*. It is to give as a reason that which is no reason. The case on which Aristotle dwells is that of introducing into an argument irrelevant premises which lead to a contradiction, and then "fathering the contradiction on the position con-

troverted." But the name may be applied to any argument containing steps without logical connection (without middle terms).

(7) Last on Aristotle's list stands the fallacy of "Many Questions" (τὸ τὰ δύο ἐρωτήματα ἓν ποιεῖν). It consists in demanding "a plain answer—yes or no"—to a question which really implies an assumption: *e.g.*, "Have you abandoned your intemperate habits yet?"

§ 2. The Aristotelian classification lays perhaps too much stress on language, the verbal expression of judgments, in making this the principle of division. But as long as we retain his Terminology as all modern text-books retain it, it is well to retain his meaning also. Confusion has been created by keeping the one without the other. Thus, the division of fallacies into logical and material, current since Whately's time, is, by Jevons and others, identified with Aristotle's division into fallacies *in dictione* and *extra dictionem*. The Aristotelian division rests on an entirely different basis. The modern division has been clearly explained by Mr Stock: "Whenever in the course of our reasoning we are involved in error, either the conclusion follows from the premises or it does not. If it does not, the fault must lie in the process of reasoning, and we have then what is called a Logical Fallacy. If, on the other hand, the conclusion does follow from the premises, the fault must lie in the premises themselves, and we then have what is called a Material Fallacy. Sometimes, however, the conclusion will appear to follow from the premises until the meaning of the terms is examined, when it will be found that the appearance is deceptive owing to some ambiguity in the language. Such fallacies as these are, strictly speaking, non-logical, since the meaning of words is extraneous to the science which deals with thought

But they are called Semi-logical. Thus we arrive at three heads, namely—(1) Formal or Purely Logical Fallacies, (2) Semi-logical Fallacies or Fallacies of Ambiguity, (3) Material Fallacies." The second class, fallacies of Ambiguity, are identical with those which Aristotle called fallacies "in the language"; the third class, Material fallacies, are the same as Aristotle's fallacies "outside the language." The first class, Formal fallacies, consist of breaches of the syllogistic rules, of which examples have already been given (ch. VI. § 3). The most important of these are (*a*) four terms, (*b*) undistributed middle, (*c*) illicit process of the major or minor. The student will see that all Formal fallacies are at bottom cases of four terms.

§ 3. "Inductive Fallacies," mistakes incident to inductive reasoning, are usually said to be of three main types:—

(*a*) erroneous observation.
(*b*) ,, analogy.
(*c*) ,, generalisation.

We shall briefly point out the nature of these inductive fallacies. At bottom they are all cases of erroneous generalisation.[1]

(*a*) Observation is at bottom sense-perception. All the possibilities of error in sense-perception arise from the fact that in perception things are not imaged in the mind as in a mirror,—the mind itself contributes to the result. There is no perception without an element of thought and *inference*, although in simple cases (*e.g.*, the perception of a colour as red) we are scarcely conscious of the inference. We need not dwell on this doctrine,

[1] For numerous instructive illustrations of what is said in this section, the student is referred to Fowler, *Inductive Logic*, ch. vi. (on Inductive Fallacies).

which is well established in modern psychology. The more elaborate and systematic the observation is, the more extensive is the work of *thought* in it. And it is in this thought-aspect of perception and observation that the possibilities of truth and of error lie. Many writers describe this source of error as "a confusion of what we *perceive* and what we *infer* from what we perceive." This suggests that the perception and the inference are two separate things, which is not the case. The confusion referred to is between the half-unconscious and instinctive inference, which experience has taught us to make correctly (*e.g.*, "that is a man"), and the more deliberate and conscious inference, by which we extend the former (*e.g.*, "that man is my friend Smith"). We often treat these secondary inferences as if they were as trustworthy as the primary ones, which is scarcely ever true.

(*b*) With regard to mistaken analogies, it must be remembered that analogy is never *strict proof;* and, as a rule, the conclusion of an argument from analogy is only problematical. The real importance of analogy is to suggest hypotheses and lines of inquiry. Hasty and insufficient analogies may suggest unscientific and even absurd hypotheses. Most primitive superstitions, characteristic of the childhood of the race, are cases of hypothesis resting on some fragment of analogy. This fact is abundantly illustrated in the anthropological writings of Tylor, Lubbock and Clodd.

(*c*) Mal-observation and false analogy are implicitly generalisations which are erroneous. Fallacies of explicit generalisation are, however, even more common —*e.g.*, to generalise from mere enumeration; to argue *post hoc ergo propter hoc*, mistaking mere succession for true causation; to generalise in neglect of "extreme

cases," which our generalisation ought to cover; to neglect counteracting causes or material conditions; to ~~neglect negative instances~~ (non-observation).

An excellent recent treatment of the whole subject of Fallacies will be found in Welton's *Logic*, vol. ii. bk. vii. (pp. 227 ff.) The Aristotelian doctrine is well stated and illustrated by De Morgan in his *Formal Logic* (out of print); some of his material is made accessible to the student by Mr Welton. The historically important treatments of logical Fallacy have been: that of Aristotle, *Sophistical Refutations* (English edition by Poste, 1866); that of Bacon, the "Idola," *Novum Organum*, Bk. I.; that of Whately, *Logic*, Bk. III.; that of Mill, *Logic*, Bk. V.

The student should examine various classifications of Fallacies also as illustrations of the Theory of Classification, different bases leading to different divisions.

EXERCISE XVIII.

State the following arguments in logical form, examine their validity, and name any fallacies they contain:—

1. Knowledge gives power; power is desirable, therefore knowledge is desirable.

2. All X's look so and so; this looks so and so, therefore this is an X. (What common psychological act does this argument formulate?)

3. I suppose that France abounds in beggars; for, from the cases of Italy, Spain, &c., I infer that all Catholic countries do so.

4. If a man be rightfully entitled to the produce of his labour, then no one can be rightfully entitled to anything which is not the produce of his labour. [St. A.]

5. (*a*) Provided he has been properly taught, he can himself teach; for experience makes experts.

(*b*) The historical novel is an impossibility; for it proposes to combine fiction with fact, and these are contradictories.

(*c*) He is innocent, for he has faced his accusers; a guilty man would run away. [L]

6. The sea was the place where the incidents of my story occurred; there is the sea, therefore my story is true. [C.]

7. All responsible beings are rational; responsibility increases with increase of rationality; some dogs are more rational than some men, therefore some dogs are more responsible than some men. [L.]

8. The theory of evolution is true, because it is accepted by every scientific biologist. [L.]

9. The object of war is durable peace, therefore soldiers are the best peace-makers. [Whately.]

10. Improbable events happen almost every day; but what happens almost every day is a very probable event, therefore improbable events are very probable events. [Whately.]

11. Books are a source both of instruction and amusement; this is a book, therefore it is a source both of instruction and amusement. (This example can be treated without substituting "some books" for "books" in the major.)

12.[1] Reason is identical in nature wherever it is found, and all men are rational, therefore all men should be treated as equal.

13. To make a good bargain is an advantage, therefore the more good bargains we make the better.

14. A person found guilty by a jury may be innocent; this man has been found guilty, therefore he may be innocent.

15. Wisdom is inseparable from benevolence, hence all benevolent persons are wise.

16. He condemns Napoleon and Cæsar; but Napoleon and Cæsar are great men, therefore he condemns great men.

17. The earth cannot be round; for if it were, the water in the Suez canal would flow out at both ends.

* 18. It is absurd to encourage such a disposition as emulation *in moderation;* for if it is a good thing, we cannot have too much of it; and if it is a bad thing, we cannot have too little of it.

* 19. Nothing but M is a cause of P; hence if M occurs, P may follow.

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[1] Examples 12 to 17 are from the Pass, and examples 18 to 22 from the Honours, papers of the University of London Intermediate Examination in Arts, 1900-1903.

* 20. All scientific statements should be capable of proof. But it is impossible to prove everything, for that would involve a *regressus ad infinitum;* it would seem, then, that science is impossible.

* 21. Nothing is possible unless all the conditions of its existence are fulfilled; but where all these conditions are present, it actually exists; therefore whatever is possible is actual.

* 22. It is not necessarily true that one plus one equals two; for the world might have been so constituted that whenever one thing was combined with another, a third thing came into being.

* 23. The unknowable is the unthinkable, and therefore cannot exist.

* 24. To say that there is *no* rule without an exception, is self-contradictory.

* CHAPTER XI.

THE PROBLEMS WHICH WE HAVE RAISED.

§ 1. THROUGH all the preceding treatment of the more elementary doctrines of Logic, we have been expounding the essentials of what is rightly called **the Traditional Logic**; and, in order to make many of its doctrines and phrases more intelligible, we have connected them with their Aristotelian fountainhead. But we have stopped short of developing some further issues which they involve, although we have frequently come within sight of these issues.

In the present chapter we propose to examine the bearings of these more fundamental questions in a way which, it is hoped, will afford some guidance to the student who wishes to pursue further the study of **Modern Logic** in its philosophical aspects. We shall take up the questions in the order in which they have been raised in the previous exposition.

We have said that Logic deals with the principles which regulate valid or correct thought, and on which the validity of the thought depends (ch. I. § 1). We may call them **postulates** of knowledge, because without them not only science but everyday thought cannot even begin to work. If they are untrustworthy, every fabric of knowledge falls to pieces, for they are the general bonds of connection which hold it together,

and only through them has our knowledge such coherence as it now possesses. We have discussed some of the more fundamental of these principles—*e.g.*, the primary Laws of Thought (ch. II. §§ 10-14), the Aristotelian Canon of Deductive Reasoning (ch. VI. § 6), and the Law of Universal Causation (ch. VIII. § 7). They have been called axioms or "self-evident truths" (see p. 45 above). The chief object of Modern Logic is to state these principles as completely and systematically as possible, in the light of the idea that the general activity of Thought may be compared to the activity of a living organic body. "In this case"—as the writer has expressed it elsewhere—"the intellectual postulates appear as the vital processes or functions—*e.g.*, digestion, circulation, respiration—by which the life of the organism is preserved and its growth effected; they are the *vital functions of thought.* It is useless to discuss the 'certainty' of any one of these principles when considered in isolation; the very fact that we are separating it and considering it by itself precludes us from seeing its real significance. Its true character only appears through the function it performs in the growth of intelligence and the attainment of knowledge; and to discuss this function is to treat it not in isolation but in relation to other similar principles,—to inquire into its place in our intellectual activity as a whole."[1]

Hence we see in what sense Logic is "formal" (ch. I. § 2). It discusses the general characteristics of the thinking process without regard to the particular details which form the *objects* of the thinking. But for a similar reason "all science is formal, because all science consists in tracing out the universal characteristics of things,—the structure that makes

[1] *Philosophical Criticism and Construction*, ch. i. p. 12.

them what they are." To say that a science is formal is only to say that a definite kind of properties comes under the point of view from which that science looks at things; and in dealing with these properties, we *abstract from* the full environment in which they occur in reality. In fact, to say that a science is "abstract" and to say that it is "formal," mean the same thing, and mean simply that the science deals with a particular subject-matter, distinguished from the subject-matter of other sciences and from things in general. Logic is "formal" or "abstract" only in the sense that its subject-matter is the general nature of *thinking* as a type to be conformed to.

A numerous and influential school of logicians have treated the subject as "formal" in another sense, and one which cannot be justified. Because Logic deals with Thought without reference to the details of the objects thought about, it does not follow—as these writers assume—that it can treat of Thought while disregarding *all* reference of Thought to the real world. Hamilton says (*Logic*, vol. i. p. 16): "In an act of thinking, there are three things which we can discriminate in consciousness. There is the mind or *ego*, which exerts or manifests the thought. There is the object about which we think, which is called the *matter* of thought. There is a relation between subject and object of which we are conscious,—a relation always manifested in some determinate mode or manner,—and this is the *form* of thought. Now of these three, Logic does not consider either the first or the second." That is, Logic neglects what we shall see to be the most important characteristic of Thought,—to have an objective reference *of some kind*. Such a Logic—which has been described as "subjectively formal"—places itself within a closed circle of "ideas," dealing with ideas without any outlet upon the facts. It is true that this abstraction simplifies the subject; it removes all the harder problems of Logic at the cost of taking away most of its value as an investigation of real thinking. In the present treatment of Logic we have avoided this easy abstraction.

§ 2. The next point of importance which has arisen is the relation of the Law of Identity to the Judgment,

where the subject and predicate are different and yet are united.

We have already indicated one obvious meaning of the Law of Identity (ch. II. § 11),—that the terms by which we identify our thoughts must logically have fixed meanings. In its fundamental meaning, however, the principle states that what is true must be *consistent with* itself; and this is one of the necessary tests of truth. This principle was laid down by Aristotle, though he does not attempt to cast it into the form of a Law of Identity (*An. Prior.*, i. 32): "All truth must be consistent with itself in every direction." Aristotle is here thinking specially of the consistency of a conclusion or consequence with the premises; but the principle may be made universal. If any system of doctrines or set of statements is true, they must be consistent among themselves.

Some eminent thinkers, reviving an ancient doctrine, have insisted that the Judgment must be interpreted in strict accordance with the principle of Identity *understood in its most abstract sense*—i.e., understood as asserting that a thing must be "identical with itself and with nothing else": *a* must be *a* and cannot be anything else than *a*. Thus Lotze says (*Logic*, § 55): "*Every* predicate P that differs in any way whatever from S is entirely irreconcilable with it; *every* judgment of the form 'S is P' is impossible, and in the strictest sense we cannot get beyond saying 'S is S' and 'P is P.'"

We have shown (ch. IV. § 2, pp. 103, 104) that if S and P are to be strictly identical, the judgment becomes an assertion of nothing, SP = SP, or $a = a$. There must be *some difference* between the meaning of S and that of P, and therefore no judgment can be expressed in the form $a = a$. On the other hand, there must be *some identity* between the meanings of S and of P, for the judgment, as Aristotle said, asserts that they are "in a

sense *one*" (*De Anima*, III. 6). In what sense are they one? They may be one without being identical in the way in which *a* is identical with *a*. The two meanings are one in the sense that they are united in that portion of the real world to which the judgment refers. If we find that a statement can be reduced to the form $a = a$, we regard it as a "tautology," as saying nothing at all—*i.e.*, we deny its right to be called a judgment. When we make an affirmation, we predicate something *of something else*; and this difference in the elements of the judgment is not inconsistent with a unity of reference. Speaking broadly, what the judgment does is to distinguish S and P in intension, and unite them in extension. The judgment can combine unity and variety, identity and difference, just because the meaning of every term has the two sides of *extension and intension*. "The relation between intension and extension," says Adamson, "springs from the ultimate nature of thinking, as a process at once abstractive and at the same time having a constant reference to reality; the distinction has reference to the concrete instances on the one hand, and the relatively abstract marks or meaning on the other."

§ 3. We are now in a position to estimate the merits of Hamilton's "comprehensive" view of the Judgment,—that the proposition asserts that the subject-concept includes in it, or "comprehends" in it, the predicate-concept (ch. IV. § 5, p. 111). Taken strictly and literally, this is true only of Definitions. But Mill's accusation, in his *Examination of Hamilton*, that the "comprehensive" view ignores the distinction of analytic and synthetic propositions, is really without foundation; for the criticism assumes that there is an absolute distinction between analytic and synthetic judgments.

If every real judgment involves both identity and difference between S and P, and if every term has both intension and extension, there cannot be one class of analytic and another class of synthetic judgments, though either the analytic or the synthetic aspect may be prominent in this or that judgment. A judgment may be analytic to the teacher and synthetic to the learner; but if it is *merely* synthetic—*i.e.*, if *no link is seen* between subject and predicate—it is a mere grouping of phrases. The increase of knowledge is never like the addition of new stones to a heap, or new bricks to a wall; it is an expansion of old material which can only be compared to organic growth, as in the case of a living thing,—say the germination of a seed. Thus, in every real judgment we have a development or expansion of the subject in the predicate. And the judgment is synthetic because there is an expansion—*i.e.*, something new—a statement of a new fact; it is analytic because the "something new" makes the subject itself more definite. Hamilton's "comprehensive" view applies to the *analytical* aspect of the judgment. He himself takes a proposition with a *singular term* (a proper name) for subject, to illustrate his view (see *Lectures on Logic*, vol. i. p. 220); and his interpretation would hold even for statements of "accidental" facts about such a subject. For instance, if I read the *Phædrus* for the first time and learn that Socrates went for a walk by the Ilissus, this expands my notion of Socrates; it is not (so far as it *means* anything to me) a mere tacking on of something irrelevant. Thus, even statements which give us information about a real subject are not *merely* synthetic. If a schoolboy learns by rote that "Julius Cæsar was killed in the year 44 B.C.," the statement may be indeed entirely synthetic to him,

and for that very reason may never enter as a new piece of information into the body of his knowledge; he may forget all about it, and in an examination make Julius Cæsar the Cæsar to whom St Paul appealed. But if he has realised the *general* period in which Cæsar lived,—that he was a contemporary of Cicero, earlier than Virgil, predecessor of Augustus under whom Christ was born, &c.,—then the statement of the precise date simply makes more definite some knowledge already existing: to that extent the judgment is analytic as well as synthetic.

§ 4. We have seen that every judgment is both synthetic and analytic—synthetic in its reference to real facts, analytic in connecting the knowledge of those facts with previous knowledge. We must dwell further on both these points.

In every Judgment there is a reference to reality. Our judgments, says Professor Minto, "express beliefs about things and relations among things *in rerum natura:* when any one understands them and gives his assent to them, he never stops to think of the speaker's state of mind, but of what the words represent. When states of mind are spoken of, as when we say that our ideas are confused, or that a man's conception of duty influences his conduct, those states of mind are viewed as objective facts in the world of realities. Even when we speak of things which have in a sense no reality, as when we say that a centaur is a combination of man and horse, or that centaurs were fabled to live in the vales of Thessaly, it is not the passing state of mind expressed by the speaker as such that we attend to or think of; we pass at once to the objective reference of the words [to the world of Greek mythology]."

This is the view that Mr Bradley and Prof. Bosanquet

have expressed by saying that **the ultimate subject of every judgment is Reality.** We speak of the *ultimate* subject because it may not explicitly appear in the judgment when this is expressed in words. But when we examine the implications of what is asserted, we find that, directly or indirectly, an *objective system* is referred to—as explained above—which is here called Reality. Thus, when we say "The centaur is a fabulous creature, half man and half horse, that lived in the vales of Thessaly," we touch Reality in referring to the popular mythological imaginations of some of the Greeks. This is the *ultimate* subject; and it scarcely appears in the proposition, where the subject is the centaur. The "reference to Reality" is easiest to trace in Judgments referring directly to something in the world of Mind or Matter.

Mr Bradley has expressed our result thus: **Judgment proper is the act which refers an ideal content,[1] recognised as such, to a Reality beyond the act.** Judgment is spoken of as "the act," because every judgment is a thought of the mind, and hence may be called a "mental act." In judging, we use "an ideal content, recognised as such"—*i.e.*, a universal meaning (ch. II. § 6), which, until it is asserted, is only a *recognised meaning*, a "wandering adjective." And we refer the adjective to "a Reality beyond the act"—an objective Reality which does not depend on any thoughts about it. In every judgment I assert a meaning, and assert that meaning of Reality. Mr Bradley applies this principle throughout to all kinds of judgment; hence he comes to take both the subject and predicate of

[1] The phrase "ideal content" is not a happy one. It simply means "conceptual content," and has nothing to do with the "ideal" or perfect.

the proposition adjectivally. The whole proposition expresses but one idea, and I attach this idea to the nature of the real. Thus, take the following propositions: "Sir Christopher Wren was the architect of St Paul's Cathedral"; "It was proposed to hold an Exhibition at Glasgow in 1901"; "The planets move round the sun in ellipses"; "Ozone is produced by the passage of electric sparks through the air." The subject in each of these is Reality, and the respective predicates (referred to Reality) are: "The designing of St Paul's Cathedral by Sir C. Wren"; "The proposal (of certain persons) to hold an Exhibition in Glasgow in 1901"; "The elliptic paths of the planets round the sun"; "The production of ozone by electric sparks passing through the air."

This theory of Mr Bradley's is the most important of recent investigations. It seems to be borne out by some of our familiar ways of expressing propositions—*e.g.*, "Once upon a time *there was* a giant . . ."; "Now *it came to pass* that . . ."; "*It is* meet and right and our bounden duty . . ." These all bring out the reference to some Reality outside the S and P of the ordinary analysis. But this does not dispense with or interfere with the ordinary analysis, which must be used whenever the judgments form part of an inference. In fact, Mr Bradley goes too far in again dissolving the subject of the proposition into a mere adjective, as in the examples we have given. My assertions are not usually made of Reality as a whole, as Mr Bradley suggests; they are made of some particular portion of Reality, which is taken (for the time at least) as a separate or individual thing, and which is the true logical subject of the judgment. In our given propositions the real subjects are respectively, "Sir C. Wren" (as a historical individual);

"The holding of an Exhibition in Glasgow in 1901" (as an idea entertained); "The planets" (as a class of heavenly bodies); "Ozone" (as a substance or gas existing in Nature). The subjects of our judgments have very different degrees of permanence or individuality, as when we make assertions about "that cloud," "the sun," "the present king," "the plays of Shakespeare"; but any such subject is referred to in judgment as having an existence distinct from other things, and as having features or characteristics which may be predicated of it.

Prof. Bosanquet gives a modified statement which seems to agree with what we have just said: **Judgment is the reference of a significant idea to a Subject in Reality by means of an identity of content between them.** The "Subject in Reality" is the individual thing (or things) of which we have spoken; the "identity of content" may be explained by an example which Prof. Bosanquet gives. "When I say, 'This table is made of oak,' the table is given in perception; . . . among its qualities it has a certain grain and colour in the wood. I know the grain and colour of oak-wood, and if they are the same as those of the table, then the meaning or content 'made of oak' coalesces with this point in reality; and . . . I am able to say, 'This table is made of oak-wood'" (*Essentials of Logic*, p. 70). We have before our mind, in perception or otherwise, a real subject, about which we judge; having also before our mind a previously formed concept which is identical with certain features or aspects of the subject, we attach it as predicate.

§ 5. When we examine the relation of affirmative to negative judgments (ch. III. § 1, p. 52), we see that

even a negative judgment refers to reality, and implies that reality is inconsistent with a suggested assertion.

Aristotle says emphatically, "There is one primary assertive λόγος, affirmation; then there is denial;" "affirmation is prior in thought to denial" (*De Int.*, c. 5, *An. Post.*, i. 25; cf. *Poetics*, c. 20). This states a fact which will be evident on a little reflection. *Negative propositions have the function of simply averting error.* In real thought and speech we never make a denial unless there has been some affirmation suggested, imagined, or actually made, and we wish to deny it; and the reason why we deny it is that we believe we have grounds for another assertion which is incompatible with what we deny. In other words, we deny a proposition only because we have in our minds an affirmative counter-proposition which excludes the former one. The principle of Contradiction expresses the nature and character of the negative by saying that it cannot be true *together with* the affirmative. If I assert of a distant object that "it is not red," I do so because I think the question of its being red has been or may be raised, and also because I think that it is some other colour which is incompatible with red. If I make the statement "a republic does not necessarily secure good government," I make it because I think that the contradictory, "all republics necessarily secure good government," is an opinion actually or possibly held, and also because I think there are cases where republican governments have been bad and corrupt. Thus every negative judgment has a *positive implication.*

When we have expressed a negative judgment in the form "S is not P," *the negative does not belong to the predicate.* The forms of the proposition to which the

processes of obversion, &c., lead, are artificial; they do not naturally occur, for we never affirm "not-P" of S. If we did make such an affirmation, we must have passed through a denial to reach it; if S accepts "not-P," we must already have learnt that it rejects P. In fact, as we implied when first dealing with this subject (ch. II. § 4), "not-P" is a purely formal conception, summing up and containing under it *any* possible contrary. We never make the bare idea of the contradictory the predicate of a judgment. There is no motive for making such assertions. Prof. Bosanquet has observed that though "what we say always approaches the contradictory, what we mean always approaches the contrary."

Thus, our result is that "S is not P" denies the *suggested* affirmation "S is P," and is asserted on the positive basis that S is something which excludes P. It is indeed obvious, from common language, that no one ever thinks it worth while to deny things except with reference to some actual or possible affirmation. If I say to a man, "You cannot jump over the moon," he might think me mad; but if I say, "You cannot jump as high as *that*," he might either accept the challenge or reply, "Well, I never said I could."

We now turn to a related question. Does every affirmation involve the idea of a negation? Whenever we affirm anything we affirm a *significant idea*,—a meaning, or concept, in the logical sense. How is the concept formed? By *comparison*, as we have seen. Now comparison is impossible without *distinction*. This is a very obvious fact; I cannot compare things together, or thoughts together, except by keeping them distinct in my mind; if I have them distinct, then I can note their resemblances And distinction in-

volves separation, exclusion, and, in other words, negation. An affirmation, as "S is P," involves the general idea of negation, but not the negation of that particular connection of S and P. It involves the general idea of negation, because we can only think of S and P by distinguishing them respectively from things which are not S and not P, and we can only think of the relation "S is P" by distinguishing it from different relations. This has been excellently stated by Professor Minto: "Nothing is known absolutely or in isolation; the various items of our knowledge are inter-relative; everything is known by distinction from other things. Light is known as the opposite of darkness, poverty of riches, freedom of slavery, in of out; each shade of colour by contrast to other shades. . . . It is in the clash or conflict of impressions that knowledge emerges; every item of knowledge has its illuminating foil, by which it is revealed, over against which it is defined." The things distinguished are of the same kind, otherwise the distinction would not be made; we are not concerned to distinguish "honest" from "triangular," or "round" from "sick." We make a thing definite by distinguishing it from a variation of the same thing; "we do not differentiate our impression against the whole world, but against something nearly akin to it,—upon some common ground. . . . We find that this is practically assumed in Definition: it is really the basis of definition *per genus et differentiam*. When we wish to have a definite conception of anything, to apprehend what it is, we place it in some class and distinguish it from species of the same. In obeying the logical law of what we ought to do with a view to clear thinking, we are only doing with exactness and conscious method what we all do and cannot help

doing with more or less definiteness in our ordinary thinking."

There is a principle, celebrated in the history of Philosophy, that *omnis determinatio est negatio;* the sense in which it is true is the sense in which affirmation involves the general idea of negation.

§ 6. Returning to our fundamental fact, that every Judgment is both analytic and synthetic, we proceed to discuss a most important and fundamental illustration of it.

We have seen that greater stress may be laid, now on the synthetic, and now on the analytic side of the Judgment: in other words, now on the objects or groups of objects which are referred to, and now on the connection of attributes or general qualities which is asserted. According as greater stress is laid on the one aspect or on the other, we have a distinction of two kinds of universal judgments (cf. ch. VII. §§ 3, 6, pp. 216, 230). When this distinction is firmly grasped, few difficulties remain for the student in the higher developments of Modern Logic.

(*a*) In the judgment "all S is P," the "all S" may refer to a group, a definite number of cases actually observed or recorded in history or other narrative. Such judgments are the result of a "complete enumeration." I suppose myself to have counted the S's, then, observing that they *all* have the quality, I say "all S are P." Such judgments belong to history or narrative; in this they resemble the singular judgment "This S is P," and the particular judgment "Some or many S are P." We know (ch. II. § 1, *a*) that the singular judgment is characterised by being limited *by indications of time and place to a single object.* The universal judgment (of the kind now under consideration) is limited *in the same way*

to a whole group; and if the indications of time and place which limit it are not expressed they are implied. "All leopards are spotted"—*i.e.*, the collection consisting of every specimen hitherto observed of the species. The place is anywhere where a leopard has been found; the time is "up to the present." "All the men of the regiment were captured"—*i.e.*, at some engagement the time and place of which are supposed to be understood. "Every book on these shelves treats of Logic": here the place is indicated, and the time is as long as the collection remains there unaltered. The particular judgment is limited in the same way to part at least of a group: "Nearly all the Dublin Fusiliers lost their lives."

Judgments resting on *observation or narrative* may be called "empirically valid," for they are true only of certain times and places. And judgments of this type which refer to the whole of a group so limited may be called "empirically valid universals." The "all" is numerical, and practically makes the subject, "all S," a collective term. Hence Professor Bosanquet has called them simply "collective judgments." The singular, the "particular," and the collective universal judgments assert the existence of particular things, and set forth their qualities and relations to other things. In all these judgments much more stress is laid on the extension of the subject (the reference to particular things) than on the intension. In such judgments also the *synthetic* aspect is predominant.

(*b*) There is another and different type of universal judgments, where the main stress is laid on the side of intension, and the *analytic* aspect is predominant. The judgment makes an assertion regarding the connection of the attributes which the subject and predicate signify, not regarding the existence of any particular group of objects. In this case the form "all S is P" is hardly

satisfactory; for the meaning is that the attributes of S necessarily carry with them the attribute P, and hence we should rather say "S is necessarily P" or "S must be P." Professor Creighton has explained this type of judgment as follows: "When we say, 'ignorant people are superstitious,' the proposition does not refer directly to any particular individuals, but states the necessary connection between ignorance and superstition. Although the *existence* of ignorant persons who are also superstitious is presupposed in the proposition, its most prominent function is to assert a connection of attributes. . . . So, in the proposition 'all material bodies gravitate,' the main purpose of the judgment is evidently to affirm the necessary connection of the attributes of materiality and gravitation." Professor Bosanquet distinguishes these as *generic* judgments, for they rest "on a connection of content or presumption of causality"—*i.e.*, they assert that given attributes necessarily result in certain others. We may say, therefore, that they assert a "general law." These judgments *do not depend on an enumeration of instances.* In Martineau's words, "The foresight of its particular cases is not included in the meaning or in the evidence of a general rule; and a person may reasonably assent to the Law of Refraction without any suspicion of the vast compass of facts over which its interpretation ranges. There are grounds—whatever account we may give of them—for ascribing attributes to certain *natures* or *kinds* of being, without going through the objects included under them or having any prescience of their actual contents. It is not necessary to know the natural history of all the varieties of mankind before we can venture to affirm mortality of human beings in general."

The simplest instances of this type of universal judg-

ment are found in mathematics—*e.g.*, "The three interior angles of any triangle are together equal to two right angles"; "The circumference of a circle is incommensurable with its diameter." Here there is an assertion of a *necessary* connection. In every instance, the property stated in the subject of the propositions has as a consequence the property stated in the predicate. The statements do not rest upon an enumeration of instances, but on the connection of the concept of the angles of a triangle with that of two right angles, and on the connection of the concept of a circular line with that of a straight line. And this connection may be asserted as true without any limitation of time and place,—in other words, without reference to any particular instance or instances. For this reason, the plural with "all" is not an adequate expression of the judgment; as it does not rest upon enumeration, the sign of numerical quantity should be dropped. The proper form is that of the so-called indesignate judgment, "S is P"; or, to emphasise the necessity of the connection, "S must be P." Sometimes the emphasis laid on the connection of attributes is so strong that the reference to *particular* things or instances may be dropped, and the judgment assumes the conditional (hypothetical) form "if anything is S it is P." This statement asserts only the reality of the general law that the attributes of S necessarily involve those of P.

This distinction between collective and generic judgments was clearly explained by Aristotle in his *Posterior Analytics*. The generic judgment he calls "universal" (καθόλου) in the proper sense; the collective judgment asserts merely what is common or generally applicable to a group (κοινόν or κατὰ παντός). "By *universal* [*i.e.*, universal predication] I mean what belongs to all,

and belongs essentially, and belongs to the thing *as such.* It is plain, therefore, that all universals belong *necessarily* to their subjects; and to belong to a thing essentially, and to belong to it as such, are the same. For example, the triangle *as such* has its three interior angles together equal to two right angles, and these angles together are *essentially* equal to two right angles. The universal must hold of *any* thing of a certain kind, and also of that kind *first* [*i.e.*, of no kind constituting a wider genus]." This is exactly the "generic" universal, holding of any thing *of a certain kind* just because the thing is of that kind and of no other (*An. Post.*, i. 4).

The traditional four-fold scheme (A, E, I, O) has its most glaring defect in making no distinction between the merely collective and the generic universal judgment. Closely connected with this is the gratuitous paradox of treating the singular judgment as a universal. It must further be noticed that the scheme fails to provide a place for the simplest forms of judgment—*e.g.*, the "impersonal" ("it rains," "bad!" "how hot!" &c.) and the "demonstrative" or "perceptive" ("this is . . ." or "that is . . ."). These have to be treated as "particulars." But the true "particular" proposition always indicates an exception with reference to the whole of a class—*e.g.*, certain individuals which are S ("swans") differ from the rest in being P ("black"). Such an exception may be indicated by a single instance; but the particular judgment in this logical sense must be distinguished from any judgment which refers to an individual without any reference to a *class.*

§ 7. We have found that the "generic universal judgment," in its most abstract form, still contains a reference to reality, though not necessarily to any *particular* objects in the real world. The more abstract it becomes, the more it tends to take the hypothetical form; in that case—as Mr Bradley says—its truth lies

in its affirmation of the connection of the *then* with the *if;* that is, the affirmation of the existence in reality "of such a general law as would, if we suppose some conditions present, produce a certain result." Because the hypothetical proposition "if S is M it is P" is capable of the implication just mentioned, it is capable of being used as a significant portion of scientific knowledge (ch. VIII. § 1); in Aristotelian language, it can be used as a "major premise." But before we can "draw a conclusion" from it, its general reference to reality requires to be *particularised*, by being connected with some actual case in space and time—*i.e.*, it requires a "minor premise." In the absence of this *particular* reference, the judgment in its hypothetical form gives us no information about anything in experience; this is why the conditional form may be used to express ignorance: "*if* S is M it is P (but I do not know whether it is M or not)." But even then the ignorance is only about the particular case; the positive assertion of the general connection of P with M is evidently implied.

Now, in the **disjunctive judgment** both these sides can be detected, but both possess fuller significance (ch. VII. § 4). The *particular* reference is less indeterminate than in the hypothetical; and the *general* implication is larger, "A is either B or C." "Even if you do not know *which* of the two it is, how do you know that it *must be one?*" Evidently we cannot make such an assertion about A without knowing something of the general system of things to which A belongs, and of A's relations to other things in that system. Only on the basis of a knowledge of elementary geometry could we say that any section of a cone by a plane *must* be either a circle, or an ellipse, or a parabola, or a

hyperbola, or two intersecting straight lines, or a single straight line. Examples from mathematics might easily be multiplied; for this branch of science is sufficiently developed for us to make exhaustive disjunctions in the form "A *must* be B or C or D, &c." What the student should grasp, by reflecting upon typical concrete examples of such judgments, is this: although the disjunctive form leaves partly indeterminate the particular reference which is predicated,—so that on this side it may be used to express ignorance,—yet, when it is correctly used, it implicitly refers an individual (A) to a *system*, and implies at the same time knowledge of the general nature of the system and of the individual's place in it. If, in ordinary conversation, the disjunctive form is used to express *mere* ignorance and nothing more, it is incorrectly used; for it means, "I do not know whether A is B or C or something quite different."[1]

The disjunctive judgment is regarded by modern logicians as expressing the real aim of Thought more fully than the previous forms: for it implies the existence of a systematically connected world. And in all real thinking we are seeking to connect facts together by means of general principles into a system. To understand this is to grasp the main clue to the solution of some of the most vexed questions of Logic.

§ 8. To illustrate the observation made at the close of the previous section, we shall consider the relation between Deductive and Inductive reasoning.

English writers on Logic have usually been content to say that Deduction reasons from general principles to particular facts, Induction from particular facts to

[1] The significance of the main forms of Judgment is concisely reviewed, from the standpoint of Modern Logic, in the author's *Philosophical Criticism and Construction*, chapter iii.

general principles. Before we can estimate the value of this statement of the distinction, we must be clear as to one point. Deduction and Induction are not two different and independent kinds of reasoning. The real process of thinking is *the same in both*—*i.e.*, to find a place for some fact as a detail within a system. In the case of syllogistic deductive reasoning (ch. VI.) our "system" is partly known beforehand, in the form of a general law under which the fact or detail is brought (ch. VI. §§ 2, 6; ch. VIII. § 1). We start, having *in our hands* the common thread which unites the various facts. But in Inductive reasoning we have to *find* the common thread. We start with certain kinds of facts which occur together in our experience. We assume that there *is some* principle which unites them (ch. VIII. § 7); and our object is to read out of these particular details the general law of their connection, and, if possible, to *explain* this connection by further connecting it with other laws: and this is to connect facts and laws into a systematic whole.

Thus the traditional English mode of distinguishing Induction and Deduction must at least be qualified by remembering that in both "kinds" of reasoning we have the essential function of thought at work—*i.e.*, to show the way in which details are connected together into a system or whole. The difference lies in the *starting-point* being different in the two cases. We have seen that both modes of inference are required together in scientific reasoning; for what we called the "complete scientific method," the Method of Explanation, necessarily includes both (ch. IX. § 13). In the present work we have not limited the meaning of Induction to that kind of reasoning where we start with facts given together and have to find their real connecting prin-

ciple; we have identified the theory of Induction with the theory of Scientific Method, and have said that Induction "includes Deduction."

In many passages in Mill's *Logic* we find Induction treated—as in the present work—as the theory of Scientific Method.

Stated in its most general terms, Induction is the *discovery and proof* of "general propositions": it is "that operation of the mind by which we infer that what we know to be true in a particular case or cases will be true in all cases which resemble the former in certain assignable respects." "In other words, Induction is the process by which we conclude that what is true of certain individuals of a class is true of the whole class, or that what is true at certain times will be true in similar circumstances at all times." (III. ii. § 1). This evidently rests on the assumption of the "uniformity of nature," which may be treated as "the ultimate major premise of all inductions" (III. iii § 1). In saying this, Mill evidently conceived that a case of Induction could be expressed as a syllogism, thus—

The same cause (or group of causes) will always produce the same effect.

The causes ABCD have been observed to have the effect E.

Therefore the causes ABCD will always have the effect E.

Hence "a single instance, in some cases, is sufficient for a complete induction" (III. iii. § 3)—*i.e.*, when the investigation of the single instance has been so thorough that we can be sure of having grasped *all* the relevant circumstances ABCD and of E being their effect. Carrying on the same line of thought, Mill says that the "main business of Induction" is to ascertain "what are the laws of causation which exist in nature,—to determine the effect of every cause and the causes of all effects" (III. vi. § 3).

The process of Induction is one of *analysis* applied to the complex mass of facts which Nature presents to us. This analysis is in the first instance *mental*, and is exemplified in *knowing what to look for*. The importance of this is ex-

cellently described by Mill (III. vii. § 1). This leads to *physical* analysis, by observation or—with far more power—by *experiment* (III. vii. §§ 2, 3, 4). The methods of physical analysis are the five Inductive Methods described by Mill in Bk. III. ch. viii., ix., x.: these methods we have restated with the necessary modifications; and we have pointed out the true place of the Method of Explanation, which is accurately described by him (III. xiv.; esp. § 5), but which he treats only as a subordinate method, useful in helping out the others.

If the doctrines implied in the passages to which we have just referred were consistently worked out, the result would be a theory of Induction substantially the same as that which we have expounded. But Mill mingles it with a line of thought wholly inconsistent with it.

The student of Mill's *Logic* will see that most of the difficulties and inconsistencies in his treatment arise from a persistent attempt to found his exposition of scientific method on the theory of the origin of knowledge which is known as "empiricism."[1] This theory, which is based on that of Hume, maintains that the only source of knowledge consists in "experience," understood to mean the succession of separate facts appearing in the perceptions of our senses. The mind contributes nothing to knowledge beyond the power of receiving the facts and distinguishing them according as one is like or unlike or comes before or after another. Knowledge is the *sum* of these details of "sensation," not their connection into any kind of system.

When working out this line of thought, Mill argues that every Judgment refers to "real things," and then—as Prof. Bosanquet says—"almost takes our breath away by calling

[1] See Green's "Lectures on the Logic of J. S. Mill," in his *Philosophical Works*, vol. ii.

them [the 'real things'] 'states of consciousness'" (I. v. §§ 1, 5). From the same point of view he insists that "every general truth is an aggregate of particular truths" (II. iii. § 3), where "particular" means "unconnected" by anything common to it with others. And Induction tends to mean the process by which these disconnected details can manufacture (in our minds) general statements or laws, Hence also he maintains that the Law of Uniform Causation, which he had stated to be the presupposition of all Induction (meaning Scientific Method), "is itself an instance of Induction" (meaning the process of combining the disconnected particulars of sense-experience into general statements). It is, moreover, an instance of "Induction" in its weakest form (III. ch. xxi.); and Mill attempts to evade the resulting difficulty, as we have seen, by a flagrant though unconscious *petitio principii* (see above, ch. VIII. § 7; ch. X. § 1, p. 355).

From the same line of thought came the view that "all reasoning is from particulars to particulars" (II. iii. § 4); and the denial of the name of Induction to the generalisations of Mathematics, because "the truth obtained, though really general, is not believed on the evidence of particular instances" (III. ii. § 2). In this sense, the Methods of Scientific Inquiry expounded by Mill himself in his Third Book are not "inductive"; they do not, and can not, start with disconnected particulars, but with facts observed to be of *such and such a kind*, facts read through *conceptions*.

§ 9. The subject of the relation of Logic to other branches of Philosophy is one that has been the subject of much unprofitable discussion; nevertheless, some important questions are involved in it.[1] We shall conclude by briefly touching upon one aspect of it.

Modern Logic, as we have explained it, becomes identical with what is sometimes called the Theory of Knowledge, or Epistemology. What is the relation of

[1] The aim and scope of the various "parts" of Philosophy are considered in the author's *Philosophical Criticism and Construction*, chapter i.

the *logical* treatment of knowledge to the *psychological?* Before answering this question, we must remember that Psychology at the present day is approached from various points of view, and in particular from two fundamentally different points of view—one exemplified in the *Physiological Psychology* of Wundt and the writings of the school which he founded; another, in Stout's *Analytic Psychology* or Ladd's *Psychology, Descriptive and Explanatory*. The former treatment of Psychology has no relation whatever to Logic; for it scarcely treats ideas as cognitive—it leaves out the fact of *knowledge* and its implications. The latter treatment deals elaborately with description and analysis of the intellectual processes; but it is interested in them only as *mental facts*. Logic is interested in them as exemplifying the *regulative* principles of thought. It dwells on these principles as types to which our thought must conform itself; and hence Logic can go beyond the actual facts of the intellectual activities of mind, and can formulate an ideal of knowledge, by which the worth—that is to say, the *truth*—of our present intellectual achievements may be judged.

The ideas and aims of what we have called Modern Logic were explained by the late Professor T. H. Green in his Oxford lectures on *The Logic of the Formal Logicians* and *The Logic of J. S. Mill* (published, since the death of the author, in his collected *Philosophical Works*, vol. ii.) These views were, however, first introduced to English readers in general by Mr F. H. Bradley in his *Logic* (1886), and Mr Bernard Bosanquet in his *Logic, or the Morphology of Knowledge* (1888). The ore which these two writers worked up was mined in the *Logic* of Hegel (first published in 1818) on the one hand, and the *Logics* of Lotze (1874) and Sigwart (1873) on the other. The two last-named works have been translated into English, and are of great value to the student, more especially the work of Sigwart (translated by

Helen Dendy . two vols., London, 1896). The main points of Bosanquet's logical doctrine are stated in short form in his *Essentials of Logic* (1895). We may also refer to Creighton's *Introductory Manual of Logic* and to Welton's *Logical Basis of Education*, both of which contain introductions to Modern Logic, on the lines of Prof. Bosanquet's work; and to Welton's *Manual of Logic*, vol. ii., which treats, on the same lines, of Inductive Logic.

* References for Reading.

(1) Historical; Adamson, article *Logic* in *Encyclopedia Britannica* (comprehensive and systematic); Ueberweg, *Logic* (valuable and accurate historical information on every important point).

(2) Scope of Logic: Adamson, article *Logic* as above; more briefly in Adamson, articles *Empirical Logic* and *Formal Logic* in Baldwin's *Dictionary of Philosophy and Psychology*. See also Adamson's review of Lotze's *Logic*, "Mind," O.S., vol. x. pp. 100-114, and his review of Bradley's *Principles of Logic*, "Mind," O.S., vol. ix.

(3) Significance of the act of Naming, and of the twofold meaning of Terms: Mill, *Logic*, Bk. I. ch. i., ii.; Venn, *Empirical Logic*, ch. vi.; Bosanquet, *Logic*, vol. i. Introduction (pp. 8 ff.), and more briefly in his *Essentials of Logic*, ch. v.; Sigwart, vol. i. §§ 6-8, 40-44 (pp. 29 ff., 245 ff.); Lotze, *Logic*, vol. i. §§ 1-30 (pp. 13 ff.), and cp. Bosanquet, *loc. cit.* (vol. i. pp. 30-40).

(4) The Proposition as the expression of the Judgment: Mill, Bk. I. ch. iv. (forms), ch. v. (import); Sigwart, vol. i. § 5 (p. 25). Subject and Predicate arrangement: Venn, ch. viii., and by Bosanquet, *Essentials*, ch. vi. Schemes of classification: Venn, ch. ix.

(5) General nature of Judgment: Bosanquet, vol. i. ch. i. (pp. 72 ff.); *Essentials*, ch. ii. System of the forms of Judgment: should be read first in Sigwart, vol. i. ch. ii. (Singular Judgments), ch. iii. (Analytic and Synthetic J.), ch. iv. (Plural J. and the two kinds of Universal), ch. v. (Modality), and Venn, ch. x. (Hypotheticals and Disjunctives: notice p. 264); then in Bosanquet, vol. i. ch. v.,

vi., viii. (pp. 340-350; attempt to treat Disjunction as the goal of knowledge); cp. Mellone, *Philosophical Criticism and Construction*, ch. iii. Bosanquet's view is based on Hegel, *Wissenschaft der Logik*, *Werke*, vol. v. pp. 90 ff., the essentials of which are set forth in short form in Hegel's smaller *Logic*, Wallace's translation, pp. 297-313 (second edition); expounded by M'Taggart, "Mind," N.S., vol. vi. pp. 164 ff., and 342 ff. Hobhouse, *Theory of Knowledge*, ch. i., ii., assumes a form of "immediate apprehension" as the basis on which Judgment develops. Lotze's treatment of the forms of Judgment (*Logic*, vol. i. §§ 36 ff.) is a persistent attempt to reconcile them with the principle of Identity, abstractly understood.[1]

(6) Particular topics related to the foregoing: (*a*) Negation: Bosanquet, *Essentials*, ch. viii. (with references there given); also his *Logic*, vol. i. ch. vii. (pp. 293 ff.), including Immediate Inference; (*b*) Laws of Thought: Jevons, *Principles of Science*, Introduction; Sigwart, vol. i. §§ 23-25 (pp. 139 ff.); Bosanquet, vol. ii. ch. vii. (pp. 205-212); Hegel, Wallace, pp. 213-229 (cp. Baillie's *Logic of Hegel*, pp. 263 ff.) (*c*) Special types of Judgment: "qualitative," Bosanquet, vol. i. ch. ii.; "quantitative," ch. iii.; "numerical," ch. iv. (*d*) Inference from Hypothetical and Disjunctive propositions: Sigwart, vol. i. §§ 36, 37 (pp. 220 ff.)

(7) Inference in general: read first Sigwart, vol i. §§ 49-59 (pp. 326 ff.); James, *Principles of Psychology*, vol. ii. ch. xxii.; Bosanquet, vol. ii. ch. i.

(8) The inner meaning of the syllogistic figures (cp. ch. viii. § 3, above): Bosanquet, *Essentials*, ch. ix., x. (short statement); also in his *Logic*, for fig. iii.: vol. ii. ch. ii. (pp. 43-48); for fig. ii.: ch. iii. (pp. 83-105); for fig. i.: ch. vi. (pp. 180 ff.); Hegel, *Wissenschaft der Logik*, *Werke*, vol. v. pp. 115 ff.; and in Wallace, pp. 314-328 (cp. M'Taggart, *loc. cit.*)

(9) For a systematic discussion of the problems arising out of the foundations of knowledge, see Hobhouse, *Theory of Knowledge*, and (in special connection with a critique of modern scientific constructions) Ward, *Naturalism and Ag-*

[1] For criticism of this principle, see Mellone, *Philosophical Criticism and Construction*, ch. iii.

nosticism. The main problems of Philosophy, as they shape themselves at the present time, are summarised in Mellone, *Philosophical Criticism and Construction.* Advanced students will find Bradley's *Principles of Logic* a valuable introduction, from the logical side, to metaphysical problems, which may be further pursued in the same writer's *Appearance and Reality* (those who find the arrangement of this work confusing should first read Taylor's *Elements of Metaphysics*, which in any case is a trustworthy guide to the general subject). Hodgson's *Metaphysic of Experience*, and Royce's *World and the Individual* (two series), are suggestive surveys of the ground.

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